

# Unemployment Fluctuations

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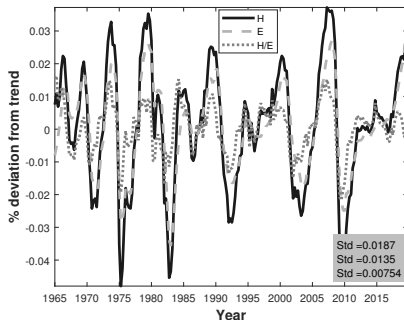
UC3M

Macroeconomics II

- So far, we study fluctuations in labor by total hours worked.
- We will see now that most fluctuations in aggregate hours result from fluctuations in number of persons employed.
- Hence, we will develop a theory of unemployment and fluctuations in unemployment.

## Data on unemployment

# Fluctuations at the extensive and intensive margin



- Total hours fluctuate because hours per worker fluctuate and because total number of workers fluctuate.
- It turns out that both contribute to the fluctuations in hours.
- However, quantitatively, fluctuations in the number of workers dominates.

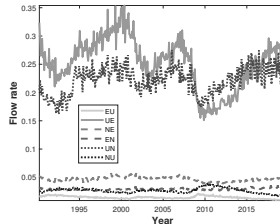
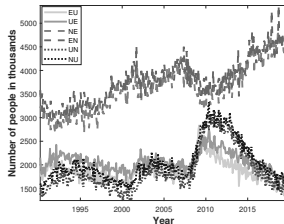
## **We start with defining some terms:**

- Non-institutional civilian population: All people older than 16 who are not in school, the army, prison ...
- Labor force: Those people who want to work.
- Employed: Those people who currently have a job.
- Unemployed: Those people who do not have a job but search for a job.

# Labor states in the US

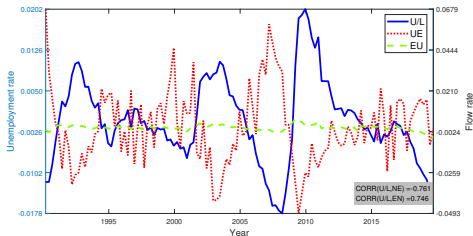
<b>TOTAL</b>	
Civilian noninstitutional population <sup>(1)</sup>	260,742
Civilian labor force	160,078
Participation rate	61.4
Employed	147,543
Employment-population ratio	56.6
Unemployed	12,535
Unemployment rate	7.8
Not in labor force	100,664

# Labor flows



- On average, more people go from out of the labor force to employment than from unemployment to employment.
- Yet, the UE rate is much higher than the NE rate suggesting that these are two different states.

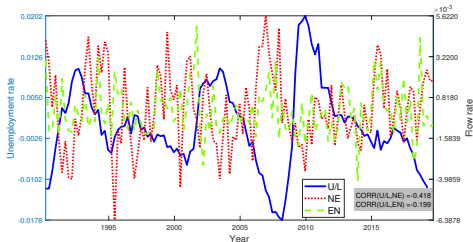
# Cyclical flows



- The UE rate moves strongly counter the unemployment rate.
- The EU rate moves together with the unemployment rate.
- The UE rate is much more volatile than the EU rate.

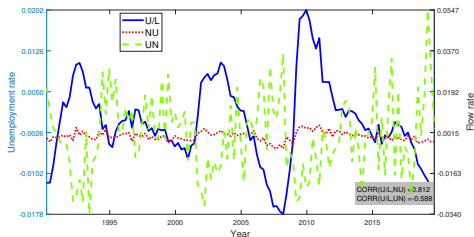


# Cyclical flows II



- Also movements from out of the labor force to employment show cyclical fluctuations.
- Both the NE and EN rate move counter the unemployment rate but the link with the former is stronger.
- Volatilities are an order of magnitude smaller than the UE fluctuations.

# Cyclical flows III



- When unemployment is high, few people flow from unemployment to out of the labor force.
- When unemployment is high, many people flow from out of the labor force to unemployment.
- Particularly the UN rate is highly volatile.

