Optimal public sector wages*

Pedro Gomes†

Universidad Carlos III de Madrid

October 11, 2013

Abstract

I build a dynamic stochastic general equilibrium model with search and matching frictions in order to determine the optimal public sector wage policy. Public sector wages are crucial to achieve efficient allocation of jobs. High wages induce too many unemployed to queue for public sector jobs, in turn raising unemployment. The optimal wage depends on the frictions in the two sectors. Following technology shocks, public sector wages should be procyclical, and deviations from the optimal policy significantly increase the volatility of unemployment.

JEL Classification: E24; E62; J45.
Keywords: Public sector employment; public sector wages; unemployment; optimal policy.

---

*I would like to thank an anonymous referee and participants in seminars at the London School of Economics, Universidad Carlos III, CREI, Nova University of Lisbon, University of Vienna, Science Po, Goethe University of Frankfurt, Queen Mary, University of Bonn, Lisbon Technical University, CEMFI, Universiteit van Amsterdam, European Central Bank, Bank of England, Bank of Spain and Bank of Portugal; and at the American Economic Association Annual Meeting, the SED annual meeting, the RES conference, the IZA summer school, the 5^{th} European Workshop in Macroeconomics, and the 24^{th} Annual Congress of the EEA. I want to give particular thanks to Chris Pissarides, Frank Cowell, António Afonso, Thijs Van Rens, Davide Debortoli, Jordi Gali, Francesco Caselli, Rachel Ngai and Bernardo Guimarães, Stephen Millard, Luís Costa, Clare Leaver, Mathias Trabandt, Wouter den Haan, Monika Merz, Antonella Trigari, Juan Dolado, Javier Fernandez-Blanco, Albert Marcet, Holly Holder and Jill Miller. Pedro Gomes acknowledges financial support from FCT and the Bank of Spain's Programa de Investigación de Excelencia. Part of the work was carried out at the European Commission (ECFIN), under the visiting fellows contract ECFIN/118/2010/SI2.563865.

†Department of Economics, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe, Spain. Tel:+34 91 624 5732, Email: pgomes@eco.uc3m.es.

1 Introduction

Most papers on the macroeconomics of fiscal policy consider government consumption as goods bought from the private sector.[1] However, the main component of government consumption is compensation to employees. In the United States, the public sector wage bill represents around 60 percent of government consumption expenditures. Government employment is not just an important aspect of fiscal policy, but also a sizable element of the labour market. In the United States, around 16 percent of all employees work in the public sector. Given the proportion of this type of expenditure, it seems plausible that fiscal policy is at least partly transmitted through the labour market.

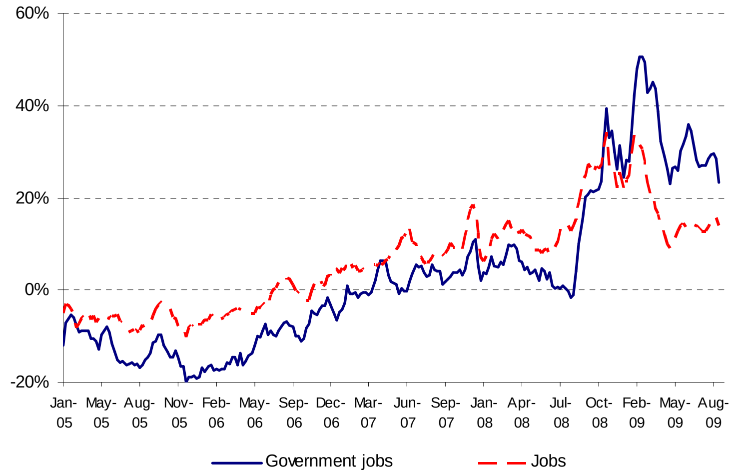
Employment and wage levels in the public sector are relevant, not just because of their weight in the economy or the government's budget but also because they play an important role in the business cycle. Since 2004, the Internet search engine *Google* has released a weekly index of keyword searches. Figure 1 shows the growth rate of keyword searches for the terms '*Jobs*' and '*Government jobs*' within the United States, relative to the previous year. From August 2008, as the recession worsened, the number of searches for jobs increased dramatically, but it is clear that from February 2009, people increasingly specified government jobs. Repeating the exercise for the United Kingdom reveals a similar picture. Indeed, this change in the search patterns during the recession gained such proportions that it was noticed by the press. The following quote is particularly insightful regarding its causes:

> Wall Street may be losing its luster for new U.S. college graduates who are increasingly looking to the government for jobs that enrich their social conscience, if not their wallet. In the boom years, New York's financial center lured many of the brightest young stars with the promise of high salaries and bonuses. However, the financial crisis has tainted the image of big banks, and with fewer financial jobs available, Uncle Sam may be reaping the benefit. (Reuters, 11^{th} of June 2009)

This quote hints that during the recession, more people searched for public sector jobs for two reasons. First, as wages in the private sector fell, more people turned to the public sector, where wages are insulated from market forces. Second, fewer jobs were available

[1] At least this is the approach taken by most articles that study the aggregate effects of government spending. For instance, Barro (1990) studies the effects of productive and unproductive spending in an endogenous growth model. Baxter and King (1993) examine their effects in a Neo-Classical setting, Linnemann and Schabert (2003) extends this model to the New Keynesian model and Galí, López-Salido, and Vallés (2007) introduces rule of thumb agents. All of these papers consider government spending as goods bought from the private sector.

Figure 1: Growth rate of *Google* keyword searches in the United States



Note: The growth rate of the four-week average index of keyword searches, relative the same four weeks in the previous year.

in the private sector while the government continued to hire. Indeed, in the United States, government employment has increased during 9 out of the last 11 recessions. These two facts suggest that both government employment and wages are important elements in explaining the business cycle fluctuations of unemployment.

This study aims to provide a comprehensive yet simple framework to study optimal public sector employment and wage policy both in the steady state and over the business cycle. I build a dynamic stochastic general equilibrium model with search and matching frictions along the lines of Pissarides (2000), with both public and private sectors. The key mechanism builds upon the observation that the unemployed direct their search to one sector or the other, depending on the probabilities of finding a job, the wages and the separation rates. Because the unemployed have a choice of where to search, public sector wages have a crucial role in achieving optimality. First, I solve the social planner's problem to find the constrained efficient allocation. I then solve the decentralized equilibrium and determine the public sector wage consistent with the optimal steady-state allocation. If the government sets a higher wage, for example, due to strong public sector unions, it induces too many unemployed to queue for public sector jobs and raises private sector wages, thus reducing private sector job creation and increasing unemployment. Conversely, if the government sets a lower wage, due to budgetary constraints, for example, few unemployed want a public sector job and the government faces recruitment problems. The optimal wage premium primarily depends on the difference in the labour market friction parameters between the two sectors. For instance, a lower separation rate in the public sector induces many unemployed to concentrate their search on this sector; thus, the government should offer lower wages to offset this. For the chosen calibration, the optimal wage is 2.5 percent lower than in the private sector.

I also examine the properties of the model when subject to technology shocks. Optimal government policy consists of counter-cyclical vacancy posting and procyclical wages. If public sector wages are acyclical, recessions make them more attractive relative to wages in the private sector, inducing more unemployed to queue for public sector jobs. This further dampens job creation in the private sector and amplifies the business cycle. Deviations from the optimal policy can entail significant welfare losses. If, for instance, public sector wages do not respond to the cycle, unemployment volatility doubles. While the result that public sector wages should be procyclical is very robust, the result of counter-cyclical vacancy posting is not; it depends on both preferences and the type of shock driving the business cycle.

This paper adds to the literature examining the transmission mechanisms of public sector employment with search and matching frictions.[2] According to Holmlund and Linden (1993), an increase in public employment has a direct negative effect in unemployment but crowds out private employment due to an increase in wages. Hörner, Ngai, and Olivetti (2007) study the effect of turbulence on unemployment when wages in the public sector are insulated. They conclude that an increase in turbulence induces the more risk-averse unemployed to search for jobs in public companies, resulting in higher aggregate unemployment than if the companies were managed privately. This paper is more related to Quadrini and Trigari (2007). However, despite having similar models, I analyse optimal government wage and vacancy policies both in the steady state and over the business cycle, rather than just looking at the effects of exogenous business cycle rules on volatility. By focussing on optimal policy, I find that some of their results are not general. While they find that the public sector's presence increases unemployment's volatility, I show that this volatility crucially depends on what business cycle policy the government follows. If the government follows the optimal business cycle policy, then the public sector's presence reduces unemployment volatility. Furthermore, while in their setting to stabilize total employment, the government's best policy is to have procyclical public sector employment, the optimal policy is actually counter-cyclical.

A few other recent papers use alternative search approaches to study the effect of public sector employment on private sector wage distribution. Burdett (2012) includes a public sector in a Burdett and Mortensen (1998) framework where firms post wages. Bradley, Postel-Vinay, and Turon (2013) further introduce on-the-job search and transitions between

[2]For examples with frictionless labour markets see Finn (1998), Pappa (2009) or Ardagna (2007).

the two sectors. Albrecht, Navarro, and Vroman (2013) consider heterogeneous human capital and match specific productivity in a Diamond-Mortensen-Pissarides model. While these papers assume that unemployed randomly search for jobs across sectors, I assume that the unemployed can direct their search towards the private or the public sector. Microeconometric studies provide evidence that individuals deliberately choose the private or public sector based on the expected wage differential. Nevertheless, I also show that the main qualitative results on optimal wages hold if job searches are random. Directed searches just amplify the costs of not following the optimal policy through the endogenous reaction of the unemployed.

2 Model

2.1 General setting

The model is a dynamic stochastic general equilibrium model with public and private sectors. The only rigidities present are due to search and matching frictions. Public sector variables are denoted by the superscript g, while private sector variables are denoted by p. Time is denoted by $t = 0, 1, 2, ...$

The labour force consists of many individuals $j \in [0,\ 1]$. A subset of the labour force is unemployed (u_t), while the remainder work in either the public (l_t^g) or the private (l_t^p) sector.

$$1 = l_t^p + l_t^g + u_t. \tag{1}$$

Total employment is denoted by l_t. The presence of search and matching frictions in the labour market prevents some unemployed from finding jobs. The evolution of employment in both sectors depends on the number of new matches m_t^p and m_t^g and on the separations. In each period, jobs are destroyed at constant fraction λ^i, potentially different across sectors.

$$l_{t+1}^i = (1-\lambda^i)l_t^i + m_t^i,\ i = p, g. \tag{2}$$

The new matches are determined by two Cobb-Douglas matching functions:

$$m_t^i = \mu^i (u_t^i)^{\eta^i} (v_t^i)^{1-\eta^i},\ i = p, g. \tag{3}$$

I assume the unemployed choose the sector in which they concentrate their search; thus, u_t^i represents the number of unemployed searching in sector i. Vacancies in each sector are denoted by v_t^i. The parameter η^i is the matching elasticity with respect to unemployment and μ^i the matching efficiency. An important part of the analysis focuses on the behaviour of those unemployed specifically searching for a public sector job, defined as: $s_t = \frac{u_t^g}{u_t}$.

From the matching functions, we can define the probabilities of vacancies being filled as q_t^i, the job-finding rates conditional on searching in a particular sector as p_t^i, and the unconditional job-finding rates as f_t^i:

$$q_t^i = \frac{m_t^i}{v_t^i}, \; p_t^i = \frac{m_t^i}{u_t^i}, \; f_t^i = \frac{m_t^i}{u_t}, \; i = p, g.$$

The assumption of directed search implies that the number of vacancies posted in one sector affects the contemporaneous probability of filling a vacancy in the other sector only through the endogenous reaction of s_t.

2.2 Households

In the presence of unemployment risk, we should observe consumption differences across different individuals. Following Merz (1995), I assume all household members pool income so that private consumption is equalised. The household is infinitely lived and has preferences over private consumption goods, c_t, and public goods g_t. It also has $\nu(u_t)$ utility from unemployment, which captures leisure and home production.

$$E_t \sum_{t=0}^{\infty} \beta^t [u(c_t, g_t) + \nu(u_t)], \tag{4}$$

where $\beta \in (0, 1)$ is the discount factor. The budget constraint in period t is given by:

$$c_t + B_t = (1 + r_{t-1})B_{t-1} + w_t^p l_t^p + w_t^g l_t^g + \Pi_t, \tag{5}$$

where r_{t-1} is the real interest rate from period $t-1$ to t, and B_{t-1} are the holdings of one period bonds. $w_t^i l_t^i$ is the total wage income from household members working in sector i. Finally, Π_t encompasses the lump sum taxes that finance the government's wage bill as well as possible transfers from private sector firms. I assume there are no unemployment benefits.

The household chooses c_t to maximize the expected utility subject to the sequence of budget constraints, taking public goods as given. The solution is the Euler equation:

$$u_c(c_t, g_t) = \beta(1 + r_t)E_t[u_c(c_{t+1}, g_{t+1})]. \tag{6}$$

2.3 Workers

The value of each member to the household depends on their current state. The value of being employed in sector i is given by:

$$W_t^i = w_t^i + E_t\beta_{t,t+1}[(1-\lambda^i)W_{t+1}^i + \lambda^i U_{t+1}], \ i = p, g, \tag{7}$$

where $\beta_{t,t+k} = \beta^k \frac{u_c(c_{t+k}, g_{t+k})}{u_c(c_t, g_t)}$ is the stochastic discount factor. The value of being employed in a specific sector depends on the current wage, as well as the continuation value of the job, which depends on the separation probability. Under the assumption of directed search, unemployed are searching for a job in either the private or the public sector, with value functions given by:

$$U_t^i = \frac{\nu_u(u_t)}{u_c(c_t, g_t)} + E_t\beta_{t,t+1}[p_t^i W_{t+1}^i + (1-p_t^i)U_{t+1}], \ i = p, g. \tag{8}$$

Beside the marginal utility from unemployment, the value of being unemployed and searching in a particular sector depends on the probability of finding a job and the value of working in that sector. Optimality implies that movements between the two segments guarantee no additional gain for searching in one sector *vis-à-vis* the other:

$$U_t^p = U_t^g = U_t. \tag{9}$$

This equality determines the share of unemployed searching in each sector. We can re-write this as

$$\frac{m_t^p E_t\beta_{t,t+1}[W_{t+1}^p - U_{t+1}]}{(1-s_t)} = \frac{m_t^g E_t\beta_{t,t+1}[W_{t+1}^g - U_{t+1}]}{s_t}, \tag{10}$$

which implicitly defines s_t. An increase in the value of employment in the public sector, driven by either wage increases or a separation rate decrease, raises s_t until no extra gain exists for searching in that sector. Under the directed search assumption, wages in the public sector play a key role in determining s_t. If unemployed randomly search between sectors, public sector wages would still have an effect through the value of unemployment, but it would be weaker.

2.4 Private sector firms

The representative firm hires labour to produce private consumption goods. The production function is linear for labour, but part of the resources produced have to be used to pay for posting vacancies $\varsigma^p v_t^p$.

$$y_t = a_t^p l_t^p - \varsigma^p v_t^p. \tag{11}$$

At time t, the level of employment is predetermined with the firm only able to control the number of vacancies it posts. The value of opening a vacancy is given by

$$V_t = E_t \beta_{t,t+1} [q_t^p J_{t+1} + (1 - q_t^p) V_{t+1}] - \varsigma^p, \tag{12}$$

where J_t is the value of a job for the firm, given by

$$J_t = a_t^p - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda^p) J_{t+1}]. \tag{13}$$

Free entry guarantees that the value of posting a vacancy is zero ($V_t = 0$); therefore, we can combine the two equations into

$$\frac{\varsigma^p}{q_t^p} = E_t \beta_{t,t+1} [a_{t+1}^p - w_{t+1}^p + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p}]. \tag{14}$$

The condition states that the expected cost of hiring a worker must equal its expected return. The benefit of hiring an extra worker is the discounted value of the expected difference between the worker's marginal productivity and his or her wage, plus the continuation value, knowing that with a probability λ^p the match is destroyed.

