Monetary and Financial Macroeconomics: Homework II

UC3M 2019

Exercise 1: Optimal quantity of money in General Equilibrium

Suppose the representative agent maximizes utility given by

$$V = u(C_1, m_1) + \frac{1}{1+\rho} u(C_2, m_2)$$

subject to

$$P_1 Y + M_0 + P_1 T_1 = P_1 C_1 + M_1$$
$$P_2 Y + M_1 + P_2 T_2 = P_2 C_2 + M_2$$

Here $P_t T_t$ are nominal lump-sum transfers received from the government.

Assume a simple monetary policy of constant money growth rate of μ.

1. With this information, write down the budget constraint for the government. Briefly explain it.

2. Write down the problem of the household. Discuss, will the demand for money be positive every period? why?

3. Write down the Lagrangian and solve the problem and show that (please also provide an interpretation of the 2 optimality conditions)

$$\frac{U_C(C_1, m_1)}{P_1} = \frac{U_m(C_1, m_1)}{P_1} + \frac{1}{1+\rho} \frac{U_C(C_2, m_2)}{P_2}$$

$$U_C(C_2, m_2) = U_m(C_2, m_2)$$

4. Let the utility function be $u(C, m) = ln(C) + ln(m)$, use these equations to find the optimal m_1^* and m_2^*. Use the feasibility constraint to express them as a function of Y and parameters (first find m_2^* and then use it to get m_1^*).

5. Do a few comparative statics: what happens with the money demand in the first period when the growth rate of money increases (what happens with the nominal price level)? what to the money in the second period?

Now, let's move to the second part of the exercise. What is the optimal rate of money growth? Plugging in the results from the previous sections

$$V = u(Y) + v(m_1^*(\rho, Y, \mu)) + \frac{1}{1+\rho}\left(u(Y) + v(m_2^*(\rho, Y))\right)$$

1. Find the optimal rate of money growth that an utilitarian government would choose. Specify the set of first order conditions and recover the optimal growth rate. If you did all right, then you just found the so called "satiation result".

Exercise 2: Comparison OLG Planner vs Decentralization

Consider the following economy, $N_t = 100$ for all t. Assume: $\beta = 0.95$

$$[y_{1,t}, y_{2,t}] = [2, 1]$$
$$U[c_{1,t}, c_{2,t}] = c_{1,t}[c_{2,t}]^{\beta}$$

- Solve for the Planner's allocation. Write down the planners problem, Discuss the welfare function of the planner and assume the planner want's to maximize the utility of all future generations.
- Plot the planner's allocation. Be as explicit as possible in the description of the figure.
- Solve now the market allocation without money. Find optimal consumption and savings as a function of the interest rate. Discuss and solve for the general equilibrium and plot a figure.
- From now on, suppose there is $M_t = 500$, for all t
- Find savings function (individual and aggregate)
- Find the interest rate
- Find the consumption each period
- Make a picture

Exercise 3: Seignoriage collection versus lump-sum taxation

Consider the following economy, $N_t = 1000$ for all t. Assume: $\beta = 0.9$

$$[y_{1,t}, y_{2,t}] = [5, 1]$$
$$U[c_{1,t}, c_{2,t}] = ln(c_{1,t}) + \beta ln(c_{2,t})$$

Suppose there is $M_t = 500$, and the growth rate of money is 0.03.

- Solve now the market allocation. Find optimal consumption and savings as a function of the interest rate. Discuss and solve for the general equilibrium and plot a figure.
- Compute the seignoriage collected by the government in period $t+1$
- Compute the welfare of the representative household in the economy
- Now suppose the government decides to change the way to collect resources and sets $\mu = 0$ and replace them by lump sum taxes such that it obtains the same number of goods than under seignoriage. Recompute the general equilibrium and the welfare of the representative household.
- Discuss, when is welfare higher? Why?

Exercise 4: Capital in a 2 period OLG model

Before doing this exercise you will benefit from reading chapter in Champ Freeman and Haslag. Assume that in the OLG economy, the households can save only using capital (a productive asset that creates goods in the future). Households maximize utility subject to

$$c_{1,t} + k_t \leq y_1$$
$$c_{2,t} \leq y_2 + (1+x)k_t$$

where x is the net return of capital investment.