- Unemployment and out of the labor force are two distinct states, yet, both are important to understand labor market flows.
- What is more, particularly the NE and UN rates are cyclical.
- Hence, to understand the full dynamics of employment and unemployment, we require a 3-state model.
- However, for simplicity, we are going to ignore the out of the labor force state.
- Fluctuations in the UE rate are much larger than in the EU rate. Hence, we will focus on the former.

# A simplified model

- Let us assume that the labor force,  $L$ , is constant.
- Each period, we have  $L = E_t + U_t$ .
- In this model, only two states are relevant, employment and unemployment.
- Hence, it is irrelevant whether we study fluctuations in employment or unemployment.

- We will study the process of workers flowing between employment and unemployment. This is called the flow approach to unemployment.
- Focusing on flows makes sense given the high job finding rates, i.e., it is not always the same person who are employed and unemployed.
- Moreover, you will see that this flow approach carries important insights about unemployment.
- We start with the model in steady state.
- Afterward, we are going to introduce fluctuations in TFP to understand business cycle fluctuations.

## Labor market flows

## Why workers lose their jobs:

- Labor demand by firms change, i.e., they reduce employment.
  - Changes in demand for their products.
  - Changes in technology such as automatization.
- Individual reasons given a fixed firm labor demand.
  - The firm recognizes that the worker is a poor match with the job.
  - The job tasks may change requiring a different worker.

For the moment, we will assume that there is a constant job destruction rate,  $\delta$ , that is common to all workers.

## **Why do not all unemployed find a job instantaneously?:**

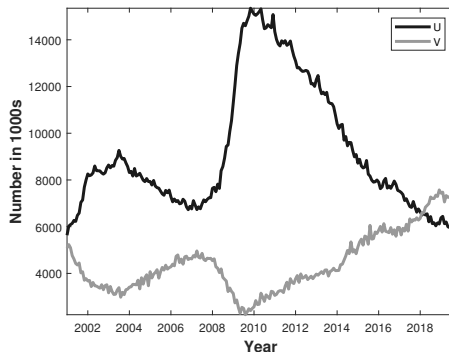
- In the labor market, we have a labor demand curve that is decreasing in the real wage.
- We have a labor supply curve that is upward sloping in the real wage.
- In a standard competitive market, the wage adjusts to clear the market, i.e., all unemployment find a job.
- Hence, to rationalize job finding rates below one and, thus, persistent unemployment, we require some friction in the labor market.



## How workers become employed:

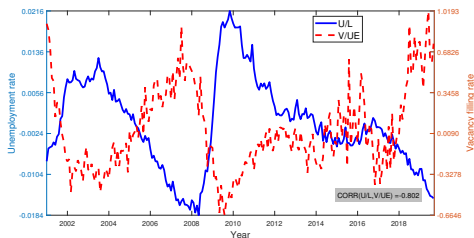
- Workers search for jobs. This search takes time:
  - They need to collect information about different job opportunities.
  - They select the best job among potential multiple options.
- Firms search for workers for their open vacancies:
  - Firms wait for workers to apply to their vacancies or search themselves for suitable workers.
  - They collect information on potential candidates and select the one that fits the job best.
- Importantly, search takes time and this search friction is at the heart of unemployed not finding new work instantaneously.
- This idea has been formalized by Diamond, Mortensen, and Pissarides who have received the [Nobel price](#) for their analysis. The resulting model is often referred to as the DMP model.

# Job vacancies



- Firms search for workers by posting job openings, so called vacancies.
- At any point in time, there are almost as many job vacancies available as unemployed searching for a job.

# Job vacancies II



- Define the vacancy filling rate as the ratio of total vacancies and the number of workers becoming employed.
- This vacancy filling rate moves strongly counter the unemployment rate.

