ADVANCED MICROECONOMICS - 2013/2014

Sheet 1: Exchange Economies

Instructions: The exercises you have to deliver are marked in green. The cover sheet can be downloaded from the course webpage (and it is compulsory). Fill the data and staple this cover page, together with the rest of the exercises, in the upper left-hand corner. This material will not be returned, so make photocopies and keep the originals for yourself.

1 Consider an exchange economy with two agents, two goods and Cobb-Douglas utility preferences given by

$$u_1(x,y) = x^\alpha y^{1-\alpha} \text{ and } u_2(x,y) = x^\beta y^{1-\beta},$$

with $0 \leq \alpha, \beta \leq 1$. The initial endowments are $\omega^1 >> 0$, $\omega^2 >> 0$.

(a) Compute the Pareto efficient allocations.

(b) Compute the equilibrium prices.

2 Suppose that the utility functions of the agents are $u_1(x,y) = \max\{x,y\}$ and $u_2(x,y) = \min\{x,y\}$, and that the endowments are $\omega = (5,2)$. Represent the Pareto efficient allocations.

3 Suppose that the utility functions of the agents are $u_1(x,y) = 2x+4y$ and $u_2(x,y) = \min\{x,y\}$, and that the initial endowments are $\omega = (3,1)$. Represent the Pareto efficient allocations.

4 Consider and exchange economy with two agents and two goods. The consumption set of each agent is $X_i = \mathbb{R}^2_+$. The aggregated initial endowments are $\omega = (2,2)$. Determine the set of Pareto efficient allocations for each pair of the following utility functions:

(a) $u_1(x,y) = \ln x + \ln y$ and $u_2(x,y) = \ln x + \ln xy$.

(b) $u_1(x,y) = x$ and $u_2(x,y) = y$.

(c) $u_1(x,y) = xy$ and $u_2(x,y) = y$.

(d) $u_1(x,y) = \ln x + 2\ln y$ and $u_2(x,y) = xy$.

(e) $u_1(x,y) = \min\{x,y\}$ and $u_2(x,y) = \min\{x,y\}$.

5 For each of the following exchange economies with consumption set $X_1 = X_2 = \mathbb{R}^2_+$

- Compute the demand function of each agent and the competitive prices and allocations.
- Determine the set of Pareto optima allocations.

(a) $u_i(x,y) = 2\ln x + \ln y, (i = 1,2)$, $\omega_1 = (1,1)$, $\omega_2 = (1,1)$.

(b) $u_i(x,y) = x^\rho + y^\rho, (i = 1,2)$, $0 < \rho < 1$, $\omega_1 = (1,0)$, $\omega_2 = (0,1)$.

(c) $u_1(x,y) = \sqrt{xy}$, $u_2(x,y) = (xy)^2$, $\omega_1 = (1,0)$, $\omega_2 = (0,1)$.

(d) $u_1(x,y) = xy$, $u_2(x,y) = \ln x + \ln y$, $\omega_1 = (18,4)$, $\omega_2 = (3,6)$.

(e) $u_1(x,y) = x + \frac{y^2}{2}$, $u_2(x,y) = x + y^2$, $\omega_1 = (0,4)$, $\omega_2 = (4,2)$.

6 The utility functions of the agents are $u_1(x,y) = x + y$ and $u_2(x,y) = \max\{x,y\}$, and that the initial endowments are $\omega_1 = \omega_2 = (1,1)$.

(a) Represent the situation in an Edgeworth box.

(b) In the competitive equilibrium, What is the relation between the prices?

(c) What is the competitive equilibrium allocation?

$\boxed{7}$ The utility functions of the agents are $u_1(x,y) = x + 2y$ and $u_2(x,y) = \sqrt{xy}$, and the initial endowments are $\omega_1 = (1,2)$ and $\omega_2 = (1,3)$. Compute the competitive equilibrium equilibrium.

$\boxed{8}$ The utility functions of the agents are $u_1(x,y) = u_2(x,y) = xy^2$. The initial endowments are $\omega_1 = (40,160)$, $\omega_2 = (240,120)$.

(a) Compute the competitive equilibrium.

(b) Show that the allocation $x_1 = (80,80)$, $x_2 = (200,200)$ is Pareto efficient.

(c) What prices would support the allocation found in (b)?

(d) Can you redistribute the initial endowments so that the allocation in (b) becomes a competitive equilibrium? Is this redistribution unique?

