ADVANCED MICROECONOMICS - 2013/2014

Solutions Sheet 1: Exchange Economies

1 (a)

$$x_1 = \left(t, \frac{(1-\alpha)\beta t w^2}{\alpha(w^1 - t - \beta w^1) + \beta t}\right)$$

$$x_2 = \left(w^1 - t, \frac{\alpha(1-\beta)(w^1 - t)w^2}{t(\alpha - \beta) - \alpha w^1 (1-\beta)}\right)$$

(b)

$$x_{11} = \frac{\alpha(1-\beta)w_{12}w_{21} + w_{11}(w_{12} + \beta w_{22})}{\alpha w_{12} + \beta w_{22}}$$

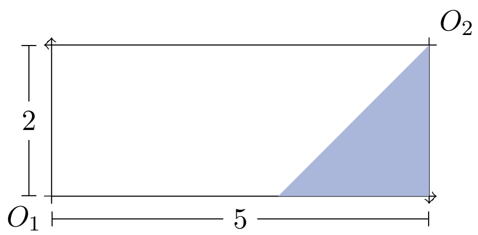
$$x_{12} = \frac{(\alpha - 1)((1-\beta)w_{12}w_{21} + w_{11}(w_{12} + \beta w_{22}))}{(\alpha - 1)w_{11} + (\beta - 1)w_{21}}$$

$$x_{21} = \frac{(\beta - 1)((w_{11} + w_{21})w_{22} + \alpha(w_{12}w_{21} - w_{11}w_{22}))}{\alpha w_{12} + \beta w_{22}}$$

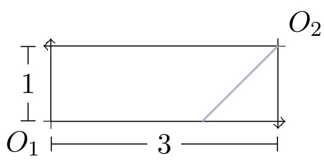
$$x_{22} = \frac{(\beta - 1)((w_{11} + w_{21})w_{22} + \alpha(w_{12}w_{21} - w_{11}w_{22}))}{(\alpha - 1)w_{11} + (\beta - 1)w_{21}}$$

$$p_2 = \frac{(w_{11} - \alpha w_{11} + w_{21} - \beta w_{21})}{\alpha w_{12} + \beta w_{22}} p_1$$

2 Pareto efficient allocations:



3 Pareto efficient allocations:



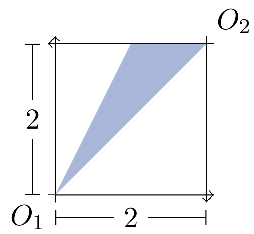
4 (a) $x_1 = (t, t)$ and $x_2 = (2-t, 2-t)$, where $t \in [0, 2]$.

(b) $x_1 = (2, 0)$ and $x_2 = (0, 2)$.

(c) $x_1 = (t, 2)$ and $x_2 = (2-t, 0)$, where $t \in [0, 2]$.

(d) $x_1 = \left(t, \frac{4t}{t+2}\right)$ and $x_2 = \left(2-t, \frac{4-2t}{t+2}\right)$, where $t \in [0, 2]$.

(e) The efficient allocations are:



5 (a) ◦ Demand functions:

$$z_1(p) = z_2(p) = \left(\frac{2(p_1+p_2)}{3p_1}, \frac{p_1+p_2}{3p_2}\right)$$

◦ Equilibrium: $x_1 = (1,1)$, $x_2 = (1,1)$, $p_1 = 2p_2$.

◦ Pareto efficient allocations: $x_1 = (t,t)$ and $x_2 = (2-t, 2-t)$, where $t \in [0,2]$

(b) ◦ Demand functions:

$$z_1(p) = z_2(p) = \left(\frac{p_1^{\frac{2-\rho}{1-\rho}}}{p_1^{\frac{2-\rho}{1-\rho}} + p_2^{\frac{2-\rho}{1-\rho}}}, \frac{p_1}{p_2} - \frac{p_1}{p_2}\frac{p_1^{\frac{2-\rho}{1-\rho}}}{p_1^{\frac{2-\rho}{1-\rho}} + p_2^{\frac{2-\rho}{1-\rho}}}\right)$$

◦ Pareto efficient allocations: $x_1 = (t,t)$ and $x_2 = (1-t, 1-t)$, where $t \in [0,1]$

(c) ◦ Demand functions:

$$z_1(p) = z_2(p) = \left(\frac{p_1}{p_1+p_2}, \frac{p_2}{p_1+p_2}\right)$$

◦ Equilibrium: $x_1 = \left(\frac{1}{2}, \frac{1}{2}\right)$, $x_2 = \left(\frac{1}{2}, \frac{1}{2}\right)$, $p_1 = p_2$.