# Matching unemployed and vacancies

- We need to think about the process of unemployed workers and vacancies matching with each other.
- We will take a rather abstract view: A matching function brings the two together.
- Note, we assume that we can aggregate all unemployed and all vacancies into two numbers.
- Hence, the total amount of matches in a period is:  $m_t = f(u_t, v_t)$ .
- Next, we need to decide on the properties of this function.

# Properties of the matching function

$$m_t = f(u_t, v_t) \quad (1)$$

- When no vacancies or no unemployed exist, no matches can be formed:  $f(0, v_t) = f(u_t, 0) = 0$ .
- More unemployed and more vacancies searching for a partner increases the number of matches:  $\frac{\partial f}{\partial v_t} > 0$  and  $\frac{\partial f}{\partial u_t} > 0$ .
- The function has constant returns to scale:  $f(\lambda u_t, \lambda v_t) = \lambda f(u_t, v_t)$ .

# Properties of the matching function II

Many functions satisfy these criteria. We will choose a Cobb-Douglas function:

$$m_t = \varphi u_t^\alpha v_t^{1-\alpha}. \quad (2)$$

- $\varphi$  is the so called matching efficiency which we assume to be time-invariant.
- Shifts in the matching efficiency represent shifts in the Beveridge curve.
- $\alpha$  is the elasticity of matches with respect to unemployment.

# Labor market flow rates

Define *labor market tightness* as  $\theta_t = \frac{v_t}{u_t}$ .

The job finding rate:

$$f_t = \frac{m_t}{u_t} = \frac{\varphi u_t^\alpha v_t^{1-\alpha}}{u_t} = \varphi \theta^{1-\alpha}. \quad (3)$$

The vacancy filling rate:

$$q_t = \frac{m_t}{v_t} = \frac{\varphi u_t^\alpha v_t^{1-\alpha}}{v_t} = \varphi \theta^{-\alpha}. \quad (4)$$

Relative rates:

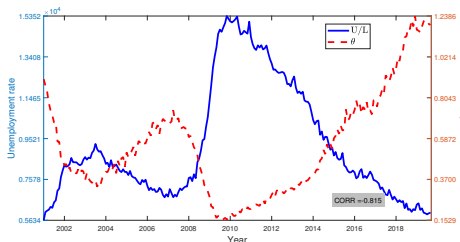
$$\frac{f_t}{q_t} = \frac{m_t}{u_t} \frac{v_t}{m_t} = \theta. \quad (5)$$

# The importance of constant returns to scale

- CRS imply that we can write the flow rates all in terms of the ratio of the two inputs.
- This will prove very useful because only the ratio will matter for the long-run equilibrium.
- This is as in the Solow Model which we solve in terms of the  $K/L$  ratio.

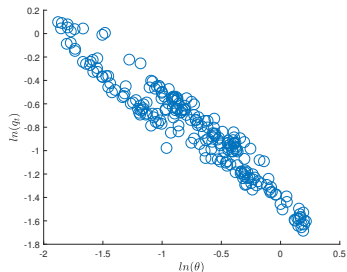
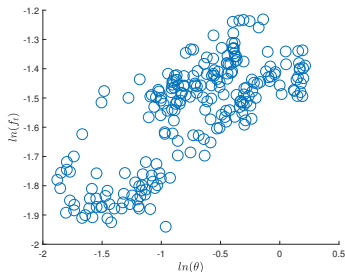


# Labor market tightness



- Labor market tightness moves strongly against the unemployment rate.

# Constant returns to scale in the data?



- Constant returns to scale imply that, for constant  $\varphi$ ,  $\alpha$ , the log job finding rate and the log vacancy filling rate are linear in  $\ln \theta_t$ .
- The data does not clearly reject this implication.