Finally, I consider private sector wages as the outcome of Nash bargaining between workers and firms. The sharing rule is given by

$$(1 - b)(W_t^p - U_t) = b J_t, \tag{15}$$

where b is the workers' bargaining power.

2.5 Government

The government produces its goods using a linear technology on labour. This type of good is different from private consumption goods: it is non-rival and supplied to the representative family for free. As in the private sector, the costs of posting vacancies are deducted from production.

$$g_t = a_t^g l_t^g - \varsigma^g v_t^g. \tag{16}$$

The government collects lump sum taxes to finance its wage bill:

$$\tau_t = w_t^g l_t^g. \tag{17}$$

The numeraire of this economy is the private consumption good. As a public good is not sold, it has no actual price. However, there is an implicit relative price given by the marginal rate of substitution. The formulation of the production function (16) implies that the cost of recruiting is given in units of the public good. Alternatively, if the cost was included in the budget constraint, it would be expressed in units of private consumption.

The government sets a policy for the sequence of vacancies and wages $\{v_t^g,\ w_{t+1}^g\}_{t=o}^{\infty}$. Wages are set one period in advance, at the time it posts the vacancies. As s_t is determined on the basis of expected future wages in the two sectors, the current public sector wage does not affect any decision variable. I will compare the optimal policy, which arises from the social planner's problem, with policies that are exogenously constrained, for instance, due to strong public sector unions or budgetary constraints.

2.6 Decentralised equilibrium

Definition 1 *A decentralised equilibrium is a sequence of prices $\{r_t, w_t^p\}_{t=o}^{\infty}$ such that, given a sequence of government vacancies and wages $\{v_t^g, w_{t+1}^g\}_{t=o}^{\infty}$, the household chooses a sequence of consumption $\{c_t\}_{t=o}^{\infty}$, and the fraction of unemployed members searching in the public sector s_t and firms choose private sector vacancies v_t^p, such that (i) the household maximises its lifetime utility; (ii) the share of unemployed searching in the public sector is such that the values of searching in both sectors equalise (equation 10); (iii) private sector vacancies satisfy the free entry condition (14); (iv) the private wage level w_t^p solves the bargaining condition (15); (v) the private goods market clears: $c_t = y_t$; and (vi) lump sum taxes τ_t are chosen to balance the government's budget (equation 17).*

2.7 Social planner's solution

Many decentralised equilibria are possible depending on the government's wage and vacancy policies. In my evaluation, I use the constrained efficient solution as a benchmark. The social planner's problem is to maximize consumers' lifetime utility (4) subject to the labour market and technology constraints (1-3, 11 and 16). The first-order conditions are given by

$$\frac{\varsigma^p}{q_t^p} = \beta E_t\{\frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)}[(1-\eta^p)a_{t+1}^p - (1-\eta^p)\frac{\nu_u(u_{t+1})}{u_c(c_{t+1}, g_{t+1})} + (1-\lambda^p)\frac{\varsigma^p}{q_{t+1}^p} - \frac{\eta^p \varsigma^p v_{t+1}^p}{(1-s_{t+1})u_{t+1}}]\}, \tag{18}$$

$$\frac{\varsigma^g}{q_t^g} = \beta E_t \{ \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} [(1-\eta^g) a_{t+1}^g - (1-\eta^g) \frac{\nu_u(u_{t+1})}{u_g(c_{t+1}, g_{t+1})} + (1-\lambda^g) \frac{\varsigma^g}{q_{t+1}^g} - \frac{\eta^g \varsigma^g v_{t+1}^g}{s_{t+1} u_{t+1}}] \}, \quad (19)$$

$$\frac{u_g(c_t, g_t) \varsigma^g v_t^g \eta^g}{(1-\eta^g) s_t} = \frac{u_c(c_t, g_t) \varsigma^p v_t^p \eta^p}{(1-\eta^p)(1-s_t)}. \quad (20)$$

Constrained efficient allocation consists of a triplet of sequences $\{v_t^p, v_t^g, s_t\}_{t=0}^{\infty}$. Conditions (18) and (19) describe optimal private and public sector vacancies. On the left hand side, we have the expected cost of hiring an extra worker. The right hand side gives us the marginal social benefit of hiring this additional worker. It consists of the worker's expected marginal productivity minus the utility cost of working, weighted by the matching elasticity with respect to vacancies, plus continuation value. This last element enters with a negative sign, reflecting that hiring an additional worker makes it harder for both sectors to recruit a worker in the future.

The optimal split of the unemployed between sectors, pinned down in (20), depends on the following factors: i) the marginal utility of consumption of both goods, ii) the number of vacancies and their costs and iii) the matching elasticity with respect to unemployment in both sectors.

In order to implement the first-best allocation, the government can directly set the optimal path of vacancies as well as an appropriate path for public sector wages in order to induce the optimal proportion of unemployed searching for public sector jobs. But what about private sector vacancies?

Proposition 1 *In the steady state, if the government sets the optimal level of public sector vacancies and sets wages such that the optimal share of unemployed is searching for public sector jobs, then if workers' bargaining power equals the matching elasticity with respect to unemployment in the private sector ($b = \eta^p$), the level of vacancies in the private sector is optimal.*

The proof can be seen in the companion appendix. In a one-sector model, a firm's vacancy posting behaviour entails both a positive and a negative externality: it increases the probability of an unemployed finding a job but reduces other firms' probability of filling a vacancy. The decentralised equilibrium is efficient if the share of the match's surplus that goes to the firm $(1-b)$ equals the importance of the vacancies in the matching process $(1-\eta^p)$, in what is usually called the Hosios condition. When we include the public sector, externalities arising from directed search join those of public sector vacancies. If more unemployed search in the public sector, the probability of filling a vacancy is higher in the public sector but lower in private sector firms. This proposition states that, in the steady state, if the government is able to internalise the externalities in $\bar{v}^g$ and $\bar{w}^g$, then private sector vacancies will also be efficient, provided that the Hosios condition is satisfied. If it is not, then the government would need another instrument, such as a vacancy subsidy or tax, to align private vacancies with the first best.[3]

2.8 Characterization of optimal policy in the steady state

The main question of interest is how to set optimal public sector wages. Under some restrictions, we can derive an expression for optimal public sector wages in the steady state. Assume that unemployed does not bring any utility to the household: $\nu_u(u_t) = 0$ and that the costs of posting vacancies are proportional to productivity $\varsigma^i = \hat{\varsigma}^i \bar{a}^i$.[4] The upper bar denotes the variables in the steady state. If we combine optimality conditions, we can derive an expression characterizing the optimal tightness in each sector, $\bar{\theta}^i \equiv \frac{\bar{v}^i}{\bar{u}^i}$:

$$\eta^i \bar{\theta}^{i*} + (\bar{\theta}^{i*})^{\eta^i} (\beta^{-1} - 1 + \lambda^i) = \frac{(1-\eta^i)\mu^i}{\hat{\varsigma}^i}, \; i = p, g. \tag{21}$$

The equation shows that optimal tightness is independent of both technology and preferences, depending only on the friction parameters of the corresponding sector. Combining this expression with the competitive equilibrium equation (10) that pins down the split of searching across sectors, we can show that

$$\bar{w}^{g*} = \frac{(1-\eta^g)}{(1-\eta^p)} \frac{\left(\frac{1}{\hat{\varsigma}^g \bar{\theta}^g} + 1\right)}{\left(\frac{1}{\hat{\varsigma}^p \bar{\theta}^p} + 1\right)} \bar{w}^p. \tag{22}$$

This equation provides a simple rule for setting optimal public sector wages. They should be indexed to private sector wages, but adjusted by a factor that reflects friction parameters differences: η^i, $\hat{\varsigma}^i$, λ^i and μ^i. For symmetrical frictions, wages should be the same in each sector. As in a frictionless labour market, the optimal wage ratio is independent of preferences or the sectors' relative productivity (Finn, 1998). Differences in preferences or productivity should not be reflected in the relative wages, but via the relative total number of employees. Because a sector's productivity is embodied in the job rather than the worker, any change is reflected by the social planner in terms of quantities (i.e. the amount of public sector goods produced) rather than prices (i.e. the wages received by workers). This becomes clear if we

[3]Because future public sector wages affect the unemployed's decision regarding where to search today, a time inconsistency problem could exist. In this setting, the lump sum nature of taxes indicates that the government does not gain from setting a current wage different than promised. On the other hand, with distortionary taxes, the government has incentives to deviate and offer a lower wage than promised.

[4]This is equivalent to writing the production functions as: $g_t = a_t^g(l_t^g - \hat{\varsigma}^g v_t^g)$ and $c_t = a_t^p(l_t^p - \hat{\varsigma}^p v_t^p)$.

solve for the optimal share of unemployed searching in the public sector:

$$\frac{\bar{s}}{1-\bar{s}} = \frac{\bar{a}^g u_g}{\bar{a}^p u_c} \frac{\eta^g(1-\eta^p)\mu^p \bar{m}^g}{\eta^p(1-\eta^g)\mu^g \bar{m}^p} \frac{\hat{\varsigma}^g(\bar{\theta}^g)^{\eta^g}}{\hat{\varsigma}^p(\bar{\theta}^p)^{\eta^p}} \tag{23}$$

The optimal search number depends both on both productivity and preferences. If we assume log preferences $u(c_t, g_t) = \log c_t + \zeta \log g_t$, we can further simplify to

$$\frac{\bar{s}}{1-\bar{s}} = \zeta \frac{\eta^g(1-\eta^p)}{\eta^p(1-\eta^g)} \frac{\left(\frac{\mu^p}{\hat{\varsigma}^p \lambda^p (\bar{\theta}^p)^{\eta^p}} - 1\right)}{\left(\frac{\mu^g}{\hat{\varsigma}^g \lambda^g (\bar{\theta}^g)^{\eta^g}} - 1\right)}. \tag{24}$$

With symmetric labour market frictions, the optimal number of unemployed searching in the public sector depends only on preferences for the actual size of the public sector $\bar{s} = \frac{\zeta}{1+\zeta}$ and the relative size of the public sector is $\frac{\bar{l}^g}{\bar{l}^p} = \frac{\bar{v}^g}{\bar{v}^p} = \zeta$.

Without these assumptions, no closed-form solution exists for the wage ratio; therefore, we have to solve the model numerically. The equations (22) and (23) show how important calibration of the two sectors' friction parameters is for predicting the optimal wage ratio. In order to ensure an accurate prediction, I explore information from several sources.[5]

3 Calibration

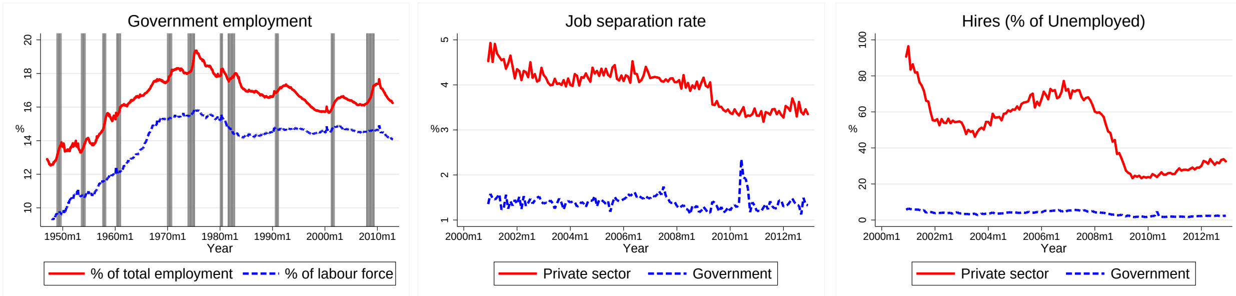
To solve the model, I assume a CES utility function in logs, which allows us to address the different elasticities of substitution between the two consumption goods. The utility of unemployment is linear.

$$u(c_t, g_t) = \frac{1}{\gamma} ln[c_t^\gamma + \zeta g_t^\gamma], \ \nu(u_t) = \chi u_t.$$

The model is calibrated to match the US economy at a monthly frequency. I do not assume in my calibration that the government is following optimal wage policy. Instead, I

[5]An alternative analysis would directly follow from a multi-sector version of Mortensen-Pissarides, where public sector wages were also bargained. Adapting the concept of Nash bargaining to the public sector is not a straightforward process because this sector produces a different good that is non-rival and is also not sold. The numeraire of the economy is the private sector good in which public sector wages are paid, but the government raises revenue through lump sum taxes, a process which has, effectively, no cost. We could still define the surplus from public sector employment by using the marginal product of labour evaluated at the shadow price for public goods. As typical in these problems, if the Hosios condition is satisfied in the public sector, we would have efficient public wages, characterized by equation (22). If high union bargaining power would imply higher wages, lower bargaining power would imply lower wages. By considering wages as a policy instrument, we can bypass this analysis and evaluate the consequences of both higher and lower wages without explicitly modelling what causes deviations from optimality.

Figure 2: Evidence for the United States



Note: The government employment series is taken from the Current Employment Statistics survey (Bureau of Labor Statistics). The grey bars indicate the NBER recession dates. The job-separation and job-finding rates are calculated from the Job Opening and Labour Turnover Survey.

calibrate the public sector wage premium that is consistent with microeconometric estimations. These estimates have proven quite sensitive to a worker's education level and gender or the subsector of government. The survey by Gregory and Borland (1999) places the premium between 0 and 10 percent. I set it close to the lower boundary, at 2 percent ($\pi \equiv \frac{\bar{w}^g}{\bar{w}^p} = 1.02$).

The first graph in Figure 2 shows government employment in the United States since 1947. Under the baseline calibration, steady-state vacancies in the public sector are such that employment therein corresponds to the sample's average of 16 percent of total employment ($\bar{l}^g = 0.15$).

The second graph shows the monthly separation rate for both sectors, taken from the Job Opening and Labour Turnover Survey (JOLTS). The separation rate in the private sector is almost three times higher than in the government: 4.0 against 1.4 percent. The last graph plots the new hires of each sector as a share on total unemployed, which serves as a proxy for the job-finding rate. The probability of finding a job in the government sector is only 3.6 percent compared to 50.6 percent in the private sector.

For an approximation of the matching elasticity with respect to vacancies, I regress the log of the job-finding rate (the ratio between hires in that sector and unemployment) on the log of tightness (the ratio between job openings in that sector and unemployment) for each sector. The estimated coefficients are 0.62 for the private sector and 0.76 for the public sector, which suggest that vacancies are more important determinants of matches in the public sector. These numbers might be biased because we are omitting unemployed that are effectively searching in each sector. If we include a measure of relative search from *Google trends* as a proxy, the estimated elasticities change to 0.67 and 0.87 respectively. These numbers are similar if we estimate the matching function with the log of new matches as a function of log vacancies and log unemployment, imposing constant returns to scale. I

Table 1: Baseline calibration

Parameters											
a^p	1	η^p	0.4	ς^p	1.5	μ^p	1.080	λ^p	0.0397	$\bar{l}^g$	0.15
a^g	1	η^g	0.2	ς^g	0.8	μ^g	0.699	λ^g	0.0139	π	1.02
γ	0	ζ	0.183	χ	0.544	β	0.9955	b	0.4		
Steady-state variables											
$\bar{u}$	0.06	$\bar{q}^g$	1	$\bar{f}^g$	0.035	$\bar{p}^g$	0.167	$\bar{s}$	0.21	$\frac{\varsigma^p \bar{v}^p + \varsigma^g \bar{v}^g \frac{u_g}{u_c}}{\sum \bar{w}^i \bar{l}^i}$	0.037
$\bar{l}^p$	0.79	$\bar{q}^p$	1.5	$\bar{f}^p$	0.523	$\bar{p}^p$	0.660	$\frac{\nu_l}{u_c \bar{w}^g}$	0.509	$\frac{\bar{W}^g - \bar{U}}{\bar{W}^p - \bar{U}}$	3.96

set public sector matching elasticity with respect to unemployment, η^g at 0.15 and η^p at 0.4, slightly higher than the estimated value but in line with estimates from the literature (Petrongolo and Pissarides (2001)). Elasticity with respect to vacancies of 0.85 in the public sector is less extreme than in Quadrini and Trigari (2007). They use the minimum of vacancies and unemployment as a matching function, which implies a matching elasticity of 1.

