Monetary and Financial Macroeconomics: Homework III

UC3M 2019

Question 1: Private debt

We will study here the existence of private debt in the OLG context (IOUs). To this end, let's assume that half of the population has an endowment y when young and the other half has an endowment of y when old (and zero in the other period, respectively).

Households want to maximize present discounted utility, given by $u(c_{1t}) + \beta u(c_{2t})$.

Suppose there is a loan market. The lender lends l and the borrower borrows b in this market at the rate $1 + r$.

1. Write down the first and second period budget constraints for the lender and the borrower.

2. Assume logarithmic utility and solve for the allocation of the too agents in the model without money.

3. Draw the solution to the problem of the borrower and lender in the space $c_{1t}$and c_{2t} for a given $1 + r$.

4. Repeat the previous pictures for different values of r. Derive a loan supply and loan demand curves.

5. Assume that $\beta = 1$, $y = 100$ and constant population $N = 1000$. Solve for the general equilibrium. Find interest rate, consumption and loans.

Question 2: Private debt with capital

Suppose now that to the previous model we add capital. That is, savers can lend to the borrowers or save in capital. Assume capital pays $1+x$ per period. Suppose that $1+x > 1+r^*$, that is the return of capital is larger than the equilibrium return you computed for the previous question.

1. Represent this in the figure for the equilibrium between Supply and Demand of loans.

2. Would there be loans in equilibrium? If so, how many goods will be lent and how many will be stored in capital? Denote total goods lent by $\bar{L}$

3. Suppose now that the government restricts total loans to $\bar{\bar{L}} < \bar{L}$ what will happen with total capital savings?

4. Suppose that the amount of loans supply increases. What is the effect on capital and interest rate? What if there is no capital?

5. Suppose that x decreases, what is the effect on the capital stock and the amount of borrowing?

Question 3: Budget constraints

Consider the budget constraints for the government and central bank seen in class,

$$G_t + i_{t-1}B^T_{t-1} = T_t + \left(B^T_t - B^T_{t-1}\right) + RCB_t,$$

and

$$\left(B^M_t - B^M_{t-1}\right) + RCB_t = i_{t-1}B^M_{t-1} + (M_t - M_{t-1})$$

On top of the baseline assumptions, consider the case in which the government issues debt in domestic and foreign currency, and the central bank accumulated foreign reserves (that pay a positive return) and buy and sell private assets. Use ϵ_t to denote the nominal exchange rate defined as the price in domestic currency of one unit of foreign currency. Use $P^A_t a_t$ to define the value of assets in nominal terms and i^A_t its nominal return. Rewrite the budget constraints in real terms, nominal terms, identify the impact on both agent after a domestic currency depreciation.

Then consolidate the two budget constraints into a government-central bank constraint in real terms. Is it the effect of currency depreciation clear? What about a strong drop in the value of private sector assets in hands of the central bank?

Question 4: Open market operation in the Ricardian economy

Let's study the consolidated government-central bank budget constraint in a simplified form, where we follow the notation introduced in class:

$$(1 + r_{t-1})b_{t-1} = t_t + b_t + m_t - \frac{m_{t-1}}{1 + \pi_t}$$

Assume that the budget constraint of a representative agent is given by

$$c_t + m_t + b_t = y + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} - t_t$$

Introduce a simple fiscal policy rule. Assume the government sets taxes as

$$T_t = \psi(1 + r_{t-1})b_{t-1}$$

Where T_t is the present discounted value of taxes

1. Show that $T_t = t_t + \psi b_t$. Explain carefully each step.

2. Show that
$$t_t = \psi(R_{t-1}b_{t-1} - b_t)$$
and
$$s_t = (1-\psi)(R_{t-1}b_{t-1} - b_t)$$
and interpret economically. Explain carefully each step.

3. Substitute t_t in the households budget constraint and discuss. How does model implications change with the value of ψ?

4. Rewrite the budget constraint assuming this change of variables $w = m + (1-\psi)b$. Also suppose that money demand is proportional to consumption $m_t = \delta c_t(1+i_t)/i_t$ and that a consumption rule for agents is $c_{t+1} = \beta R_t c_t$. Also if this economy is closed, no investment and no government spending, $y = c_t$

5. Let's focus in the steady state (a situation where all t can drop). Show that the price level depends both on M and B.

6. How does open market operations that leave total liabilities fixed affect prices?

Question 5: Bank runs

Consider the model of demand deposits described in class. Suppose $N = 1000$, $y = 10$, $v^k - \theta = 0.6$, and $X = 1.3$. Let each person have a $\frac{1}{3}$ chance of being a type 1 and a $\frac{2}{3}$ chance of being a type 2.

a What bank portfolio can guarantee the rate of return 1 to all type 1 people and the rate of return 1.3 to all type 2 people? How many goods are placed in storage? In capital?

b Now suppose the type 2 people pretend to be type 1 people and withdraw early. How many people can be paid before the bank runs out of assets? Assuming that all type one are repaid, what is the fraction of type II agents that are not refunded?

c Suppose now that because of a government reform, capital investors are required to provide a more transparent documentation on the initial capital investment, which reduces the effort cost of verifying the quality of capital investment, $v^k - \theta = 1$.

  c.1 How this change would affect your answer to section b)?

  c.2 Is the economy going to experience a bank run in equilibrium? Explain.

d Assume $v^k - \theta = 0.99$ and repeat point c.