# Cobb Douglas and the Beveridge curve

In equilibrium, the inflow from unemployment needs to equal the outflow from unemployment:

$$\delta E_t = m(U_t, V_t) \quad (6)$$

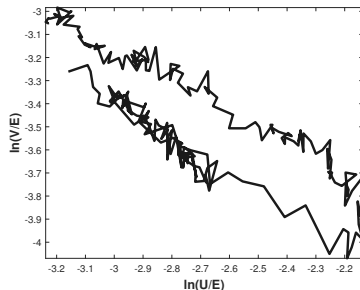
$$\delta = m\left(\frac{U_t}{E_t}, \frac{V_t}{E_t}\right), \quad (7)$$

where the latter follows from constant returns to scale. With our Cobb Douglas function, it follows that

$$\ln(\delta) = \ln(\varphi) + \alpha \ln\left(\frac{U_t}{E_t}\right) + (1 - \alpha) \ln\left(\frac{V_t}{E_t}\right). \quad (8)$$

Hence, there should exist a constant, negative relationship between  $\ln\left(\frac{U_t}{E_t}\right)$  and  $\ln\left(\frac{V_t}{E_t}\right)$ .

# Beveridge curve



- Indeed we find such a constant, negative relationship. This is called the Beveridge curve.
- It is not stable, however. It started to shift out in the great recession.

# Beveridge curve II

Rewriting the Beveridge curve yields:

$$\ln \left( \frac{V_t}{E_t} \right) = \frac{1}{1-\alpha} [\ln(\delta) - \ln(\varphi)] - \frac{\alpha}{1-\alpha} \ln \left( \frac{U_t}{E_t} \right). \quad (9)$$

A shift in the curve results from

- An increase in the job separation rate.
- A decrease in the matching efficiency rate.

## The model in steady state

# Further assumptions

- Time is discrete, agents have an infinite horizon and discount the future with factor  $\beta$ .
- Utility is linear in income (perfect insurance).
- The total labor force is of size 1.
- Labor is the only factor of production.

# The worker problem

An employed worker receives a wage  $w$  per period. Her value function is:

$$W = w + \beta \left[ \delta U + (1 - \delta) W \right].$$

An unemployed worker receives a flow benefit  $b$  per period. Her value function is:

$$U = b + \beta \left[ (1 - f(\theta)) U + f(\theta) W \right].$$

We will assume  $b < w$ , i.e., the worker accepts job offers.



# The firm problem

An open vacancy,  $v$ , costs a firm  $\nu$  every period. Hence, the value of an unfilled vacancy is given by

$$I = -\nu + \beta \left[ (1 - q(\theta))I + q(\theta)J \right]. \quad (10)$$

There is free entry in the market for new vacancies. Free entry drives the value of a vacancy to zero:

$$I = -\nu + \beta q(\theta)J = 0. \quad (11)$$

Firms consist of single job/worker matches, i.e., productivity is linear. A filled job produces output  $A$ :

$$J = A - w + \beta \left[ (1 - \delta)J + \delta I \right] \quad (12)$$

$$J = A - w + \beta(1 - \delta)J. \quad (13)$$

## Search frictions give rise to a so called match surplus:

- Workers strictly prefer employment over unemployment.
- Firms strictly prefer a filled job over a vacant vacancy.

$$S = W - U + J - I \quad (14)$$

$$S = A - b + v + \beta(W - U)[1 - \delta - f(\theta)] + \beta J[(1 - \delta) - q(\theta)]. \quad (15)$$

# Distributing the match surplus

Note, the match surplus does not depend on the wage. The wage simply shifts match surplus from the firm to the worker.

$$S = A - b + v + \beta(W - U)[1 - \delta - f(\theta)] + \beta J[(1 - \delta) - q(\theta)].$$