A recent paper by Davis, Faberman, and Haltiwanger (2013) provides some insights into the duration of vacancies by sector. They use JOLTS data to study the behaviour of vacancies and hiring. After adjusting the data, they estimate that government vacancy remains open for 30 days while a private sector vacancy remains unfilled for only 20 days. I calibrate the matching efficiency μ^i to reproduce these numbers ($\bar{q}^p = 1$ and $\bar{q}^g = 1.5$).

Although I could not find data on public sector recruitment costs in the United States, the United Kingdom has a unique source. Every year, the Chartered Institute of Personal Development performs a recruitment practice survey covering approximately 800 organizations from the following sectors: manufacturing and production, private sector services, public sector services and voluntary, community and not-for-profit sector (CIPD (2009)). The costs of recruiting a worker, which encompass advertising and agency costs, are approximately £4000 for the median firm, corresponding to approximately 8 weeks of the median income in the United Kingdom. On an average, these costs are 40 percent lower in the public sector. I consider that these values indicate that cost per hire is lower in the public sector. I consider the cost of posting a vacancy ς^i to be 1.5 in the private sector and 0.8 in the public sector. Given that a vacancy's duration is longer in the public sector, these values imply that the public sector's average cost of recruiting expressed in the same units is 20 percent lower than in the private sector. Under this calibration, recruitment costs total 3.7 percent of total labour costs, close to the value found in Russo, Hassink, and Gorter (2005). These numbers also imply that public vacancies are only 10 percent of private sector vacancies, which is close to 9 percent found in JOLTS.

The empirical evidence relative to the substitution elasticity between private and government consumption is not conclusive. Evans and Karras (1998) find that private consumption is complementary to military expenditure and a substitute for non-military expenditure. Fiorito and Kollintzas (2004) disaggregate expenditure into ‘public goods’ (defence, public order and justice) and ‘merit goods’ (health, education and other services). They find that ‘public goods’ are substitutes and ‘merit goods’ are complements to private consumption. In the baseline case, I consider an elasticity of substitution of 1 ($\gamma = 0.0$), but also discuss the cases where goods are substitutes ($\gamma = 0.5$) and complements ($\gamma = -0.5$). The parameter ζ, which reflects the preference for government services, is chosen such that the optimal level of public sector employment is 0.15.

For the model to satisfy the Hosios condition in the private sector, the worker's share in the Nash bargaining is set at 0.4. The value of leisure in the utility function is calibrated such that the steady-state unemployment rate is 0.06, implying an outside option equivalent to 50 percent of the average wage. Technology in both sectors is normalised to 1 and the discount factor is set at 0.9955, which implies an annual interest rate of 5 percent. Table 1 summarises the baseline calibration and the implied steady-state values for the key variables.

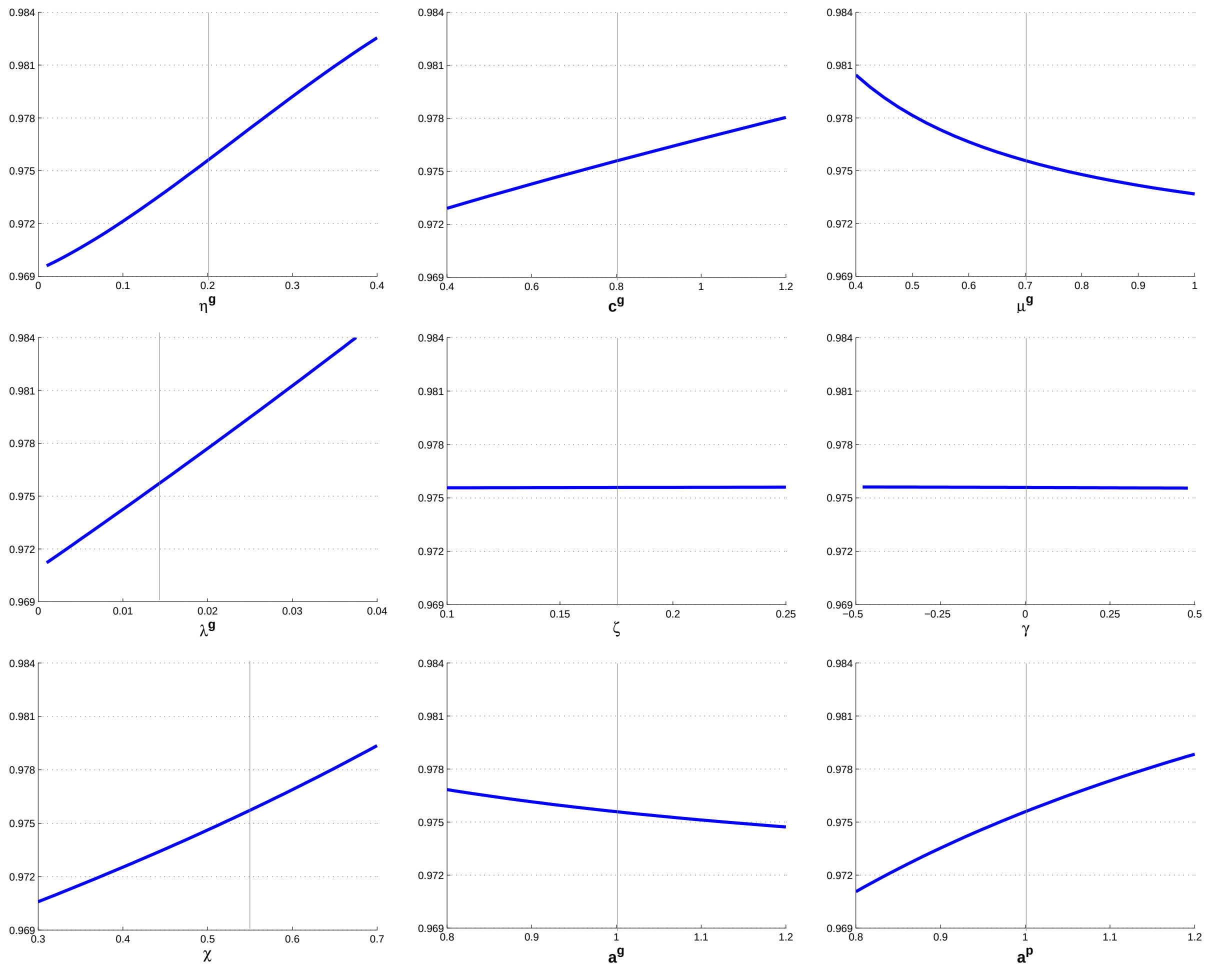
4 Optimal public sector wage in steady state

As already shown, the optimal wage ratio primarily depends on the difference between friction parameters in the public and private sectors and only equals one if the sectors are symmetric. Under the baseline calibration, the optimal public sector wage is 2.5 percent lower than in the private sector. Figure 3 shows how this optimal wage varies with the public sector's parameters.[6]

When the cost of posting vacancies is lower or when matching depends more on vacancies (lower η^g), it is more efficient for the matching to be driven by vacancies rather than by the unemployed. In order to induce fewer unemployed to search in the public sector, the government should pay less to its workers. When the separation rate decreases or matching becomes more efficient, more unemployed turn to the public sector at a time when it would be optimal to have less. As the private incentive is not efficient, the government should offer lower wages to correct this. Quantitatively, the difference between matching elasticities

[6]The companion appendix shows how the optimal share of unemployed searching in the two sectors, unemployment rate, size of the public sector and wages and tightness in both sectors vary with these parameters.

Figure 3: Optimal steady-state public-private wage ratio



explains one third of the optimal public sector wage gap, while differences in separation rates and vacancy-posting costs account for one sixth each.

The optimal wage ratio positively depends on the disutility of working (χ) and negatively on the productivity of the public sector, but it does not depend on the coefficients of the utility function, namely γ and ζ. Higher χ raises the value of employment in the private sector relative to that in the public sector because people are more likely to have another spell of unemployment if they are employed in the private sector. As this induces more unemployed to search in the private sector, the government needs to offer higher wages to offset this preference.

In the numerical simulations, the cost of posting vacancies is not proportional to productivity. In such a scenario, lower productivity in the public sector indicates that the relative cost of posting vacancies is higher because the marginal utility of public sector goods increases and hiring costs are internal. Although the social planner wants to produce fewer public sector goods, it prefers new matches be driven by the unemployment side to save vacancy posting costs, a decision which requires higher public sector wages.

The parameters of the utility function mainly affect the optimal amount of public sector goods produced, but numerically the parameters do not change the optimal wage ratio. This result occurs because the optimal tightness in the two sectors stays the same, and the different values of working in each sector as well as the value of being unemployed all remain unchanged. As in the frictionless world, preferences determine the relative employment level but not the wage ratio.

To investigate the consequences of paying more to public sector employees, I compare the unemployment rate when the public sector wage is optimal (a gap of 2.5 percent) with the baseline case (a premium of 2 percent). The unemployment rate, which was calibrated to 6 percent in the baseline steady state, falls to 5 percent when the government sets the optimal wage. This happens because many unemployed that were queuing for public sector jobs now find it more attractive to search in the private sector ($\bar{s}$ changes from 21 to 3 percent), boosting job creation. Interestingly, private sector wages do not go up in line with public sector wages. On one hand, the public sector wages increase an unemployed's outside options if they search in the public sector, exerting an upward pressure on the wage bargaining. On the other hand, such public sector wage increases also entail a negative wealth effect in the real business cycle sense. By increasing unemployment and crowding out private employment permanently, they reduce the total amount of private goods produced in the economy, raising their marginal utility. Higher marginal utility of consumption reduces the value of being unemployed and makes people willing to work at lower wages. The second effect seems to dominate. Welfare costs in the baseline scenario are equivalent to a permanent private consumption reduction of 0.6 percent. All in all, public sector wages are an important determinant of equilibrium unemployment.[7]

5 Public sector policies and the business cycle

Now I examine the effects of a 1 percent negative private technology shock on the economy under alternative government policies. I consider an AR(1) shock with autoregressive coefficient $\rho = 0.9$.

$$\ln(a_t^p) = (1-\rho)\ln(\bar{a}^p) + \rho\ln(a_{t-1}^p) + \epsilon_t^a.$$

[7]Figures can be found in the companion appendix.

Figure 4 shows the impulse responses, starting from the efficient steady state, when the government follows the optimal rule. I contrast the optimal policy with the following simple rules for vacancies and wages:

$$\log(v_t^g) = \log(\bar{v}^g) + \psi^v[\log(v_t^p) - \log(\bar{v}^p)], \tag{25}$$

$$\log(w_{t+1}^g) = \log(\bar{w}^g) + \psi^w[\log(w_t^p) - \log(\bar{w}^p)]. \tag{26}$$

Existing evidence by Lane (2003) and Lamo, Pérez, and Schuknecht (2008) suggest that public sector wages are less procyclical than private sector wages, particularly in the United States.[8] For simplicity, I consider two cases where public sector wages are acyclical ($\psi^w = 0$). In the first case, public sector vacancies proportionally decline to increases in private sector vacancies ($\psi^v = -1$). In the second, they are acyclical ($\psi^v = 0$).

After the negative productivity shock, private sector firms post fewer vacancies, resulting in reduced probability of finding a job in this sector; therefore, the unemployed increasingly search for public sector jobs. The unemployment rate increases at most by 0.06 percentage points. As pointed out by Shimer (2005), under a standard calibration, search and matching models cannot generate sufficient fluctuations in unemployment in response to technology shocks.

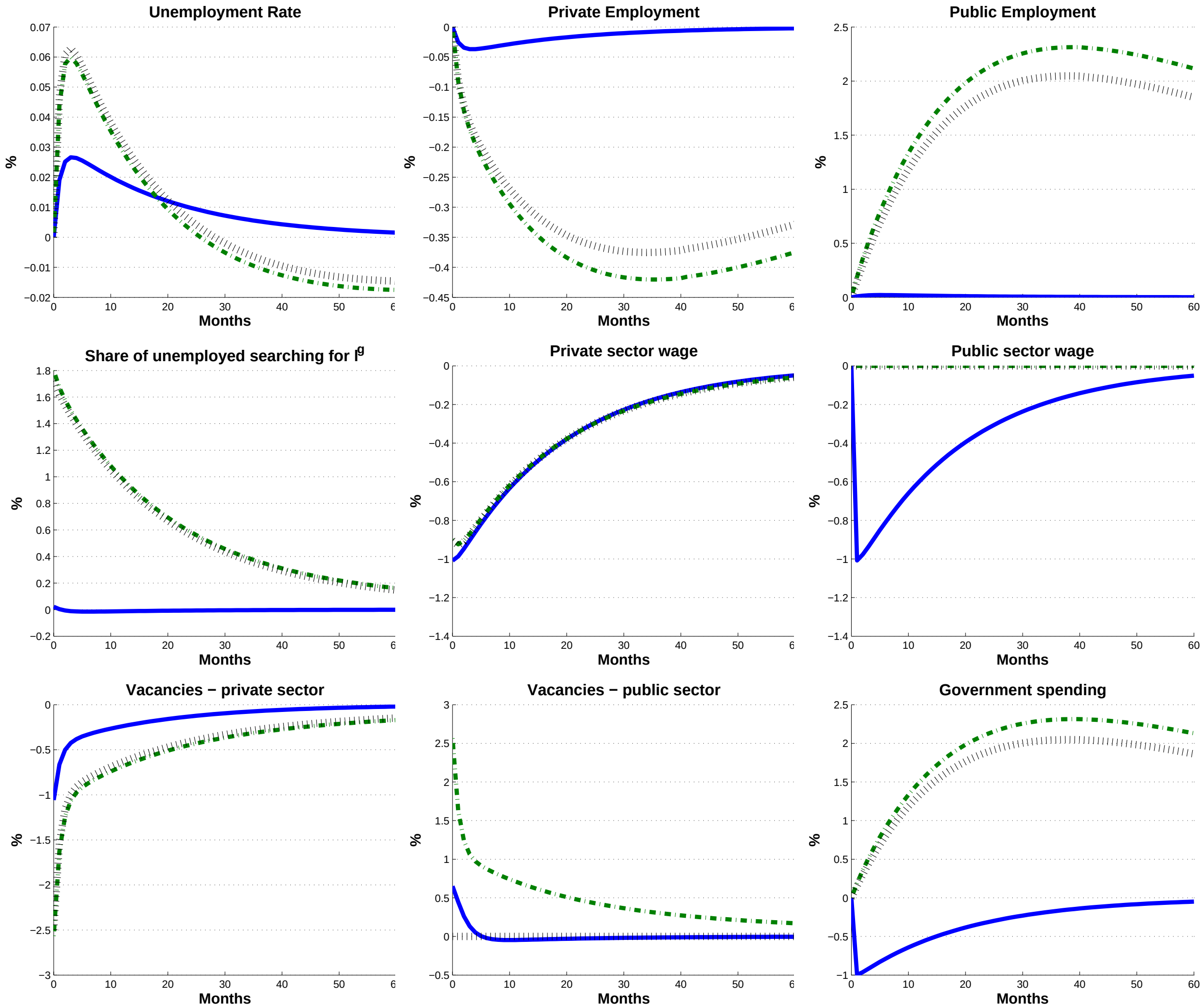
Optimal government policy will have procyclical wages and counter-cyclical vacancies. The sector reallocation argument advocates hiring more people in recessions. If the private sector has lower productivity, the economy will be better served by the public sector absorbing part of the unused labour force. This result is fragile in two dimensions. First, it depends on preferences. Figure 5 shows the optimal business cycle policy for three cases: if the goods are substitutes ($\gamma = 0.5$), complements ($\gamma = -0.5$) and the baseline case with an elasticity of substitution of 1 ($\gamma = 0.0$). The result is overturned if the goods are strong complements. In this case, during a recession, as the marginal utility of the government services falls with consumption of private goods, the government should also reduce the number of people it employs. The result of counter-cyclical vacancies also breaks down following an economy-wide technological shock or discount factor shocks. As they affect both sectors symmetrically, the argument for sector reallocation no longer holds.[9]

On the other hand, public sector wages should follow declines in private sector wages. In recessions, if the government keeps its wages constant, it becomes more attractive relative

[8] Additionally, a study by Devereux and Hart (2006) using micro data for the United Kingdom finds that for job movers in the private sector, the wages are procyclical but for the public sector they are acyclical.