$\boxed{9}$ Consider an economy with two goods and two agents whose utility functions are

$$u_1(x,y) = xy^2 \text{ and } u_2(x,y) = x^2y$$

The aggregate initial endowments are $(10,20)$.

(a) Find a Pareto efficient allocation(a,b) in which $u_2(a,b) = 8000/27$. (prove that the solution is $x_{11} = 10/3$, $x_{12} = 40/3$).

(b) Suppose that the initial endowments are $w_1 = (10,0)$, $w_2 = (0,20)$. Compute the competitive equilibrium.

$\boxed{10}$ In an exchange economy with two goods, there are two types of consumers: m_1 consumers of type 1 with utility functions $\frac{1}{2}\ln x_1 + \frac{1}{3}\ln y_1$ and one unit of each good each, and m_2 consumers of type 2 with utility functions $2x_2 + y_2$ and 12 units of each good each.

(a) Determine the competitive equilibrium when $m_1 = m_2$.

(b) How does the competitive equilibrium change as m_1 varies?

$\boxed{11}$ Consider an agent whose preference relation $\succeq$ is locally non-satiated and let x^* be a maximal element of $\succeq$ in the budget set $\{x \in X : p \cdot x \leq \theta\}$. Prove that if $y \succeq x^*$, then $p \cdot y \geq \theta$.[1]

$\boxed{12}$ Suppose that the preference relation $\succeq$ is locally non-satiated. Let x^* be a feasible allocation and let p be a price vector. Prove that the two following conditions are equivalent:

(1) If $y \succeq x^*$ then $p \cdot y \geq p \cdot x^*$.

(2) x^* is a solution of the problem

$$\left.\begin{array}{ll} \min & p \cdot x \\ \text{s.t.} & \\ & x \succeq x^* \end{array}\right\}$$

$\boxed{13}$ Consider an exchange economy with two agents and two goods. The agents $i = 1,2$ have the same preference relation, represented by the utility function $u_i(x,y) = x^2 + y^2$ The initial endowments are $w_1 = (4,2) = w_2$. Prove that there is no competitive equilibrium.

$\boxed{14}$ Consider an exchange economy in which all the agents have the preference relation which is strictly convex.[2] Prove that dividing the initial resources equally among the agents is a Pareto efficient allocation. Is this true if not all the agents have the same preferences?

$\boxed{15}$ Consider an exchange economy with 15 agents and 2 goods. An agent has the utility function $u(x,y) = \ln x + \ln y$. In a Pareto efficient allocation, the agent receives the bundle $(10,5)$. Compute the equilibrium prices that would support that Pareto efficient allocation as a competitive equilibrium.

[1] A preference relation $\succeq$ is locally non-satiated iff for all $x \in \mathbb{R}^L$ and any distance $r > 0$ there exists another allocation $y \in \mathbb{R}^L$ such that $\|x - x'\| < r$ and $x' \succ x$.

[2] A preference relation $\succeq$ is convex iff for all $x, x', y \in \mathbb{R}^L$, if $x \succeq y$ and $x' \succeq y$ then $\lambda x + (1-\lambda)x' \succeq y$ for any $\lambda \in [0,1]$.

$\boxed{16}$ Consider an exchange economy with two goods and two agents whose preferences are represented by the utility function $u_i(x, y) = x^\alpha y^{1-\alpha}$ $(i = 1, 2)$, with $0 < \alpha < 1$. The aggregated initial resources are such that $\omega_1 + \omega_2 = (10, 10)$. Prove that the allocation $x_{11} = x_{12} = x_{21} = x_{22} = 5$ is Pareto efficient. Determine a distribution of initial endowments ω_1, ω_2 with $\omega_1 \neq \omega_2$, such that, in this economy, the above allocation is a competitive equilibrium.

$\boxed{17}$ Consider an exchange economy with two goods and three agents whose preferences are determined by the following utility functions

$$\begin{aligned} u_1(x, y) &= xy \\ u_2(x, y) &= xy^2 \\ u_3(x, y) &= 5 \ln x + \ln y \end{aligned}$$

The aggregated initial endowments are $\omega^1 + \omega^2 + \omega^3 = (7, 8)$. Prove that the allocation $x_1 = (1, 2)$, $x_2 = (1, 4))$, $x_3 = (5, 2)$ is Pareto efficient. Compute the prices that would support this allocation as a competitive equilibrium.