- Write down the intertemporal budget constraint
- Compute the seignoriage collected by the government in period $t+1$
- Assume $U(c) = ln(c)$ and obtain the optimal savings and consumption.
- Now assume that there is money in the economy and set up the budget constraints for the first and second periods with money. Write down the intertemporal budget constraint and discuss the arbitrage condition between money and capital.
- Consider now the case of a return to capital that would appear in a production economy with decreasing returns to scale $f(k) = k^\alpha$ with $\alpha < 1$. Rewrite the problem of the household and find the optimal demand for capital
- In this last economy, assume the government increases the stock of money at a rate μ every period. What is the effect of μ on the capital stock?... If you answer right, then you just found the "Tobin Effect"

Exercise 5: Capital, liquidity and a 3-period OLG model

A result from the previous exercise is that the rate of return of money and capital are the same. This may not be true in general. Assume that capital and money have different degrees of liquidity. In particular assume an OLG where people live for 3 periods. They maximize utility as in $U = ln(c_{1,t}) + ln(c_{2,t}) + ln(c_{3,t})$

$$c_{1,t} + k_t + p_t^m m_t \leq y_{1,t}$$
$$c_{2,t} \leq y_{2,t} + p_{t+1}^m m_t$$
$$c_{3,t} \leq y_{3,t} + (1+x)k_t$$

Suppose that capital is not observable and hence, if you own capital you cannot issue shares of it. Also, assume that capital requires 2 periods to mature, you invest in period t and the returns are ready in period $t+2$.

- In which sense capital is illiquid here?
- Find the optimal demand for money and capital (for this you need to setup the Lagrangian, solve it, solve for equilibrium in all markets, find the saving and consumption choices and return equilibrium prices.
- Assume now that $x = 0.05$, $M_t = 500$, $y = [100, 0, 0]$. Recompute the previous solutions, what is the return on money?
- Suppose now that the government rises μ to 2%. What happens with money and capital demand
- What if $\mu = 6\%$? Do people still demand money?

Exercise 6: Population growth

Suppose an OLG economy like the one described in class. Each period, there are N_t young and N_{t-1} old. Agents receive an endowment of y_1 in their first year of life and y_2 in the second. Households decide how much to consume ($c_{1,t}$, $c_{2,t}$) and how much money to demand.
Each period t, the amount of money in the economy grows at rate μ: $M_{t+1} - M_t = \mu M_t$ and the government introduces this money in the economy through lump sum subsidies to each old person.

1. Find the intertemporal budget constraint for households. What is the return of money?.
2. Impose necessary conditions for a monetary steady state equilibrium and graph the budget set.
3. Suppose there is no population growth in this economy: $N_t = N_{t+1}$. In the graph from 2), depict the feasible set for this economy and discuss about the inefficiency of inflation.

4. Now suppose there is population growth in this economy: $N_{t+1} = (1+n)N_t$. What is the return of money?. Repeat the steps done in point 2) and do a graph with the new budget set. Compare with the correspondent feasible set and discuss.

5. Suppose the same economy we had in previous point. In that case, the population grew at rate n, implying the total endowment of the economy also grew at rate n. Now assume the government creates money at rate n, i.e., $n = \mu$. What is the inflation rate and the return of money?. Does this policy maximize the utility of future generations? Discuss and graph.

Exercise 7: Seigniorage

Consider a monetary OLG model in which individuals live for two periods, young and old. Individuals are identical and we assume that they choose a symmetric consumption basket. Assume that growth rate of the at money supply is $\mu > 1$ and thus, the law of motion of of the aggregate supply of money follows $M_{t+1} = (1+\mu)M_t$. The growth rate of fiat money is used to finance government purchase of goods per young person (g) in every period. Suppose that population grows at rate n, that is $N_{t+1} = (1+n)N_t$.

1. Find the individuals budget constraints when young and when old. Combine them to form the individuals lifetime budget constraint and graph this constraint.

2. Assume that the utility function of a young cohort in the economy is $\ln(c_{1,t}) + \beta \ln(c_{2,t})$. Write the lifetime maximization problem of a young person, who tries to decide optimally her consumption as young and her consumption as old.

3. Find the optimal $c_{1,t}$ and $c_{2,t}$.

4. Find the demand for money m_t in a stationary equilibrium as a function of μ and n.

5. Find the government budget constraint at a stationary equilibrium and express it as a function of μ and n (Be clear abut the equilibrium conditions you are using).

6. Suppose that $\beta = 0.9$, $\{y_{1,t}, y_{2,t}\} = \{3, 1\}$ for all t and $n = 0.1$. What is the level of money growth rate μ^* such that the government attains the maximum level of g at equilibrium?

7. Suppose that that the current growth rate of fiat money is $\mu < \mu^*$ and the government decides to rise μ in order to increase g. How does this affect the equilibrium values of m_t, $c_{1,t}$ and $c_{2,t}$?

8. Suppose now that the government uses lump-sum taxes to finance g instead of using the growth rate of fiat money. Would agents be better off in this case? If yes, why? No algebra needed, just explain with words or/and using a graph.

Exercise 8: Money growth

Considere a 2 period OLG model where $M_{t+1} = (1+\mu)M_t$

Consider the versions of the model: one in which the new money is transfered to the old agents as $\mu M_t/N_t$ and a second one where the transfers are μm_t.

For the 2 cases solve the housheholds and the general equilibrium problem. Draw a figure and carefully explain the differences.