[9] This is shown in the companion appendix.

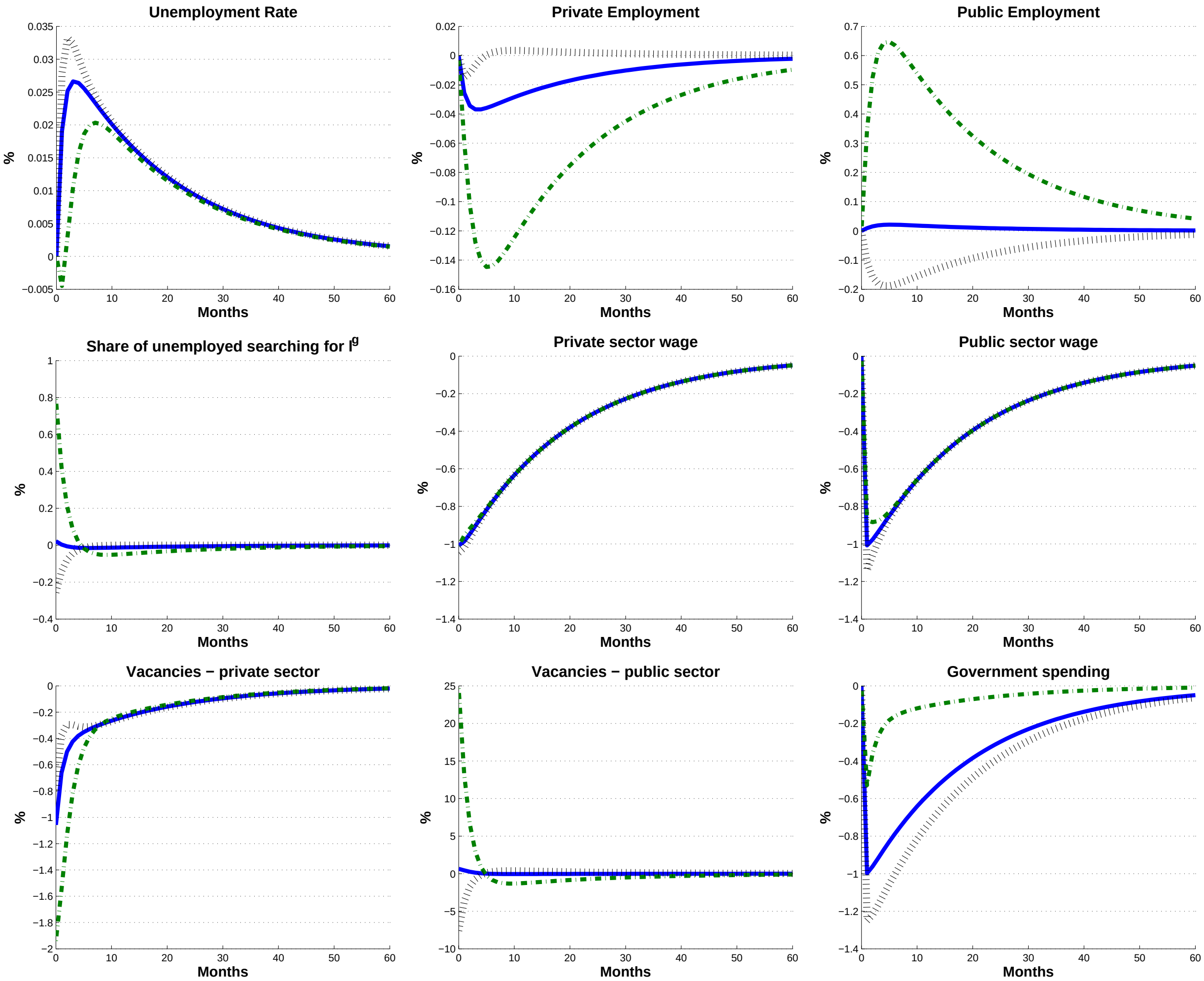
Figure 4: Response to a private sector technology shock under different policies



Note: Solid line (optimal policy); dash line (counter-cyclical vacancies and acyclical wages) and dotted line (acyclical vacancies and wages). The response of the variables is shown in percentage of their steady-state value, except for unemployment rate and the share of unemployed searching for public sector jobs, which is expressed in percentage point difference from the steady state.

to the private sector, thus increasing the share of unemployed searching for public sector jobs. This in turn reduces the probability that a vacancy in the private sector will be filled, which further dampens job creation and amplifies the business cycle. We can see that under the two exogenous rules, unemployment shows a much stronger response. The share of unemployed searching of public sector jobs increases by 1.6 percentage points, much higher than under the optimal policy (0.02 percentage points). This result is remarkably robust to alternative preference specifications or business cycle driving forces. This robustness reflects the fact that the optimal wage gap mainly depends on the friction parameters, which do not fluctuate over the business cycle. As a consequence, the two wages should have a very close

Figure 5: Optimal business cycle policy under different elasticities



Note: Solid line ($\gamma = 0.0$); dash line (substitutes, $\gamma = 0.5$) and dotted line (complements, $\gamma = -0.5$). The response of the variables is expressed in percentage of their steady-state value, except for unemployment rate and the share of unemployed searching for public sector jobs, which is shown in percentage point difference from the steady state.

comovement.[10]

Table 2 compares the standard deviation of key variables in the alternative policies, as well as when no public sector exists ($\zeta = 0$). If government follows the optimal policy, the presence of public sector employment stabilises unemployment. However, if public sector wages are acyclical, unemployment volatility increases twofold.

In their paper, Quadrini and Trigari (2007) have two conclusions contrary to mine. First, they argue that the presence of the public sector increases unemployment volatility. I show

[10] Optimal wage policy would be reinforced with endogenous job destruction. In recessions, the job-separation rate in the private sector would increase, further reducing the value of private sector employment relative to that in the public sector. This would encourage more unemployed to search in the public sector, requiring public sector wages to fall even more to maintain optimality.

Table 2: Business cycle properties under different policies

Policy	Standard deviations				Correl	Welfare
	l_t^p	l_t^g	u_t	w_t^p	(l_t^g, u_t)	cost
Optimal steady-state wage						
No government	0.0008	–	0.0007	0.021	–	0.022%
Optimal policy	0.0008	0.0004	0.0006	0.021	0.994	0.018%
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0282	0.1558	0.0016	0.0206	−0.506	0.296%
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0247	0.1350	0.0015	0.0206	−0.368	0.240%
Optimal steady-state wage						
Substitutes ($\gamma = 0.5$)						
Optimal policy	0.0035	0.0157	0.0005	0.021	0.919	0.0186%
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0274	0.1482	0.0015	0.0207	−0.252	0.184%
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0233	0.1249	0.0014	0.0208	−0.084	0.150%
Complements ($\gamma = -0.5$)						
Optimal policy	0.0002	0.0047	0.0007	0.021	−0.985	0.0231%
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0292	0.1645	0.0019	0.0205	−0.695	0.553%
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0263	0.1473	0.0017	0.0205	−0.612	0.463%
Baseline steady-state wage						
Rule ($\psi^w = 1$, $\psi^v = -1$)	0.0015	0.0051	0.0007	0.021	0.498	–
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0060	0.0218	0.0022	0.021	0.473	–
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0047	0.0147	0.0022	0.021	0.480	–

that this result is not general and that the presence of public sector employment affects unemployment volatility in a manner crucially dependant on the government's business cycle policy. Second, they argue that procyclical public sector employment is the best policy to stabilize total employment, but this is not the optimal policy. In their model, the government does not choose wages optimally, in either the steady state or along the business cycle. Procyclical employment can be optimal if public sector wages are not. But if they are, public sector vacancies and employment should be counter-cyclical.

The last column presents the welfare cost of business cycles in the different scenarios.[11] When the public sector is absent, fluctuations have a very small welfare cost – approximately 0.028 percent of steady-state consumption. This is a well known result. When the government is present and behaves optimally, the fluctuations have lower welfare costs, but under the two rules, the cost can be up to four times higher. Welfare costs are higher when the goods are complements, as a negative technology shock has further negative effects on the marginal utility of the public good. Optimal business cycle policy requires the policy to remain optimal in the steady state. Outside the efficient steady state, characterizing the optimal business cycle policy is difficult, but we can analyse the volatility of key variables

[11] See the companion appendix for details.

depending on business cycle policy. The last panel of Table 2 shows the case for the baseline steady state. In the two scenarios with acyclical public sector wages, unemployment volatility is three times higher than when the public sector wages respond one-to-one to the private sector.

In a final robustness exercise, I investigate if these conclusions change in the presence of rigid private sector wages. I considered two alternative methods of modelling: one following Hall (2005) and the second following Blanchard and Galí (2010). In both cases, rigidities are introduced by assuming an ad-hoc expression for wages. Full details and results are shown in the companion appendix. Compared to the flexible wage benchmark, unemployment rate volatility increases three times in the case of Hall (2005) specification and 15 times in the case of Blanchard and Galí (2010). In both cases, we observe reduced unemployment volatility when government follows a procyclical wage policy, but by less than the benchmark case. The standard deviation of unemployment rate decreases by only 10 percent. Rigid wages reduce the cost of not following a procyclical public wage policy. Although wage rigidity has been proposed in the literature as one solution to the Shimer Puzzle, its relevance is still under discussion. For the main mechanism of the model, only the wages of new-hires are relevant in the decisions. As been argued by Pissarides (2009), microeconometric evidence suggests that wages in new matches are more procyclical and volatile than average wages.

6 Directed versus random search

6.1 Evidence for Directed search

The theoretical model presented in this paper has two important policy prescriptions. First, government wages in the steady state should not be too high relative to the private sector. Otherwise they generate higher equilibrium unemployment. Second, government wages should track private sector wages over the business cycle or unemployment volatility will be higher due to fluctuations in the share of unemployed searching for public sector jobs. An important part of the mechanism depends on the assumption of directed search.

According to micro evidence, this assumption is very realistic. As mentioned previously, public sector wage premium substantially varies within groups. As reported in Gregory and Borland (1999), the premium is much higher for females, veterans and minorities, and is higher for federal government employees compared to state or local government employees. Differences also exist across education levels. Katz and Krueger (1991) find that in the previous two decades, more educated individuals tend to be paid less in the public sector, while individuals with less education tend to receive a higher premium. If people can direct their search, these differences should have repercussions.

Gregory and Borland (1999) report a number of studies that found the existence of queues for federal public jobs. For example, Venti (1985) finds that for each federal government job opening, 2.8 men and 6.1 times as many women want the job. Katz and Krueger (1991) find that blue collar workers are willing to queue to obtain public sector jobs, whereas the public sector has difficulty in recruiting and retaining highly skilled workers. Postel-Vinay and Turon (2007) also finds evidence in the United Kingdom of low-employability individuals queuing for public sector jobs, as they stand to gain larger potential premia from working there.

Most studies that estimate the public sector wage premium use switching regression models, which posit that the unemployed can self-select to work in the sectors offering more advantages. Blank (1985) finds that, among other factors, sectoral choice is influenced by wage comparison. Heitmueller (2006) manages to quantify this effect and finds that an increase of 1 percent in the public sector's expected wage increases the probability employment in that sector by 1.3 percent for men and 2.9 percent for women.

The micro evidence supports the directed search assumption in the steady state. However, over the business cycle, as the public sector premium is different for subgroups based on characteristics that do not change over the cycle, small changes in private sector wages might not be sufficient to change aggregate search patterns. Note that the successful functioning of this mechanism does not require every unemployed to swing between sectors, as long as a small number at the margin are so able. Anecdotal evidence presented in the introduction already suggests that the same mechanism is also relevant over the business cycle.

6.2 The case with random search

Although directed search is an important part of the mechanism, the main results are also robust if we consider search as being random. Consider the following specification in which the unemployed do not have a choice where to search:

$$m_t^p + m_t^g = \mu(u_t)^\eta (v_t^p + v_t^g)^{1-\eta}, \tag{27}$$

$$\frac{v_t^g}{v_t^p} = \frac{m_t^g}{m_t^p}. \tag{28}$$

There is one matching function that has unemployment and the total number of vacancies as inputs. In this setting, public sector wages do not have any externality on the matching function, only public sector vacancies. However, public sector wages still affect the value of unemployment and, hence, the outside option of the worker when bargaining.

$$U_t = \frac{\nu_u(u_t)}{u_c(c_t, g_t)} + E_t\beta_{t,t+1}[f_t^p W_{t+1}^p + f_t^g W_{t+1}^g + (1 - f_t^g - f_t^p)U_{t+1}]. \tag{29}$$

In this setting, optimality only requires optimal vacancies in both sectors. The government can set the optimal level of government vacancies directly, incorporating the resulting externality. As there is one less variable for efficiency, the government can now use public sector wages as an instrument to achieve the optimal level of private vacancies. This means that, for any bargaining power of the worker (even if the Hosios condition is not satisfied), only one public sector wage will achieve the optimal level of private sector vacancies. As with directed search, higher public sector wages imply both higher wages and lower vacancies in the private sector, and hence higher unemployment. This is also true over the business cycle. Unemployment volatility is higher if public wages are kept constant. In essence, directed search amplifies the costs of not following the optimal policy through the endogenous decision of where to search.[12]

7 Conclusion

This paper examines the links between public and the private sectors through the labour market. The main normative conclusion is that government wage policy plays a key role in attaining efficient allocation of jobs across sectors. In the steady state, the optimal public sector wage premium should reflect differences in labour market friction parameters. Although other reasons exist for governments to set wages, namely to induce effort or avoid corruption, they should weigh the cost of such an action in terms of labour market inefficiency, as higher wages relative to the private sector induce queues and higher equilibrium unemployment.

Over the business cycle, public sector wages should follow those in the private sector. Otherwise, in recessions, too many people queue for public sector jobs whilst in expansions, few people apply for them. Although I have abstracted from financing issues, a procyclical public sector wage has the advantage of requiring a lower tax burden in recessions. However, it also has drawbacks. First, lowering public sector wages in recessions might prove politically difficult to implement. Yet, to achieve efficiency in the labour market, the only relevant wages are those of new hires, which are potentially easier to reduce in recessions.

[12] In the companion appendix, I present details of the model with random search, the social planner solution and the results of some simulations.

Second, it does not seem plausible that government can vary wage from quarter to quarter. However, indexing annual wage increases to the evolution of their private sector counterparts is possible. Finally, I have ignored the insurance role of the government. If agents are risk averse, they would prefer to have a constant income profile throughout the business cycle, which is an argument for acyclical wages. While this line of reasoning is valid, one has to realise that the intertemporal insurance is achieved at the cost of greater fluctuations in unemployment.

The results on optimal policy are relevant in two ways. First, studies of optimality in fiscal policy have primarily focused on the taxation side because on the spending side, it is harder to dissociate optimal policy from preferences. In the model, government policy on vacancies and employment faces the same problem, which is why I give it little relevance. On the other hand, the results of government wages essentially depend on the labour market, which allow us to develop a more robust theory of optimal spending, at least for one of its components.

The result is considered relevant because despite being simple and intuitive, optimal business cycle policy does not seem to be acknowledged by policy makers who view government wages as a stabilization tool. In a recent occasional paper from the European Central Bank (Holm-Hadulla, Kamath, Lamo, Pérez, and Schuknecht (2010)), the authors argue that the government should avoid mild wage procyclicality, as increasing wages in expansions might boost aggregate demand and amplify the business cycle. As we have seen, this policy could heavily distort the labour market. To stabilise demand, the government can use either employment, purchases of intermediate goods, investment or transfers, but leave the wage to promote efficiency in the labour market.