◦ Pareto efficient allocations: $x_1 = (t,t)$ and $x_2 = (1-t, 1-t)$, where $t \in [0,1]$

(d) ◦ Demand functions:

$$z_1(p) = \left(9 + 2\frac{p_2}{p_1}, 2 + 9\frac{p_1}{p_2}\right)$$

$$z_2(p) = \left(\frac{3}{2} + 3\frac{p_2}{p_1}, 3 + \frac{3}{2}\frac{p_1}{p_2}\right)$$

◦ Equilibrium: $x_1 = \left(\frac{65}{5}, \frac{44}{7}\right)$, $x_2 = \left(\frac{39}{5}, \frac{26}{7}\right)$, $p_2 = \frac{10}{21}p_1$.

◦ Pareto efficient allocations: $x_1 = (t, \frac{10}{21}t)$ and $x_2 = (21-t, 10-\frac{10}{21}t)$, where $t \in [0,21]$

(e) ◦ Demand functions:

$$z_1(p) = \begin{cases} (0,4) & \text{if } p_1 > 2p_2 \\ \{(0,4), (\frac{4p_2}{p_1}, 0)\} & \text{if } p_1 = 2p_2 \\ (\frac{4p_2}{p_1}, 0) & \text{if } p_1 < 2p_2 \end{cases}$$

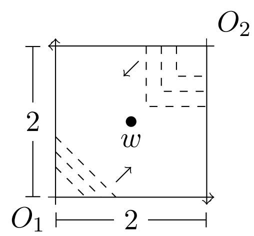
$$z_2(p) = \begin{cases} (0, \frac{4p_1}{p_2}) & \text{if } p_1 > \frac{1}{2}p_2 \\ \{(0, \frac{4p_1}{p_2}), (4,0)\} & \text{if } p_1 = \frac{1}{2}p_2 \\ (4,0) & \text{if } p_1 < \frac{1}{2}p_2 \end{cases}$$

◦ Equilibrium: $x_1 = (4,0)$, $x_2 = (0,4)$, $p_2 = p_1$.

◦ Pareto efficient allocations: Borderline of the Edgeworth box, that is,

$$\{((0,t),(4,4-t)), ((4,t),(0,4-t)), ((t,0),(4-t,4)), ((t,4),(4-t,0))\}$$

6 (a) Edgeworth box:



(b) $p_1 = p_2$.

(c) There are two competitive equilibrium allocations: $x_1 = (2,0)$, $x_2 = (0,2)$, and $x_1 = (0,2)$, $x_2 = (2,0)$.

7 There are several equilibria: $x_1 = (0,3)$, $x_2 = (2,2)$, $p_1 = p_2$, and $x_1 = (3/4, 5/2)$, $x_2 = (5/4, 5/2)$, $p_1 32/12\ p_2 = 16/13$, and $x_1 = (9/7, 0)$, $x_2 = (5/7, 5)$, $p_1 = 7p_2$.

8 (a) $x_1 = (120, 120)$, $x_2 = (160, 160)$, $p_2 = 2p_1$.

(b)

(c) $p_2 = 2p_1$.

(d) For example, $w_1 = (120, 120)$, $w_2 = (160, 160)$.

9 (a) $x_1 = (\frac{10}{3}, \frac{40}{3})$, $x_2 = (\frac{20}{3}, \frac{20}{3})$.

(b) $x_1 = (\frac{10}{3}, \frac{40}{3})$, $x_2 = (\frac{20}{3}, \frac{20}{3})$, $p_1 = 2p_2$.

10 (a) If $m_1 = m_2 = m$, then $x_1 = 9$, $y_1 = 24$, $x_2 = 15$, $y_2 = 0$, and $p_1 = 4p_2$.

(b) If does not change ($m = m_1 = m_2$).

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15 $p_2 = 2p_1$.

16 For example, $\overline{w}_1 = (4, 6)$, $\overline{w}_2 = (6, 4)$

17 $p_2 = 2p_1$.