2.4 Multiproduct Monopoly

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2.4 Multiproduct Monopoly

- The firm is a monopoly in all markets where it operates
- $i=1,\ldots.n$ goods sold by the monopolist
- $p=(p_1,\ldots.p_n)$ prices charged for each good (uniform)
- $q=(q_1,\ldots.q_n)$ quantities sold of each good
- $q_i=D_i(p)$ = demand of good i – Note that what is important here is that demand for good i may depend on the full price vector not only of p_i
- C(q1,…qn)= Cost function, depends on the quantities produced of all goods. Note, quantities here may not be added because the monopolist is producing different goods.

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Examples:

Example 1: Launching Prices – e.g.: "imagénio" by Telefónica, CNN plus (initial prices very cheap), ING 1st deposit; cable TV (some extra channels at very low prices).

Example 2: Learning-by-doing –

Example 3: New Product lines – Kmart, gas stations at certain supermarkets.

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A special case (theory)

- Suppose demands are independent i.e. they only depend on their own price p_i: $q_i=D_i(p_i)$.
- Separability in the Cost function :

  $C(q_1,\ldots q_n)=C_1(q_1)+\ldots C_n(q_n)$

In this case the monopolist's maximization problem may be written as n separate problems since the n markets are independent.

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A special case(cont.)

$$\underset{\{p_1,\dots,p_n\}}{Max}\ \Pi = \sum_{i=1}^{n} D_i(p_i)p_i - \sum_{i=1}^{n} C_i(D_i(p_i))$$

$$\text{FOC: } \frac{\partial \Pi}{\partial p_i} = 0 \text{ for } i = 1,\dots,n$$

$$\Leftrightarrow D_i(p_i) + D_i'(p_i)p_i = C_i'(D_i(p_i))D_i'(p_i)$$

$$\Leftrightarrow \underbrace{\frac{p_i - C_i'(D_i(p_i))}{p_i}}_{\text{Lerner Index}} = \frac{1}{\varepsilon_i}$$

That is, the optimal pricing strategy is to have a higher margin in those markets in which demand is less elastic. This is the same result obtained in third-degree price discrimination, except that here the goods are different while in third-degree price discrimination we were dealing with the same good

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More General case – w.l.o.g. assume n=2

$$\underset{\{p_1,p_2\}}{Max}\ \Pi = D_1(p_1,p_2)p_1 + D_2(p_1,p_2)p_2 - C(D_1(p_1,p_2), D_2(p_1,p_2))$$

$$\text{FOC: } \frac{\partial \Pi}{\partial p_1} = 0 \Leftrightarrow D_1(p) + \frac{\partial D_1(p)}{\partial p_1}p_1 + \frac{\partial D_2(p)}{\partial p_1}p_2 = \frac{\partial C(\bullet)}{\partial D_1}\frac{\partial D_1}{\partial p_1} + \frac{\partial C(\bullet)}{\partial D_2}\frac{\partial D_2}{\partial p_1}$$

$$\frac{\partial \Pi}{\partial p_2} = 0 \Leftrightarrow D_2(p) + \frac{\partial D_2(p)}{\partial p_2}p_2 + \frac{\partial D_1(p)}{\partial p_2}p_1 = \frac{\partial C(\bullet)}{\partial D_1}\frac{\partial D_1}{\partial p_2} + \frac{\partial C(\bullet)}{\partial D_2}\frac{\partial D_2}{\partial p_2}$$

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Assume additive costs

$$C(q_1, q_2) = C_1(q_1) + C_2(q_2)$$

Hence, the first FOC simplifies to:

$$D_1(p) + \frac{\partial D_1(p)}{\partial p_1} p_1 + \frac{\partial D_2(p)}{\partial p_1} p_2 = C_1'(\bullet)\frac{\partial D_1}{\partial p_1} + C_2'(\bullet)\frac{\partial D_2}{\partial p_1}$$

$$\Leftrightarrow D_1(p) + \frac{\partial D_1(p)}{\partial p_1} p_1 \times \boxed{\frac{D_1}{D_1}} + \frac{\partial D_2(p)}{\partial p_1} p_2 \times \boxed{\frac{D_2}{D_2}\frac{p_1}{p_1}} =$$

$$= C_1'(\bullet)\frac{\partial D_1}{\partial p_1} \times \boxed{\frac{D_1}{D_1}\frac{p_1}{p_1}} + C_2'(\bullet)\frac{\partial D_2}{\partial p_1} \times \boxed{\frac{D_2}{D_2}\frac{p_1}{p_1}}$$