References

Albrecht, J., L. Navarro, and S. Vroman (2013): "Public Sector Employment in an Equilibrium Search and Matching Model," Discussion paper.

Ardagna, S. (2007): "Fiscal policy in unionized labor markets," *Journal of Economic Dynamics and Control*, 31(5), 1498–1534.

Barro, R. J. (1990): "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98(5), S103–26.

Baxter, M., and R. G. King (1993): "Fiscal Policy in General Equilibrium," *American Economic Review*, 83(3), 315–34.

Blanchard, O., and J. Galí (2010): "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2(2), 1–30.

Blank, R. M. (1985): "An analysis of workers' choice between employment in the public and private sectors," *Industrial and Labor Relations Review*, 38(2), 211–224.

Bradley, J., F. Postel-Vinay, and H. Turon (2013): "Public sector wage policy and labour market equilibrium: a structural model," Discussion paper.

Burdett, K. (2012): "Towards a theory of the labor market with a public sector," *Labour economics*, 19(1), 68–75.

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39(2), 257–73.

CIPD (2009): "Recruitment, retention and turnover survey report," Available at http://www.cipd.co.uk/subjects/recruitmen/general/.

Davis, S. J., R. J. Faberman, and J. C. Haltiwanger (2013): "The Establishment-Level Behavior of Vacancies and Hiring," *The Quarterly Journal of Economics*, 128(2), 581–622.

Devereux, P. J., and R. A. Hart (2006): "Real wage cyclicality of job stayers, within-company job movers, and between-company job movers," *Industrial and Labor Relations Review*, 60(1), 105–119.

Evans, P., and G. Karras (1998): "Liquidity Constraints and the Substitutability between Private and Government Consumption: The Role of Military and Non-military Spending," *Economic Inquiry*, 36(2), 203–14.

Finn, M. G. (1998): "Cyclical Effects of Government's Employment and Goods Purchases," *International Economic Review*, 39(3), 635–57.

Fiorito, R., and T. Kollintzas (2004): "Public goods, merit goods, and the relation between private and government consumption," *European Economic Review*, 48(6), 1367–1398.

Galí, J., J. D. López-Salido, and J. Vallés (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5(1), 227–270.

Gregory, R. G., and J. Borland (1999): "Recent developments in public sector labor markets," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3, chap. 53, pp. 3573–3630. Elsevier.

Hall, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

Heitmueller, A. (2006): "Public-private sector pay differentials in a devolved Scotland," *Journal of Applied Economics*, IX, 295–323.

Holm-Hadulla, F., K. Kamath, A. Lamo, J. J. Pérez, and L. Schuknecht (2010): "Public wages in the euro area - towards securing stability and competitiveness," Occasional Paper Series 112, European Central Bank.

Holmlund, B., and J. Linden (1993): "Job matching, temporary public employment, and equilibrium unemployment," *Journal of Public Economics*, 51(3), 329–343.

Hörner, J., L. R. Ngai, and C. Olivetti (2007): "Public Enterprises And Labor Market Performance," *International Economic Review*, 48(2), 363–384.

Katz, L. F., and A. B. Krueger (1991): "Changes in the Structure of Wages in the Public and Private Sectors," NBER Working Papers 3667.

Lamo, A., J. J. Pérez, and L. Schuknecht (2008): "Public and private sector wages - co-movement and causality," Working Paper Series 963, European Central Bank.

Lane, P. R. (2003): "The cyclical behaviour of fiscal policy: evidence from the OECD," *Journal of Public Economics*, 87(12), 2661–2675.

Linnemann, L., and A. Schabert (2003): "Fiscal Policy in the New Neoclassical Synthesis," *Journal of Money, Credit and Banking*, 35(6), 911–29.

Merz, M. (1995): "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, 36(2), 269–300.

Pappa, E. (2009): "The Effects Of Fiscal Shocks On Employment And The Real Wage," *International Economic Review*, 50(1), 217–244.

Petrongolo, B., and C. A. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), 390–431.

Pissarides, C. A. (2000): *Equilibrium unemployment.* MIT press, 2nd edn.

Pissarides, C. A. (2009): "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*, 77(5), 1339–1369.

Postel-Vinay, F., and H. Turon (2007): "The Public Pay Gap in Britain: Small Differences That (Don't?) Matter," *Economic Journal*, 117(523), 1460–1503.

Quadrini, V., and A. Trigari (2007): "Public Employment and the Business Cycle," *Scandinavian Journal of Economics*, 109(4), 723–742.

Russo, G., W. Hassink, and C. Gorter (2005): "Filling vacancies: an empirical analysis of the cost and benefit of search in the labour market," *Applied Economics*, 37(14), 1597–1606.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

Venti, S. (1985): "Wages in the Federal and Private Sectors," NBER Working Papers 1641.

COMPANION APPENDIX

<u>FOR ONLINE PUBLICATION</u>

Optimal public sector wages

Pedro Gomes

I - Data

Table A1: Public sector and the labour market in OECD countries

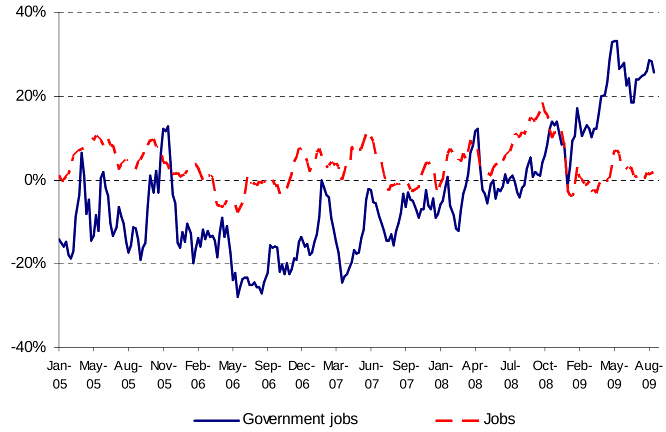
	Public wage bill (% gov. consumption)	Public Employment (% total employment)	Unemployment rate	Correlation (u_t, l_t^g)
Australia	52.2%	14.1%	6.3%	0.51
Austria	53.4%	13.1%	4.7%	0.34
Belgium	53.8%	17.9%	6.9%	0.91
Canada	59.8%	20.5%	6.8%	0.55
Denmark	67.8%	30.5%	4.4%	0.78
Finland	63.2%	24.8%	9.9%	0.76
France	58.4%	22.5%	9.4%	0.95
Germany	41.5%	11.6%	7.5%	0.82
Iceland	60.0%	19.0%	2.3%	0.74
Ireland	57.0%	12.7%	4.3%	0.84
Italy	55.6%	16.9%	10.7%	-0.40
Japan	37.7%	8.4%	4.7%	0.35
Luxembourg	49.1%	15.0%	2.6%	0.88
Netherlands	42.2%	10.9%	2.6%	0.80
Norway	63.1%	33.6%	3.4%	0.82
Portugal	72.8%	14.3%	4.0%	0.22
Spain	59.2%	14.1%	11.4%	0.13
Sweden	59.2%	31.1%	4.7%	0.33
United Kingdom	53.3%	18.0%	5.5%	0.19
United States	66.5%	15.2%	4.1%	0.66
Average	56.3%	18.2%	5.9%	0.49

Note: Public wage bill, public employment and unemployment rate refer to the year 2000. The correlation between public sector employment and the unemployment rate is computed from quarterly data (1970 to 2007). Source: OECD.

Table A2: Data on cost of recruiting - CIPD

	All sectors	**Manufacturing and production**	**Private sector services**	**Public services**	**Voluntary and not-for-profit**
Cost of recruiting (£)					
Senior managers	15123	13396	18964	10452	8534
Managers and professionals	9738	8050	12393	6067	6471
Administrative, secretarial and technical	4519	3680	5628	1935	4976
Services (costumer, personal and sales)	8996	4565	13980	2327	1399
Manual, craft workers	2381	2498	2978	1898	1379
Time to fill a vacancy (weeks)					
Senior managers	17.1	16.8	16.5	18	16.6
Managers and professionals	12.5	12.1	11.8	14.3	11.8
Administrative, secretarial and technical	6.5	6.0	7.1	9.1	7.1
Services (costumer, personal and sales)	7.0	6.7	5.6	9.9	7.4
Manual, craft workers	5.9	5.2	4.5	8.3	6.3

Figure A1: Growth rate of *Google* keyword searches in the United Kingdom

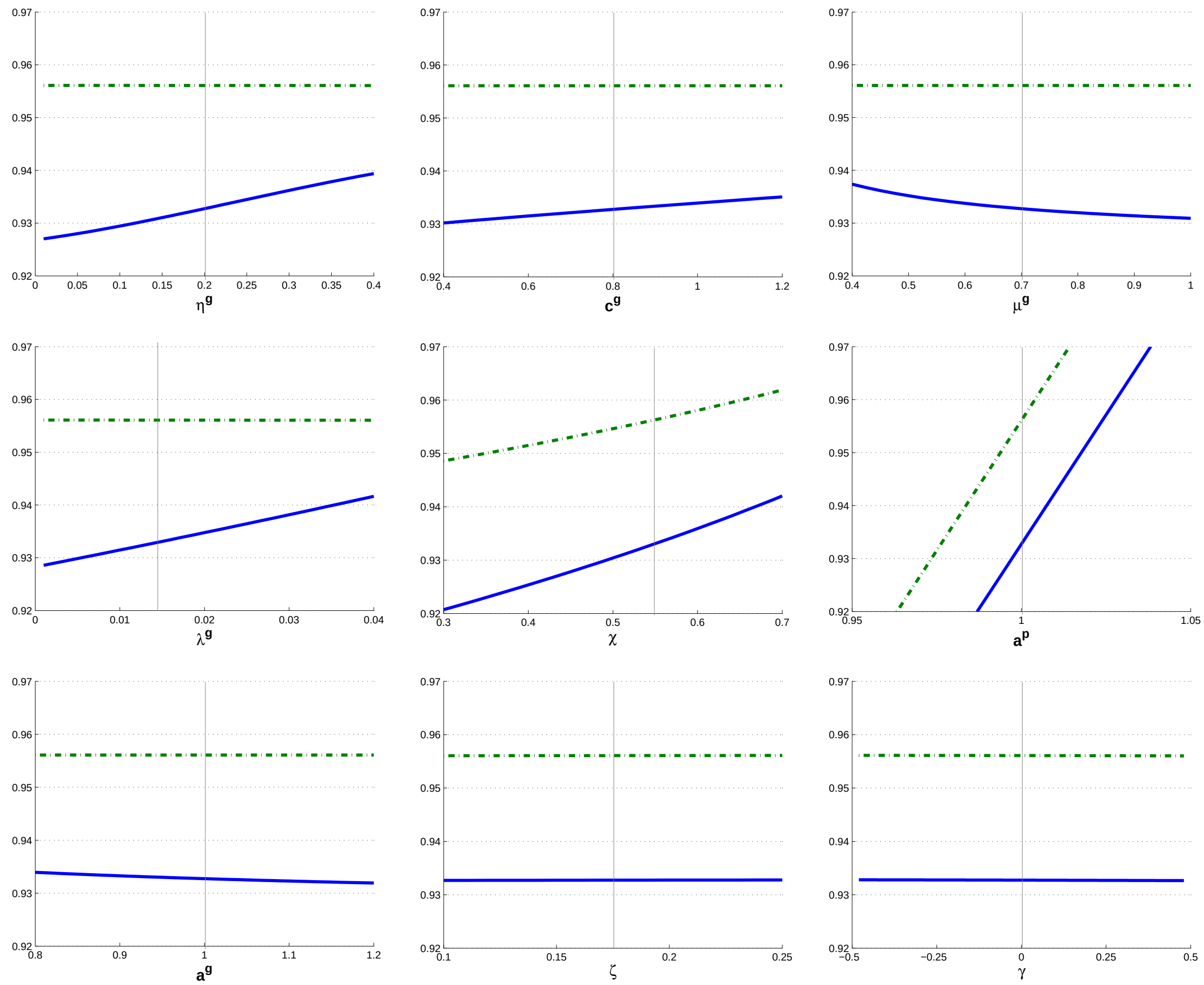


Note: The growth rate of the four-weeks average index of keyword searches, relative the same four weeks in the previous year.

II - Steady-state optimal wages, search and unemployment

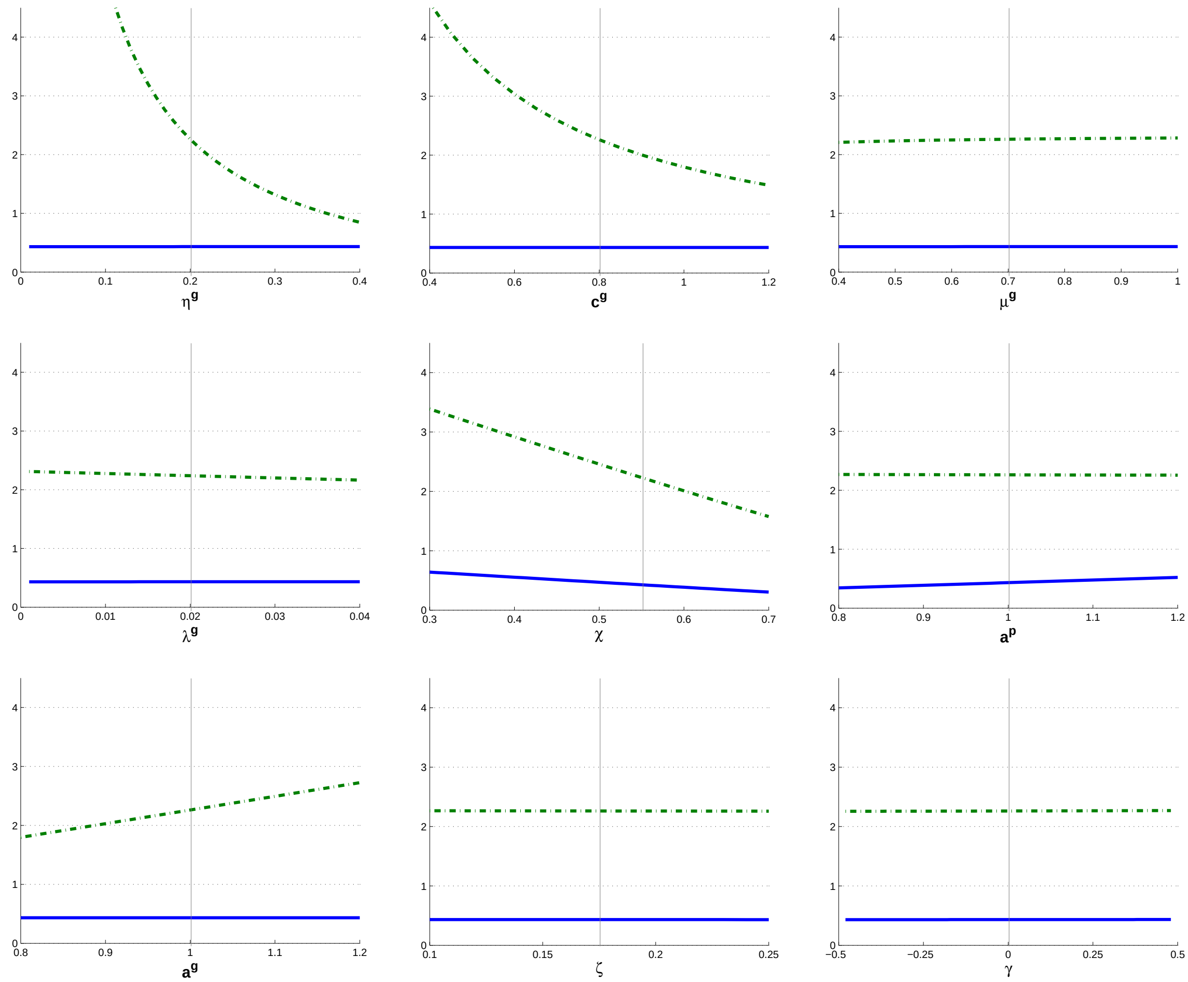
Figures A2-A6 show the optimal steady-state wages in both sectors, the optimal tightness, the optimal the share of unemployed searching in the public sector, the optimal public sector employment and the unemployment rate as a function of the labour market friction parameters in the public sector, as well as the technology and utility function parameters. Figure A7 shows how these variables change with public sector wages.

