3.5. Tacit Collusion

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Up to now firms met only once in the market. In reality, though, firms meet repeatedly. With repeated interaction, reputation and punishments can be used to induce cooperation. We will see that this offers a solution to the Bertrand paradox.

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Let's take a standard Bertrand game but where firms choose prices in T > 1 periods. This repetition may lead to what is called tacit collusion i.e. collusion that is not explicit among the oligopolists.

Assumptions:

- Homogenous goods.
- Same marginal cost, no fixed costs.
- No capacity constraints.
- Firms meet T > 1 times. In each period $t \in \{1, \ldots, T\}$ firms choose prices p_{1t} and p_{2t} simultaneously and no-cooperatively.

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The demand faced by firm i in period t is the same as Bertrand demands:

$$D_{it}(p_{it}, p_{jt}) = \begin{cases} D_t(p_{it}) & \text{if } p_{it} < p_{jt} \text{ captures all the demand} \\ \frac{1}{2} D_t(p_{it}) & \text{if } p_{it} = p_{jt} \text{ (or any other quantity)} \\ 0 & \text{if } p_{it} > p_{jt} \text{ looses all the demand} \end{cases}$$

And profits in period t are:

$$\Pi_t^i(p_{it}, p_{jt}) = (p_{it} - c) D_{it}(p_{it}, p_{jt})$$

δ is the discount factor.

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The firm's problem now is to maximize the total profit=

$$\Pi^i\left(p_i, p_j\right)=\sum_{t=1}^{T}\delta^{t-1}\Pi_t^{\,i}(p_{it}, p_{jt}) \text{ where } p_i=\left(p_{i1}, p_{i2},\ldots p_{iT}\right), p_j=\left(p_{j1}, p_{j2},\ldots p_{jT}\right)$$

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CASE I: Finite horizon ($T<\infty$):

The only subgame perfect equilibrium is that firms set $p_{1t}=p_{2t}=c$ in all periods (we would not solve the Bertrand paradox).

Proof: By backward induction:

Starting in the last period, period T. In this period, the last of the game, the game is static, i.e. coincides with the standard Bertrand game where the firms profit only depend on the actions in that period and there is no room for punishments so $p_{1T}=p_{2T}=c$.

In period T-1, firms know that in period T the Bertrand equilibrium is going to prevail so there is no room for cooperation either in period T-1, i.e there is punishment for sure in period T. Prices in period T-1 only affect the current profits, therefore the situation is equivalent to a static game. The only equilibrium is the one from the static game: $p_{1T-1}=p_{2T-1}=c$.

And we carry this argument backwards until period 1 … In the end the static Bertrand equilibrium is repeated T times, and we do not **solve the Bertrand paradox**.

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CASE II: Infinite horizon (T=∞):

1) The repetition of the static equilibrium is still an equilibrium.

Proof: Each firm sets $p_{1t}=p_{2t}=c$ independently of the history of the game up to period t. Given that $p_2=c$, the best reply is $p_1=c$ and vice-versa. Therefore, $p_1=(c,c,c,\ldots c..)$ and $p_2=(c,c,c\ldots c..)$ are an equilibrium.

2) There may be other equilibria where prices>c are sustained.

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CASE II: Infinite horizon (T=∞):

p^M=monopoly price that is the one that maximizes $\Pi=(p-c)D(p)$

Π^M= monopolist profit in one period.

$H_t=(p_{10},p_{20};p_{11},p_{21};\ldots\ldots;p_{1t-1},p_{2t-1})$ history of the game up to period t

Take the following trigger strategy: :

$$p_{it}(H_t)=\begin{cases} p^M \text{ if } H_{it}=\varnothing \text{ or if } H_{it}=(p^M,p^M;p^M,p^M;\ldots p^M,p^M) \\ c \quad \text{for any other history} \end{cases}$$

Punishment in the case firm j deviates from cooperation. Deviation in one period induces punishment forever.

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CASE II: Infinite horizon (T=∞):

If $H_t \neq (p^M, p^M; p^M, p^M; \ldots\ldots; p^M, p^M)$ both firms play c (the static Bertrand equilibrium) forever and this is always a subgame perfect equilibrium.