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$$D_1(p) + \underbrace{\frac{\partial D_1(p)}{\partial p_1}\frac{p_1}{D_1}}_{-\varepsilon_{11}} D_1 + \underbrace{\frac{\partial D_2(p)}{\partial p_1}\frac{p_1}{D_2}}_{-\varepsilon_{12}} D_2 \frac{p_2}{p_1} =$$

$$= C_1'(\bullet)\underbrace{\frac{\partial D_1}{\partial p_1}\frac{p_1}{D_1}}_{-\varepsilon_{11}}\frac{D_1}{p_1} + C_2'(\bullet)\underbrace{\frac{\partial D_2}{\partial p_1}\frac{p_1}{D_2}}_{-\varepsilon_{12}}\frac{D_2}{p_1}$$

The first FOC simplifies further to:

$$D_1(p) - \varepsilon_{11} D_1 - \varepsilon_{12} D_2 \frac{p_2}{p_1} = -C_1'(\bullet)\varepsilon_{11}\frac{D_1}{p_1} - C_2'(\bullet)\varepsilon_{12}\frac{D_2}{p_1} \quad \text{A}$$

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$$D_1(p) - \varepsilon_{11} D_1 - \varepsilon_{12} D_2 \frac{p_2}{p_1} = -C_1'(\bullet)\varepsilon_{11}\frac{D_1}{p_1} - C_2'(\bullet)\varepsilon_{12}\frac{D_2}{p_1}$$

A

Multiply both sides by p_1/D_1:

$$p_1 - p_1\varepsilon_{11} - p_2\frac{D_2}{D_1}\varepsilon_{12} = -C_1'(\bullet)\varepsilon_{11} - C_2'(\bullet)\varepsilon_{12}\frac{D_2}{D_1}$$

$$\Leftrightarrow -\left(p_1 - C_1'(\bullet)\right)\varepsilon_{11} = -p_1 + p_2\frac{D_2}{D_1}\varepsilon_{12} - C_2'(\bullet)\frac{D_2}{D_1}\varepsilon_{12}$$

$$\Leftrightarrow \left(p_1 - C_1'(\bullet)\right) = p_1\frac{1}{\varepsilon_{11}} - \left(p_2 - C_2'(\bullet)\right)\frac{D_2}{D_1}\varepsilon_{12}\frac{1}{\varepsilon_{11}}$$

$$\Leftrightarrow \boxed{\frac{p_1 - C_1'(\bullet)}{p_1} = \frac{1}{\varepsilon_{11}} - \frac{\left(p_2 - C_2'(\bullet)\right)\varepsilon_{12}D_2}{p_1\varepsilon_{11}D_1}}$$

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<u>Case 1: Independent goods ε_{12}=0,</u>

$$\boxed{\frac{p_1 - C_1'(\bullet)}{p_1} = \frac{1}{\varepsilon_{11}}}$$

<u>Case 2: Substitutes:</u>

$$\frac{\partial D_2}{\partial p_1} > 0 \Rightarrow \varepsilon_{12} < 0 \text{ because } \varepsilon_{12} = -\underbrace{\frac{\partial D_2}{\partial p_1}}_{+}\frac{p_1}{D_2} < 0$$

$$\boxed{\frac{p_1 - C_1'(\bullet)}{p_1} = \frac{1}{\varepsilon_{11}} - \underbrace{\frac{\left(p_2 - C_2'(\bullet)\right)\varepsilon_{12}D_2}{p_1\varepsilon_{11}D_1}}_{-} > \frac{1}{\varepsilon_{11}}}$$

The monopolist's margin is higher than with independent goods

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Case 2 (cont.): intuition:

$\uparrow p_1 \Rightarrow \uparrow D_2$ gives incentives to the monopolist to $\uparrow p_2$

When maximizing the joint profit, the monopolist internalizes the effects that the sale of one good has on the demand of the others. In the case of 2 substitute goods this implies that the monopolist should increase the prices of both goods relative to a situation where he treated the two goods separately.

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Case 3: Complements

$\uparrow p_1 \Rightarrow \downarrow D_2$ (because $\downarrow D_1$) then we may guess that the price of good 1 is lower than in the case in which the monopolist would treat the two goods independently. $\Rightarrow \frac{\partial D_2}{\partial p_1} < 0 \Rightarrow \varepsilon_{12} > 0$

$$\boxed{\frac{p_1 - C_1'(\bullet)}{p_1} = \frac{1}{\varepsilon_{11}} - \underbrace{\frac{\left(p_2 - C_2'(\bullet)\right)\varepsilon_{12} D_2}{p_1 \varepsilon_{11} D_1}}_{+} < \frac{1}{\varepsilon_{11}}}$$

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Case 3 (cont.): Complements

$\uparrow p_1 \Rightarrow \downarrow D_2$ (and so does $\downarrow D_1$) therefore this gives incentives to the monopolist to $\downarrow p_2$

Note: If there is strong complementarity between the two goods the monopolist sells, it may be optimal for the monopolist to sell one of the goods, say good 1, below its marginal cost in order to increase the demand for good 2.