Figure A2: Optimal steady-state public and private sector wages



Note: The solid line is the optimal public sector wage and the dash line is the optimal private sector wages.

Figure A3: Optimal steady-state public and private tightness



Note: The solid line is the optimal private sector tightness and the dash line is the optimal public sector tightness (vacancies over unemployed searching in the sector).

Figure A4: Optimal steady-state share of unemployed searching in the public sector

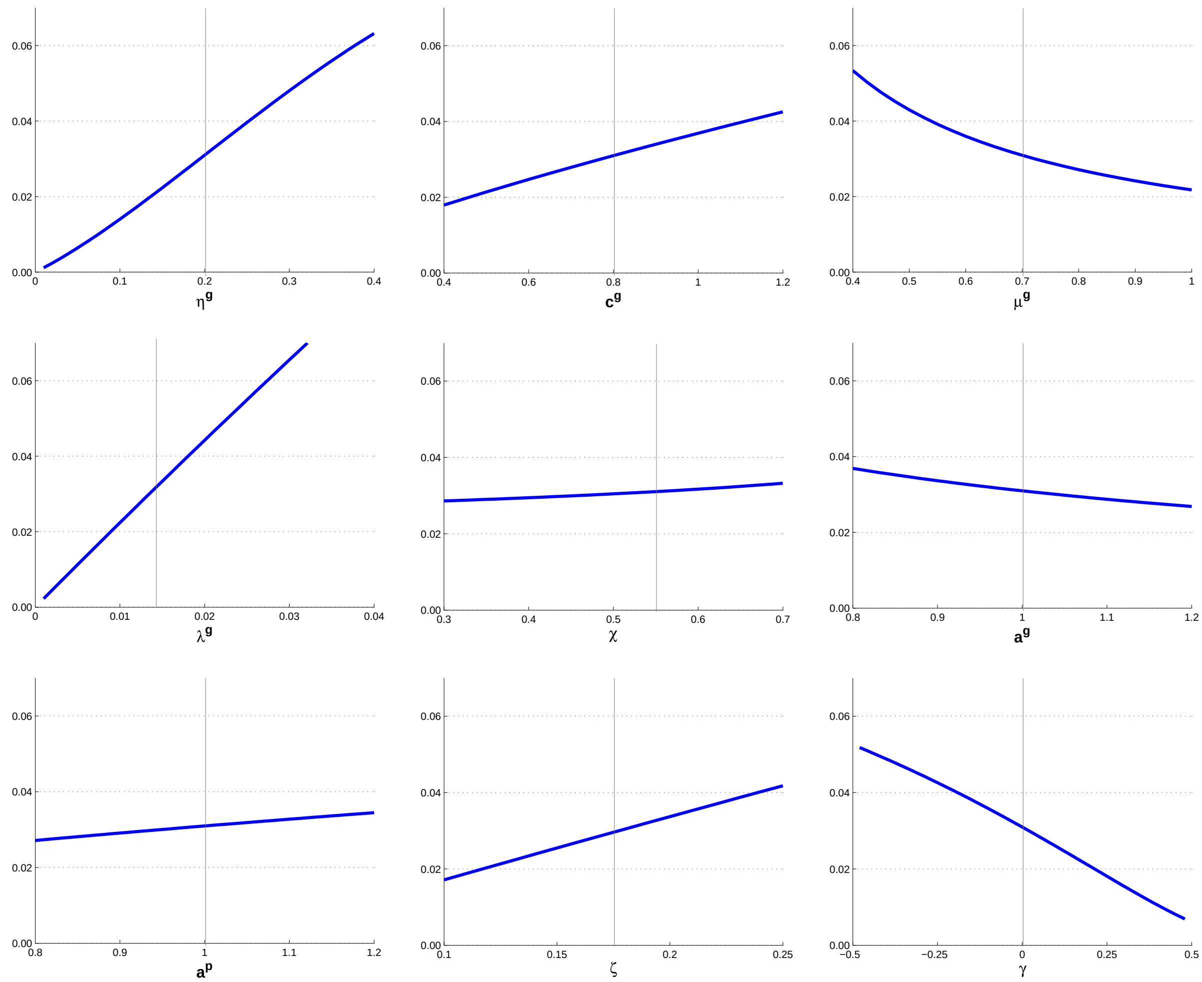


Figure A5: Optimal steady-state level of public sector employment

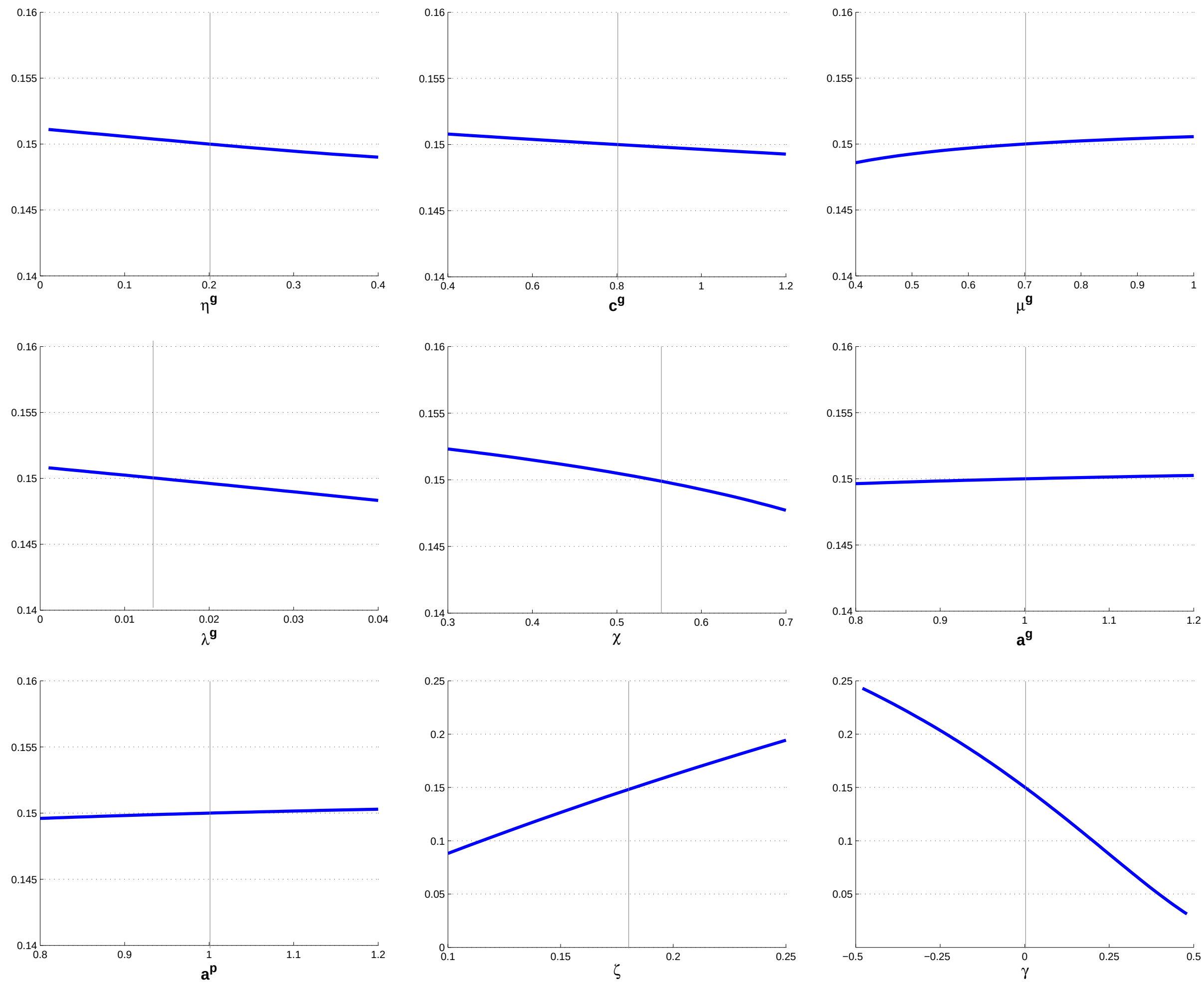


Figure A6: Optimal steady-state unemployment rate

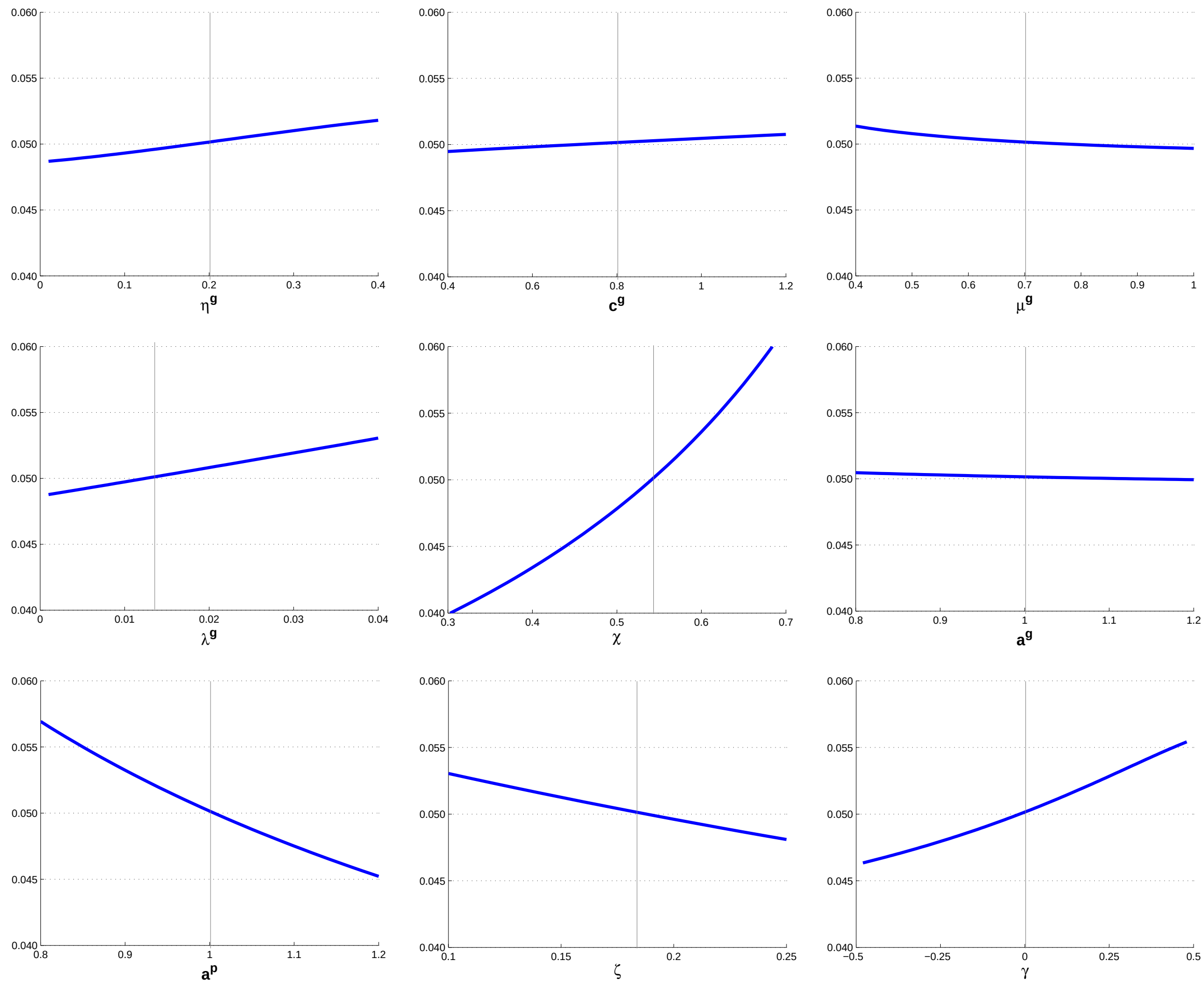
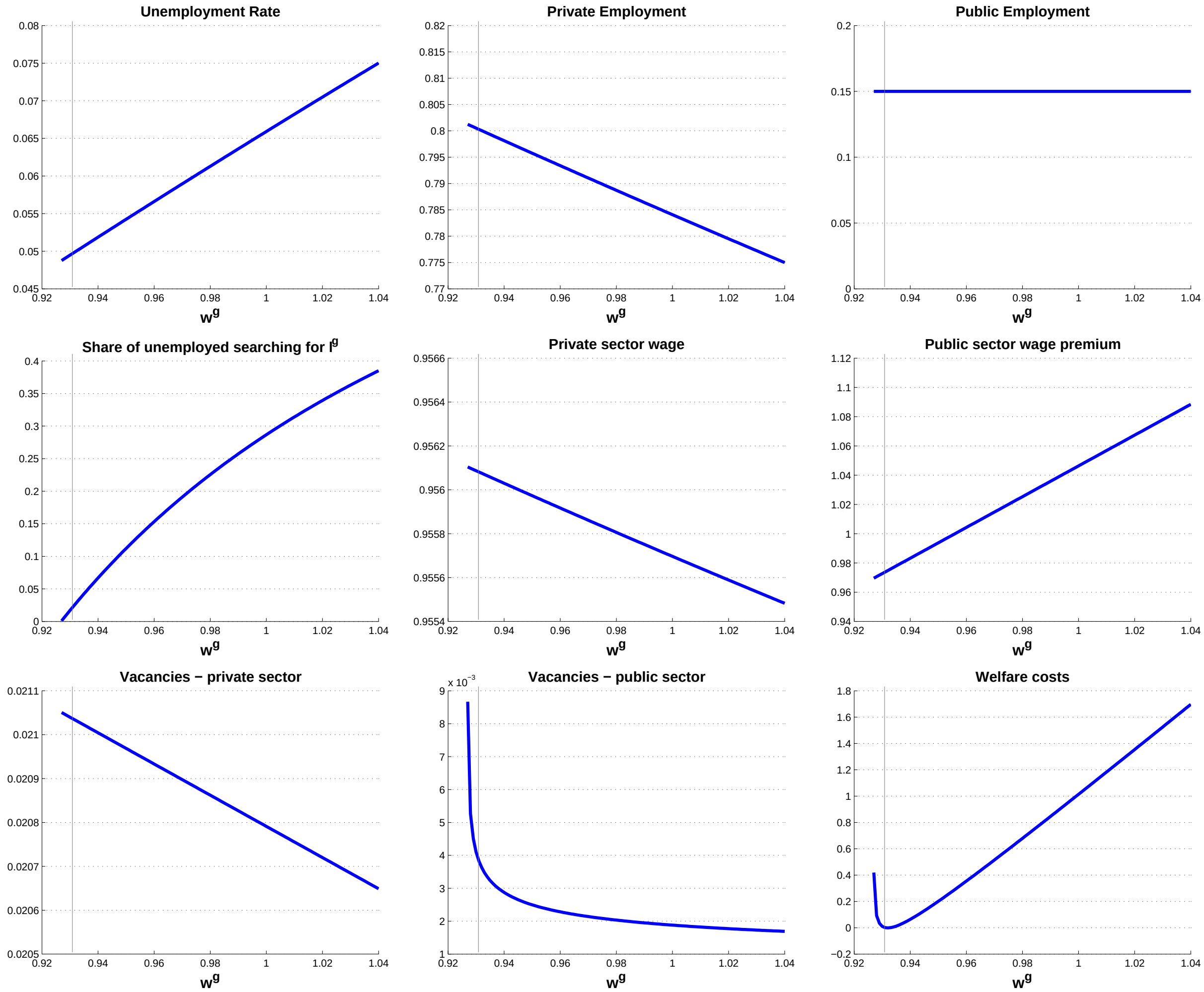
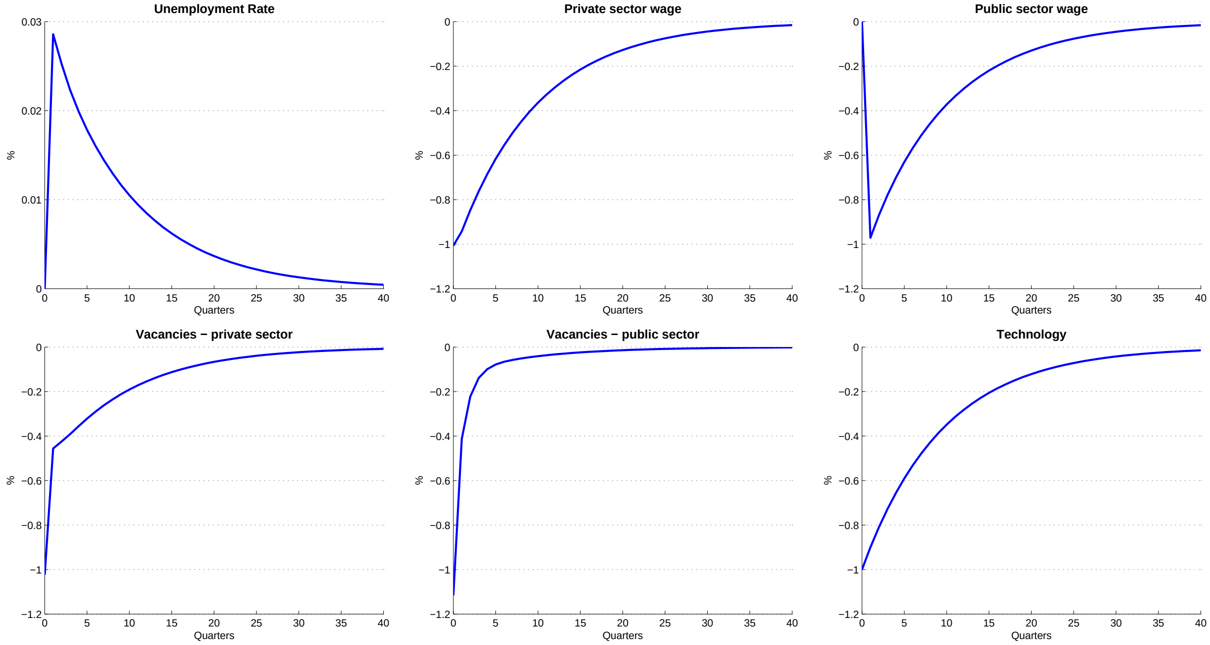