If $H_t = (p^M, p^M; p^M, p^M; \ldots\ldots; p^M, p^M)$ then each firm continues the cooperation strategy (given the rival's strategy) in which case:

$$\frac{\Pi^M}{2} + \delta\frac{\Pi^M}{2} + \delta^2\frac{\Pi^M}{2} + ... = \frac{1}{1-\delta}\frac{\Pi^M}{2}$$

If a firm deviates from cooperation (given its rival's strategy) it will set $p^M - \varepsilon$ and will gain all the demand. The profits would be in that case:

$$\approx \underbrace{\Pi^M}_{\substack{\text{profit in} \\ \text{the period} \\ \text{it deviates}}} + \underbrace{0+0+...}_{\substack{\text{punishment forever.} \\ \text{Prices are equal to c}}} = \Pi^M$$

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CASE II: Infinite horizon (T=∞):

Firms will not deviate if profits from cooperation are higher than from deviation:

$$\frac{1}{1-\delta}\frac{\Pi^M}{2} \geq \Pi^M$$

$$\Leftrightarrow \frac{1}{1-\delta}\frac{1}{2} \geq 1 \Leftrightarrow \frac{1}{1-\delta} \geq 2 \Leftrightarrow 2(1-\delta) \leq 1 \Leftrightarrow \boxed{\delta \geq \frac{1}{2}}$$

That is when they value the future enough.

Conclusion: If firms value enough the future, (i.e. $\delta \geq 1/2$) then it is possible to sustain prices higher than c, in particular p^M or any other price between c and p^M.

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CASE II: Infinite horizon (T=∞):

Take $p \in \left[c, p^M\right]$ and the previous trigger strategy

$\Pi(p) \equiv$ profit of the monopolist when price is p

If the firm collaborates :

$$\frac{\Pi(p)}{2} + \delta\frac{\Pi(p)}{2} + \delta^2\frac{\Pi(p)}{2} + ... = \frac{1}{1-\delta}\frac{\Pi(p)}{2}$$

If the firm does not collaborate and sets p'=p-ε, it gets:

$$\Pi(p) + 0 + 0 + ... = \Pi(p)$$

The firm will collaborate iff :

$$\boxed{\frac{1}{1-\delta}\frac{\Pi(p)}{2} \geq \Pi(p) \Leftrightarrow 1-\delta \leq \frac{1}{2} \Leftrightarrow \delta \geq \frac{1}{2}}$$

Same result as with p^M

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CASE II: Infinite horizon (T=∞):

Note: the simplest way to guarantee a given price is to penalize very strongly. In this case the strongest punishment is to go back to the static equilibrium where profits are zero. For the punishment to be credible it has to be an equilibrium. In equilibrium the punishment phases will never occur.

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CASE II: Infinite horizon (T=∞):

Firm i collaborates iff:

$$\frac{\Pi^M}{n}\left(\frac{1}{1-\delta}\right) \geq \Pi^M \Leftrightarrow 1-\delta \leq \frac{1}{n} \Leftrightarrow \delta \geq 1-\frac{1}{n} \xrightarrow{n\to\infty} 1$$

When ↑n the minimum value of δ to sustain collusion is higher, therefore as n increases it is harder to sustain collusion.

The intuition is that the relative gain from deviating is larger (one wins all the market instead of getting 1/nth of it) while the punishment is smaller (the difference between the cooperating equilibrium and the zero is smaller)

$$\frac{\Pi^M}{n}\left[1+\delta+\delta^2+...\right] \geq \Pi^M \Leftrightarrow \underbrace{\frac{\Pi^M}{n}\left[\delta+\delta^2+...\right]}_{=\frac{\Pi^M}{n}\frac{\delta}{1-\delta}=\text{punishment,}} \geq \underbrace{\Pi^M - \frac{\Pi^M}{n}}_{\text{gain from deviating}}$$

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Note: the collusion is more likely:

- When there are fewer firms
- The probability of detection is higher
- Firms face each other in multiple markets