Example: Price of the mobile phone with and without contract with the company

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Example 1: Launching prices, inter-temporal production and imperfect information:

- The Monopoly produces a single good
- The good is sold in 2 consecutive periods
- The first period's demand is $D_1(p_1)$ and costs $C_1(q_1)$
- Period 2: $q_2=D_2(p_2,p_1)$ and $C_2(q_2)$
- $\downarrow p1 \rightarrow \uparrow D_1$
  $\rightarrow \uparrow D_2$ then $\frac{\partial D_2}{\partial p_1} < 0$ (complements)

For example, because when there are more consumers in period 1, there is more information about the product in period 2.

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Example1: (cont.):

Note: $\frac{\partial D_1}{\partial p_2} = 0$

The profit of the monopolist is:

$$\underset{p_1, p_2}{Max}\{p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta p_2 D_2(p_1, p_2) - \delta C_2(D_2(p_1, p_2))\}$$

given that by definition $\frac{\partial D_1}{\partial p_2} = 0$ the problem in the 2nd period is standard:

FOC: $\frac{\partial \Pi}{\partial p_2} = 0 \Leftrightarrow \frac{p_2 - C_2'(.)}{p_2} = \frac{1}{\varepsilon_2} \Leftrightarrow$ monopoly price in period 2

since $\frac{\partial D_2}{\partial p_1} < 0$ (complements) $\Rightarrow \varepsilon_{12} > 0 \Rightarrow \frac{p_1 - C_1'(.)}{p_1} < \frac{1}{\varepsilon_1}$ (lower launching prices)

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Example 1: (cont.):

Conclusion: The monopolist sacrifices some short-term profits for higher long-term profits. Ex: launching prices of CNN+, cable TV.

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Example 2: Learning by Doing – it is similar to a Multi-product Monopolist with independent demands but interdependent costs, i.e. costs decrease with quantity:

- Monopolist produces a single good in two consecutive periods
- Demand in period t is $q_t=D_t(p_t)$ (independent across periods)
- C1(q1) 1st period cost function
- C2(q1,q2) second period cost function

The higher the amount produced in the 1st period, the lower are the costs in the second period → $\frac{\partial C_2}{\partial q_1} < 0; \quad \frac{\partial C_2}{\partial q_2} > 0$

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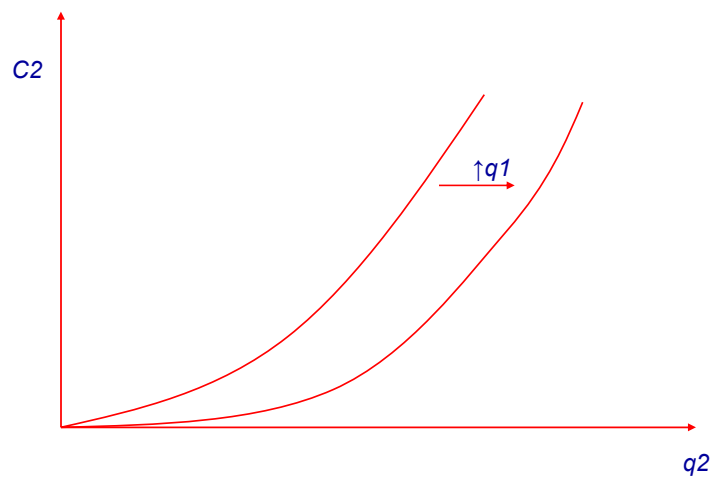

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Example 2: (cont.): Monopolist maximizes:

$$\underset{p_1,p_2}{Max}\{p_1D_1(p_1)-C_1(D_1(p_1))+\delta p_2D_2(p_2)-\delta C_2(D_1(p_1),D_2(p_2))\}$$

Again because period 2 does not have an effect in period 1's, the problem is standard:

$$\text{FOC:}\frac{\partial\Pi}{\partial p_2}=0 \Leftrightarrow \delta D_2(p_2)+\delta p_2D_2'(p_2)=\delta\frac{\partial C_2}{\partial D_2}D_2'(p_2)\Leftrightarrow \boxed{MR_2=MC_2}$$

$$\frac{\partial\Pi}{\partial p_1}=0\Leftrightarrow D_1(p_1)+p_1D_1'(p_1)=\frac{\partial C_1}{\partial D_1}D_1'(p_1)+\delta\underbrace{\frac{\partial C_2}{\partial D_1}}_{-}\underbrace{D_1'(p_1)}_{-}$$

$$\Leftrightarrow p_1+\frac{\partial p_1()}{\partial q_1}q_1=MC_1+\delta\underbrace{\frac{\partial C_2}{\partial D_1}}_{-}\Rightarrow \boxed{MR_1<MC_1}$$

q^*_1 is larger than the static optimal quantity. Short-run profits are sacrificed.