Figure A7: Cost of higher public sector wages



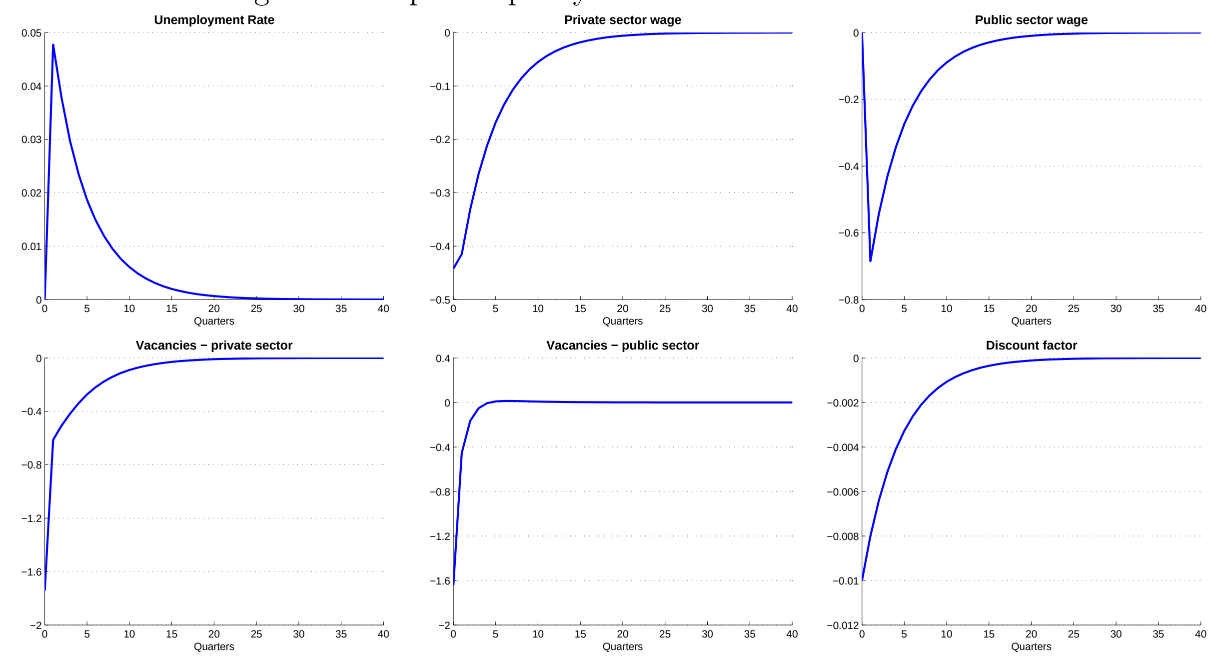
III - Optimal policy under alternative sources of fluctuations

Figure A8: Optimal policy under an economy-wide technology shock



Note: The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.

Figure A9: Optimal policy under a discount factor shock



Note: The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.

IV - Wage rigidity

Two specifications

I considered two alternative ways of modeling wage rigidity: one following Hall (2005) and the second by Blanchard and Galí (2010). In both cases the rigidities are introduced by assuming an ad-hoc expression for private sector wages.

Hall (2005): $w_t^p = \varrho w_{t-1}^p + (1-\varrho) w_t^*$

Blanchard and Galí (2010): $w_t^p = \bar{w}^p (A_t^p)^\iota$

In the first specification, the actual wage is a weighted average from the wage in the previous period and the wage that would follow from the Nash bargaining, w_t^*. In the second specification wage responds to changes in productivity but with an elasticity of ι. If $\iota = 0$ if would be the case of constant private wages. Table A3 shows the business cycle properties with wage rigidity. In all cases, constant public sector wages increase the volatility of unemployment relative to the procyclical public sector wages.

Table A3: Business cycle properties with wage rigidity

Policy	Standard deviations				Correl	Welfare
	l_t^p	l_t^g	u_t	w_t^p	(l_t^g, u_t)	cost
Baseline steady-state wage						
Rule ($\psi^w = 1$, $\psi^v = -1$)	0.0015	0.0051	0.0007	0.021	0.498	–
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0060	0.0218	0.0022	0.021	0.473	–
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0047	0.0147	0.0022	0.021	0.480	–
Rigid private wages (Hall (2005) $\varrho = 0.90$)						
Rule ($\psi^w = 1$, $\psi^v = -1$)	0.0048	0.0146	0.0024	0.0478	0.4263	–
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0036	0.0072	0.0025	0.0458	0.2065	–
Rule ($\psi^w = 1$, $\psi^v = 0$)	0.0028	0.0020	0.0021	0.0470	0.5123	–
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0035	0.0147	0.0023	0.0450	−0.1756	–
Rigid private wages (Blanchard and Galí (2010) $\iota = 0.5$)						
Rule ($\psi^w = 1$, $\psi^v = -1$)	0.0257	0.0873	0.0103	0.010	0.515	–
Rule ($\psi^w = 0$, $\psi^v = -1$)	0.0281	0.0963	0.0112	0.010	0.514	–
Rule ($\psi^w = 1$, $\psi^v = 0$)	0.0143	0.0070	0.0093	0.010	0.535	–
Rule ($\psi^w = 0$, $\psi^v = 0$)	0.0125	0.0139	0.0101	0.010	0.537	–

V - Derivations

Social planner's problem

The social planner maximises the consumer's utility (4) subject to the technology constraints (16) and (11) and the labour market frictions (1)-(3). Setting up the Lagrangian:

$$\begin{aligned}&\sum_{k=0}^{\infty}\beta^{t+k}\{u(a^p_{t+k}l^p_{t+k}-\varsigma^p v^p_{t+k},a^g_{t+k}l^g_{t+k}-\varsigma^g v^g_{t+k})+\nu(1-l^p_{t+k}-l^g_{t+k})\\&-\Omega^1_{t+k}[l^p_{t+k+1}-(1-\lambda^p)l^p_{t+k}-\mu^p((1-s_{t+k})(1-l^p_{t+k}-l^g_{t+k}))^{\eta^p}(v^p_{t+k})^{1-\eta^p}]\\&-\Omega^2_{t+k}[l^g_{t+k+1}-(1-\lambda^g)l^g_{t+k}-\mu^g(s_{t+k}(1-l^p_{t+k}-l^g_{t+k}))^{\eta^g}(v^g_{t+k})^{1-\eta^g}]\}.\end{aligned}$$

The first order conditions are given by:

$$\begin{aligned}v^c_t &: u_c(c_t,g_t)\varsigma^p=\Omega^1_t(1-\eta^p)q^p_t\\ v^g_t &: u_g(c_t,g_t)\varsigma^g=\Omega^2_t(1-\eta^g)q^g_t\\ s_t &: \frac{\Omega^2_t\eta^g m^g_t}{s_t}=\frac{\Omega^1_t\eta^p m^p_t}{1-s_t}\\ l^p_{t+1} &: \Omega^1_t=\beta\{a^p_{t+1}u_c(c_{t+1},g_{t+1})-\nu_u(u_{t+1})+\Omega^1_{t+1}(1-\lambda^p)-\Omega^1_{t+1}\eta^p\frac{m^p_{t+1}}{u_{t+1}}-\Omega^2_{t+1}\eta^g\frac{m^g_{t+1}}{u_{t+1}}\}\\ l^g_{t+1} &: \Omega^2_t=\beta\{a^g_{t+1}u_g(c_{t+1},g_{t+1})-\nu_u(u_{t+1})+\Omega^2_{t+1}(1-\lambda^g)-\Omega^1_{t+1}\eta^p\frac{m^p_{t+1}}{u_{t+1}}-\Omega^2_{t+1}\eta^g\frac{m^g_{t+1}}{u_{t+1}}\}.\end{aligned}$$

Plugging the first two equations in the third one gives the implicit expression for optimal level of search in each sector:

$$\frac{u_g(c_t,g_t)\varsigma^g\eta^g v^g_t}{(1-\eta^g)s_t}=\frac{u_c(c_t,g_t)\varsigma^p\eta^p v^p_t}{(1-\eta^p)(1-s_t)}.$$

If we rewrite the third condition as $\Omega^2_t\eta^g m^g_t+\Omega^1_t\eta^p m^p_t=\frac{\Omega^2_t\eta^g m^g_t}{s_t}=\frac{\Omega^1_t\eta^p m^p_t}{1-s_t}$, we can use it to simplify the last two conditions and get:

$$\begin{aligned}\frac{\varsigma^p}{q^p_t} &= \beta\frac{u_c(c_{t+1},g_{t+1})}{u_c(c_t,g_t)}\{(1-\eta^p)a^p_{t+1}-(1-\eta^p)\frac{\nu_u(u_{t+1})}{u_c(c_{t+1},g_{t+1})}+(1-\lambda^p)\frac{\varsigma^p}{q^p_{t+1}}-\frac{\eta^p\varsigma^p v^p_{t+1}}{(1-s_{t+1})u_{t+1}}]\},\\ \frac{\varsigma^g}{q^g_t} &= \beta\frac{u_g(c_{t+1},g_{t+1})}{u_g(c_t,g_t)}\{(1-\eta^g)a^g_{t+1}-(1-\eta^g)\frac{\nu_u(u_{t+1})}{u_g(c_{t+1},g_{t+1})}+(1-\lambda^g)\frac{\varsigma^g}{q^g_{t+1}}-\frac{\eta^g\varsigma^g v^g_{t+1}}{s_{t+1}u_{t+1}}]\}.\end{aligned}$$

Proof of Proposition 1

Plugging the steady-state expressions for the value of job, unemployment and employment in the Nash sharing rule gives us:

$$(1-b)\frac{\bar{w}^p-\frac{\nu_u}{u_c}}{1-\beta(1-\lambda^p-\frac{\bar{m}^p}{(1-\bar{s})\bar{u}})}=b\frac{\bar{a}^p-\bar{w}^p}{1-\beta(1-\lambda^p)}.$$

The decision rule for private sector vacancies is given by the free-entry condition of firms:

$$\frac{\varsigma^p}{\beta\bar{q}^p}(1-\beta(1-\lambda^p)) = [\bar{a}^p - \bar{w}^p].$$

Combining the two equations using $(\bar{a}^p - \bar{w}^p)$ we get the following expression:

$$\bar{w}^p - \frac{\nu_u}{u_c} = (1-\beta(1-\lambda^p - \frac{\bar{m}^p}{(1-\bar{s})\bar{u}}))\frac{b\varsigma^p}{(1-b)\bar{q}^p\beta}.$$

Adding it to the free-entry condition:

$$[\bar{a}^p - \frac{\nu_u}{u_c}] = (1-\beta(1-\lambda^p - \frac{\bar{m}^p}{(1-\bar{s})\bar{u}}))\frac{b\varsigma^p}{(1-b)\bar{q}^p\beta} + \frac{\varsigma^p}{\beta\bar{q}^p}(1-\beta(1-\lambda^p)).$$

We can simplify it into

$$[\bar{a}^p - \frac{\nu_u}{u_c}] = (1-\beta(1-\lambda^p))\frac{\varsigma^p}{(1-b)\bar{q}^p\beta} + \frac{b\varsigma^p\bar{m}^p}{(1-\bar{s})\bar{u}(1-b)\bar{q}^p},$$

which can be re-written as:

$$(1-\beta(1-\lambda^p))\frac{\varsigma^p}{\bar{q}^p} = \beta[(1-b)(\bar{a}^p - \frac{\nu_u}{u_c}) - \frac{b\varsigma^p\bar{v}^p}{(1-\bar{s})\bar{u}}].$$

This is equivalent to the social planner's first order condition for private vacancies if $b = \eta^p$.

