ADVANCED MICROECONOMICS - 2013/2014

Sheet 3: General equilibrium with uncertainty

Instructions: The exercises you have to deliver are marked in green. The cover sheet can be downloaded from the course webpage (and it is compulsory). Fill the data and staple this cover page, together with the rest of the exercises, in the upper left-hand corner. This material will not be returned, so make photocopies and keep the originals for yourself.

1. Consider an economy with a single good, two states in $t = 1$ and two agents with utility functions of the form:
$$u_i(x_i^1, x_i^2) = \pi_i^1 v_i(x_i^1) + \pi_i^2 v_i(x_i^2).$$
Initial endowments are $\omega_1 = \omega_2 >> 0$. Suppose that v_2 is strictly concave and agent 1 is risk neutral.

   (a) Prove that if $\pi_1^1 = \pi_2^1$, $\pi_1^2 = \pi_2^2$, then, in an interior Arrow-Debreu equilibrium, agent 2 consumes the same in both states.

   (b) Prove that if $\pi_1^1 \neq \pi_2^1$ then, in an interior Arrow–Debreu equilibrium, agent 2 does not consume the same in both states.

   (c) Prove that if $\pi_1^1 \neq \pi_2^1$ then the agent 1 gains nothing from trading.

2. Consider an economy with two periods, two agents, one good and two states in the second period, $t = 1$. The utility function of the agents is
$$u_i(x_i^1, x_i^2) = \pi_i u(x_i^1) + (1 - \pi_i)u(x_i^2) \quad i = 1, 2$$
where
$$1 > \pi_1 > \frac{1}{2} > \pi_2 > 0$$
Agents are risk averse, and there is no aggregate uncertainty ($\omega_1^1 + \omega_2^1 = \omega_1^2 + \omega_2^2$). Prove that in an Arrow-Debreu equilibrium it can be verified that
$$x_1^1 > x_1^2, \quad x_2^1 < x_2^2$$

3. Consider an economy with two periods, a single good and two agents. In $t = 1$ there are two possible states. The utility functions of the agents are
$$u_1(x_1^1, x_1^2) = \frac{1}{2}\left(\pi_1^1 \ln x_1^1 + \pi_1^2 \ln x_1^2\right)$$
$$u_2(x_2^1, x_2^2) = \frac{1}{3}\left(\pi_2^1 \ln x_2^1 + \pi_2^2 \ln x_2^2\right),$$
where $\pi_1^1 = \pi_1^2 = \pi_2^1 = \pi_2^2 = \frac{1}{2}$ and the initial endowments $\omega_1 = (1, 3)$, $\omega_2 = (3, 1)$. Suppose that there are two assets $r_1 = (1, 1)$ and $r_2 = (1, 0)$. Find the Radner equilibrium of this economy. What is the interest rate of the economy?

4. Consider an economy with two periods, a single good and two agents. In $t = 1$ there are two possible states. The utility functions of the agents are
$$u_1(x_1^1, x_1^2) = \frac{1}{2}\left(\pi_1^1 \ln x_1^1 + \pi_1^2 \ln x_1^2\right)$$
$$u_2(x_2^1, x_2^2) = \frac{1}{3}\left(\pi_2^1 \ln x_2^1 + \pi_2^2 \ln x_2^2\right),$$
where $\pi_1^1 = \pi_1^2 = \pi_2^1 = \pi_2^2 = \frac{1}{2}$ and the initial endowments $\omega_1 = (2, 2)$, $\omega_2 = (2, 2)$. Suppose that there are two assets $r_1 = (1, 1)$ and $r_2 = (1, 0)$. Find the Radner equilibrium of this economy. Compare the results with the ones in the previous exercise and comment the differences.

5 Consider an exchange economy with two agents, two periods and two goods in $t = 1$. Initial endowment of agent 1 are, in state 1, $\omega_1^1 = (1,1)$ and in state 2, $\omega_1^2 = (1,4)$. Initial endowment of agent 2 are, in state 1, $\omega_2^1 = (1,2)$ and in state 2, $\omega_2^2 = (5,2)$. The utility functions of the agents are

$$u_i(x_i) = \sum_{s=1}^{2} (2 \ln x_{i1}^s + \ln x_{i2}^s) \qquad i = 1,2$$

(a) Find the Arrow-Debreu equilibrium of the economy.

(b) Find the Radner equilibrium of the economy when the assets are $r_1 = (1,0)$ and $r_2 = (0,1)$.

6 Consider an exchange economy with two agents, two goods and two states in $t = 1$. Initial endowment of agent 1 is, in state 1, $\omega_1^1 = (a,b)$ and in state 2, $\omega_1^2 = (0,0)$. Initial endowment of agent 2 is, in state 1, $\omega_2^1 = (0,0)$ and in state 2, $\omega_2^2 = (a,b)$. The utility functions of the agents are

$$u_i(x_i) = u(x_{i1}^1) + u(x_{i2}^1) + u(x_{i1}^2) + u(x_{i2}^2), \qquad i = 1,2$$

where $u : \mathbb{R} \longrightarrow \mathbb{R}$ is a concave and differentiable utility function. There are two assets in the economy $r_1 = (1,0)$ and $r_2 = (0,1)$. Prove that in the Radner equilibrium,

$$z_{12} = \frac{u'\left(\frac{a}{2}\right) a + u'\left(\frac{b}{2}\right) b}{2u'\left(\frac{a}{2}\right)} = z_{21} = -z_{22} = -z_{11}.$$

7 Let $r = (2,3,4,2,5)$ be a primary asset. Write the expression of the buy option for a price $c \in [0,4]$.

8 Consider a sequential economy with one good, two periods and three possible states in the second period. Suppose that there are four assets $r_1 = (1,1,1)$, $r_2 = (3,0,3)$, $r_3 = (2,1,3)$ and $r_4 = (1,4,1)$, which prices are $q_1 = q_2 = 1$, $q_3 = 2$, $q_4 = 3$. Calculate all the risk–neutral probabilities. Determine if there is arbitrage in the economy. Are markets complete?

9 In an economy with two periods and two primary assets $r_1 = (64,16,4)$, $r_2 = (0,0,1)$ with prices $q_1 = 32$, $q_2 = 1$, respectively, use the notion of no-arbitrage to valuate the following derivative assets.

(a) The asset that allows the agent to buy one unit of r_1 paying the 75% of its spot price in period 1, when the state of the world is known.

(b) The asset that allows the agent to buy one unit of r_1 paying the 75% of its spot price in period 1, when the state of the world is known, only when the spot price of r_1 is above 10.

(c) The asset that allows the agent to buy one unit of r_1 paying the 75% of its spot price in period 1, when the state of the world is known, only when the spot price of r_1 is above 19.