Welfare costs of high public sector wages in steady-state

Let $\{c_{opt}, g_{opt}, u_{opt}\}$ be the steady-state private and government consumption, and unemployment under the optimal public sector wage. The $\{\bar{c}, \bar{g}, \bar{u}\}$ is the allocation under an exogenous public sector wage. We want to find what is the welfare gain as a percentage of steady-state private consumption of having the optimal steady-state public sector wage (Section 4). This is given by x that solves the following equation:

$$u(c_{opt}, g_{opt}) + \nu(u_{opt}) = u((1+x)\bar{c}, \bar{g}) + \nu(\bar{u}).$$

Using the utility function:

$$x = \frac{[\exp[\ln(c_{opt}^\gamma + \zeta g_{opt}^\gamma) + \gamma\chi(u_{opt} - \bar{u})] - \zeta\bar{g}^\gamma]^{\frac{1}{\gamma}}}{\bar{c}} - 1, \gamma \neq 0.$$

If $\gamma = 0$, the utility function is not defined, so I use the equivalent $u(c_t, g_t) = \ln(c_t) + \zeta\ln(g_t)$. The welfare cost in terms of steady state consumption is then given by:

$$x = \frac{\exp[\ln(c_{opt}) + \zeta(\ln g_{opt} - \ln\bar{g}) + \chi(u_{opt} - \bar{u})]}{\bar{c}} - 1, \gamma = 0.$$

Welfare costs of business cycles

In Section 5 I show the welfare costs of business cycles under different policies for $\{v_t^g, w_t^g\}$. Let us start by defining the variables in log-deviations from the steady-state:

$$\begin{array}{ll} \tilde{c}_t = \log(\frac{c_t}{\bar{c}}) & c_t = \bar{c}\exp(\tilde{c}_t) \\ \tilde{g}_t = \log(\frac{g_t}{\bar{g}}) & g_t = \bar{g}\exp(\tilde{g}_t) \\ \tilde{u}_t = \log(\frac{u_t}{\bar{u}}) & u_t = \bar{u}\exp(\tilde{u}_t). \end{array}$$

If we do a second-order approximation to the variables around the steady state $\{\bar{c}, \bar{g}, \bar{u}\}$:

$$\begin{aligned} c_t &= \bar{c}(1 + \tilde{c}_t + \tfrac{1}{2}\tilde{c}_t^2) + o(3), \\ g_t &= \bar{g}(1 + \tilde{g}_t + \tfrac{1}{2}\tilde{g}_t^2) + o(3), \\ u_t &= \bar{u}(1 + \tilde{u}_t + \tfrac{1}{2}\tilde{u}_t^2) + o(3). \end{aligned}$$

The second-order approximation of the utility function gives:

$$\begin{aligned} U(c_t, g_t, u_t) \;=\;& U(\bar{c}, \bar{g}, \bar{u}) + U_c(\bar{c}, \bar{g}, \bar{u})[c_t - \bar{c}] + U_g(\bar{c}, \bar{g}, \bar{u})[g_t - \bar{g}] + U_u(\bar{c}, \bar{g}, \bar{u})[u_t - \bar{u}] + \\ & \frac{1}{2}U_{cc}(\bar{c}, \bar{g}, \bar{u})[c_t - \bar{c}]^2 + \frac{1}{2}U_{gg}(\bar{c}, \bar{g}, \bar{u})[g_t - \bar{g}]^2 + \frac{1}{2}U_{uu}(\bar{c}, \bar{g}, \bar{u})[u_t - \bar{u}]^2 + \\ & U_{cg}(\bar{c}, \bar{g}, \bar{u})[c_t - \bar{c}][g_t - \bar{g}] + U_{cu}(\bar{c}, \bar{g}, \bar{u})[c_t - \bar{c}][u_t - \bar{u}] + U_{gu}(\bar{c}, \bar{g}, \bar{u})[g_t - \bar{g}][u_t - \bar{u}] + o(3). \end{aligned}$$

But for it to be correct, we have to plug in the second-order approximation of the variables. Given the additive separability of the utility functions, we can drop the cross-terms between the consumption goods and unemployment.

$$\begin{aligned} U(c_t, g_t, u_t) \;=\;& U(\bar{c}, \bar{g}, \bar{u}) + U_c(\bar{c}, \bar{g}, \bar{u})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)] + U_g(\bar{c}, \bar{g}, \bar{u})[\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)] + U_u(\bar{c}, \bar{g}, \bar{u})[\bar{u}(\tilde{u}_t + \frac{1}{2}\tilde{u}_t^2)] \\ & + \frac{1}{2}U_{cc}(\bar{c}, \bar{g}, \bar{u})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)]^2 + \frac{1}{2}U_{gg}(\bar{c}, \bar{g}, \bar{u})[\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)]^2 + \frac{1}{2}U_{uu}(\bar{c}, \bar{g}, \bar{u})[\bar{u}(\tilde{u}_t + \frac{1}{2}\tilde{u}_t^2)]^2 \\ & + U_{cg}(\bar{c}, \bar{g}, \bar{u})[\bar{c}(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2)][\bar{g}(\tilde{g}_t + \frac{1}{2}\tilde{g}_t^2)] + o(3). \end{aligned}$$

Collecting terms and substituting the derivatives,

$$\begin{aligned} U(c_t, g_t, u_t) \;=\;& U(\bar{c}, \bar{g}, \bar{u}) + u_c\bar{c}\tilde{c}_t + u_g\bar{g}\tilde{g}_t + \nu_u\bar{u}\tilde{u}_t + \\ & \frac{\bar{c}}{2}(\bar{c}u_{cc} + u_c)\tilde{c}_t^2 + \frac{\bar{g}}{2}(\bar{g}u_{gg} + u_g)\tilde{g}_t^2 - \frac{\bar{u}}{2}(\bar{u}\nu_{uu} + \nu_u)\tilde{u}_t^2 + u_{cg}(\bar{c}, \bar{g})\bar{c}\bar{g}\tilde{c}_t\tilde{g}_t + o(3). \end{aligned}$$

Taking the unconditional expectation, we can write the welfare cost in terms of the moments of the variables:

$$\begin{aligned} E[u(c_t, g_t) + \nu(u_t) - u(\bar{c}, \bar{g}) - \nu(\bar{u})] \;\approx\;& u_c\bar{c}E[\tilde{c}_t] + u_g\bar{g}E[\tilde{g}_t] + \nu_u\bar{u}E[\tilde{u}_t] + \frac{\bar{c}}{2}(\bar{c}u_{cc} + u_c)E[\tilde{c}_t^2] + \\ & \frac{\bar{g}}{2}(\bar{g}u_{gg} + u_g)E[\tilde{g}_t^2] + \frac{\bar{u}}{2}(\bar{u}\nu_{uu} + \nu_u)E[\tilde{u}_t^2] + u_{cg}(\bar{c}, \bar{g})\bar{c}\bar{g}E[\tilde{c}_t\tilde{g}_t] \equiv \Xi. \end{aligned}$$

I solve the model up to a second-order using perturbation methods and compute the moments of the variables to find the value of Ξ. To express the welfare costs as a percentage of steady-state consumption, we solve the following equation:

$$u((1-x)\bar{c},\bar{g}) - u(\bar{c},\bar{g}) = \Xi.$$

For the CES function, the derivatives are given by:

$$u_c(\bar{c},\bar{g}) = \frac{\bar{c}^{\gamma-1}}{\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma}},$$

$$u_g(\bar{c},\bar{g}) = \frac{\zeta\bar{g}^{\gamma-1}}{\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma}},$$

$$u_{cc}(\bar{c},\bar{g}) = \frac{(\gamma-1)\bar{c}^{\gamma-2}}{\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma}} - \frac{\gamma\bar{c}^{2\gamma-2}}{(\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma})^2},$$

$$u_{gg}(\bar{c},\bar{g}) = \frac{(\gamma-1)\zeta\bar{g}^{\gamma-2}}{\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma}} - \frac{\zeta^2\gamma\bar{g}^{2\gamma-2}}{(\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma})^2},$$

$$u_{cg}(\bar{c},\bar{g}) = \frac{-\gamma\zeta\bar{g}^{\gamma-1}\bar{c}^{\gamma-1}}{(\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma})^2},$$

$$\nu_u(\bar{u}) = \chi,$$

$$\nu_{uu}(\bar{u}) = 0.$$

And the expression for the welfare cost is:

$$x = 1 - \frac{\{\exp[\gamma\Xi + \ln(\bar{c}^{\gamma}+\zeta\bar{g}^{\gamma})] - \zeta\bar{g}^{\gamma}\}^{\frac{1}{\gamma}}}{\bar{c}}, \gamma \neq 0.$$

If $\gamma = 0$ the solution is given by:

$$x = 1 - \frac{\exp\{\Xi + \ln\bar{c}\}}{\bar{c}}.$$

VI - Model with random search

Setting

Equation 1-2, 4-7 and 11-17 are the same. As there is no directed search, we drop equation 10 and the matches in each sector are given by the relative vacancies:

$$m_t^p + m_t^g = \mu(u_t)^\eta (v_t^p + v_t^g)^{1-\eta}, \tag{R1}$$

$$\frac{v_t^g}{v_t^p} = \frac{m_t^g}{m_t^p}, \tag{R2}$$

$$U_t = \frac{\nu_u(u_t)}{u_c(c_t, g_t)} + E_t\beta_{t,t+1}[f_t^p W_{t+1}^p + f_t^g W_{t+1}^g + (1 - f_t^g - f_t^p)U_{t+1}]. \tag{R3}$$

Social planner's problem

The social planner maximises the consumer's utility (4) subject to the technology constraints (16) and (11) and the labour market frictions R1-R3. For convenience, instead of private and public vacancies, I set the choice variables of the government as total vacancies (v_t) and the share of vacancies that are public, defines as o. Setting up the Lagrangian:

$$\begin{aligned}
&\sum_{k=0}^{\infty} \beta^{t+k}\{u(a_{t+k}^p l_{t+k}^p - \varsigma^p v_{t+k}(1-o), a_{t+k}^g l_{t+k}^g - \varsigma^g v_{t+k} o) + \nu(1 - l_{t+k}^p - l_{t+k}^g) \\
&-\Omega_{t+k}^1[l_{t+k+1}^p - (1-\lambda^p)l_{t+k}^p - \mu(1 - l_{t+k}^p - l_{t+k}^g)^\eta (v_{t+k})^{1-\eta}(1-o)] \\
&-\Omega_{t+k}^2[l_{t+k+1}^g - (1-\lambda^g)l_{t+k}^g - \mu(1 - l_{t+k}^p - l_{t+k}^g)^\eta (v_{t+k})^{1-\eta} o]\}.
\end{aligned}$$

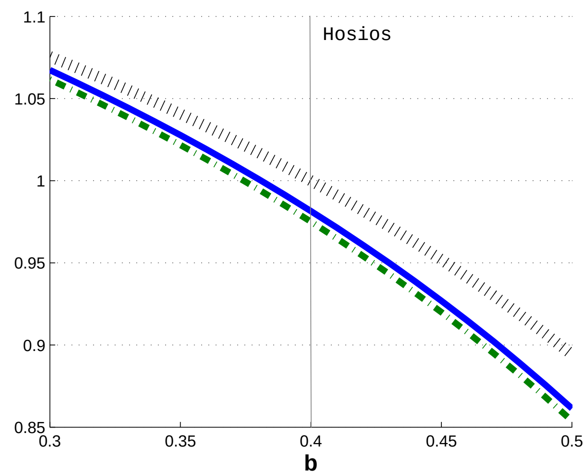
The first order conditions are given by:

$$\begin{aligned}
v_t &: \; u_c(c_t, g_t)\varsigma^p(1-o) + u_g(c_t, g_t)\varsigma^g o = \Omega_t^1(1-\eta)q_t(1-o) + \Omega_t^2(1-\eta)q_t o \\
o &: \; \varsigma^p v_t u_c(c_t, g_t) - \varsigma^g v_t u_g(c_t, g_t) = (\Omega_t^1 - \Omega_t^2)(\mu(1 - l_t^p - l_t^g)^\eta (v_t)^{1-\eta}) \\
l_{t+1}^p &: \; \Omega_t^1 = \beta\{a_{t+1}^p u_c(c_{t+1}, g_{t+1}) - \nu_u(u_{t+1}) + \Omega_{t+1}^1(1-\lambda^p) - \Omega_{t+1}^1 \eta^p(\mu(1 - l_t^p - l_t^g)^{\eta-1}(v_t)^{1-\eta})(1-o) \\
&: \; -\Omega_{t+1}^2 \eta^p(\mu(1 - l_t^p - l_t^g)^{\eta-1}(v_t)^{1-\eta})(o)\} \\
l_{t+1}^g &: \; \Omega_t^2 = \beta\{a_{t+1}^g u_g(c_{t+1}, g_{t+1}) - \nu_u(u_{t+1}) + \Omega_{t+1}^2(1-\lambda^g) - \Omega_{t+1}^1 \eta^p(\mu(1 - l_t^p - l_t^g)^{\eta-1}(v_t)^{1-\eta})(1-o) \\
&: \; -\Omega_{t+1}^2 \eta^p(\mu(1 - l_t^p - l_t^g)^{\eta-1}(v_t)^{1-\eta})(o)\}.
\end{aligned}$$

Figure A10 shows the optimal public-private wage ratio in steady-state as a function of the bargaining power of workers, for three different calibrations. If the frictions are symmetric across the two sectors and the Hosios condition is satisfied the government has to set the same wage as in the private sector to guarantee an efficient level of private vacancies. If the bargaining power of workers is higher than the Hosios condition, the government has to set lower wages to reduce the value of unemployment enough to increase the surplus of the firms and bring their vacancies in line with optimality.

For the asymmetric calibrations in which the separation rate is lower in the public sector the optimal public sector wage should be lower than the public sector. Lower separation rate raises the value of unemployment and hence the threat point in the bargaining. As in directed search, the government should offer lower wages to compensate.

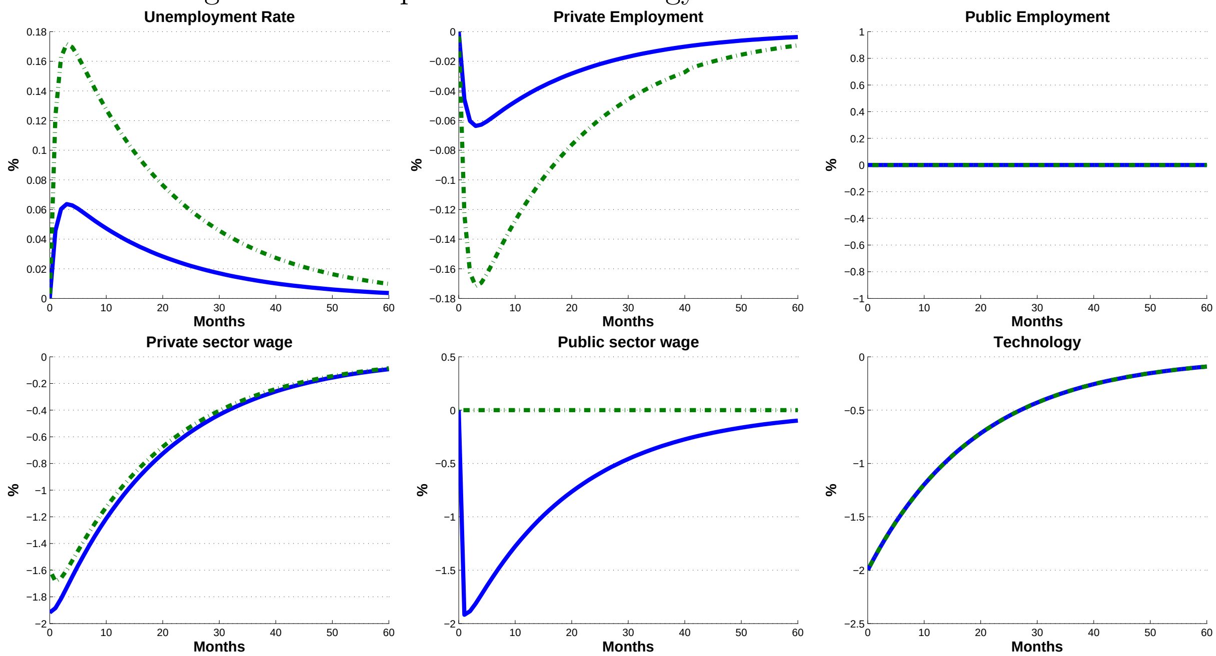
Figure A10: Optimal steady-state public-private wage ratio with random search



Note: the thick dotted line is for a calibration with symmetric frictions ($\lambda^g = \lambda^p = 0.0397$ and $\varsigma^g = \varsigma^p = 1.5$); the solid line ($\lambda^g = 0.0139$ and $\varsigma^g = \varsigma^p = 1.5$) and the dash line ($\lambda^g = 0.0139$ and $\varsigma^g = 0.8$)

Figure A11 shows the response to a technology shock when the government follows a procyclical and acyclical public sector wage policies. In both cases, the government maintains public employment constant. When the government keeps the wage constant, private sector vacancies do not fall by as much, leading to a larger fall in private vacancies and a stronger increase in unemployment. The response of unemployment is twice as strong.

Figure A11: Response to a technology shock with random search



Note: Solid line (procyclical public sector wage policy) and dash line (acyclical public sector wage policy). The response of the variables is in percentage of their steady-state value, except for unemployment rate which is in percentage points difference from steady-state.