But how much match surplus should workers and firms each receive? We will assume that the actors bargain every period over the surplus by means of Nash-bargaining. Let  $\gamma$  be the relative bargaining power of workers. Then Nash-bargaining solves:

$$\max_w \left\{ (J - I)^{1-\gamma} (W - U)^\gamma \right\}.$$

# Solution to Nash-bargaining

$$\max_w \left\{ J^{1-\gamma} (W - U)^\gamma \right\}.$$

The solution is identical to the solution of the log transformation:

$$\max_w \left\{ (1 - \gamma) \ln(J) + \gamma \ln(W - U) \right\}. \quad (16)$$

This solves for

$$(1 - \gamma)[W - U] = \gamma J. \quad (17)$$

Or in terms of the match surplus:

$$W - U = \gamma S \quad (18)$$

$$J = (1 - \gamma)S. \quad (19)$$

# Solution to Nash-bargaining II

$$(1 - \gamma) \left[ w - b + \beta(W - U)[1 - \delta - f(\theta)] \right] = \gamma \left[ A - w + v + \beta J[(1 - \delta) - q(\theta)] \right]. \quad (20)$$

Rearranging yields:

$$(1 - \gamma)(w - b) - \gamma(A - w + v) = \gamma\beta J[(1 - \delta) - q(\theta)] - (1 - \gamma)\beta(W - U)[1 - \delta - f(\theta)]. \quad (21)$$

# Solution to Nash-bargaining III

Using the fact that

$$W - U = \frac{\gamma}{1 - \gamma} J$$

yields

$$w - (1 - \gamma)b - \gamma(A + v) = \gamma\beta J[1 - \delta - q(\theta)] - \beta\gamma J[1 - \delta - f(\theta)]. \quad (22)$$

Summarizing yields:

$$w = (1 - \gamma)b + \gamma(A + v) + \gamma\beta J[f(\theta) - q(\theta)]. \quad (23)$$

# Solution to Nash-bargaining IV

From the free entry condition for vacancies, (11), we have

$$J = \frac{\nu}{\beta q(\theta)} \quad (24)$$

Substituting in gives

$$w = (1 - \gamma)b + \gamma(A + \nu) + \gamma\nu \left[ \frac{f(\theta)}{q(\theta)} - 1 \right] \quad (25)$$

$$w = (1 - \gamma)b + \gamma A + \gamma\nu\theta, \quad (26)$$

which is the solution for the wage for a given labor market tightness  $\theta$ .

# The wage function

$$w = (1 - \gamma)b + \gamma A + \gamma\nu\theta$$

- Higher unemployment benefits increase a worker's outside option and, hence, her wage.
- Higher productivity increases match surplus of which the worker receives a share  $\gamma$ .
- Higher labor market tightness increases the worker's outside option and, hence, her wage.



# Equilibrium labor market tightness

Using the equilibrium wage together with the value of the firm yields

$$J = (1 - \gamma)[A - b] - \gamma\nu\theta + \beta(1 - \delta)J$$
$$J = \frac{1}{1 - \beta(1 - \delta)} \left[ (1 - \gamma)[A - b] - \gamma\nu\theta \right].$$

Using the free entry condition gives:

$$q(\theta) = \frac{\nu}{\frac{\beta}{1 - \beta(1 - \delta)} \left[ (1 - \gamma)[A - b] - \gamma\nu\theta \right]}. \quad (27)$$

which is a non-linear equation in labor market tightness  $\theta$ .

# Equilibrium labor market tightness II

$$q(\theta) = \frac{\nu}{\frac{\beta}{1-\beta(1-\delta)} \left[ (1-\gamma)[A-b] - \gamma\nu\theta \right]}.$$

- One may think of this as a labor demand equation. It tells us how many vacancies firms are willing to create until the value of an additional vacancy is zero.
- Vacancy creation decreases in vacancy posting costs  $\nu$ .
- Vacancy creation is higher when firm profits are larger:
  - When the flow match surplus,  $A - b$ , is large.
  - When firms receive a large share of this surplus, i.e.,  $\gamma$  is small.

# Equilibrium unemployment

Unemployment (rate),  $u_t$ , moves over time according to

$$u_{t+1} = (1 - f(\theta))u_t + \delta e_t.$$

In steady state, we have  $u_{t+1} = u_t = u^*$ :

$$f(\theta)u^* = \delta e^*$$

$$f(\theta)u^* = \delta[1 - u^*]$$

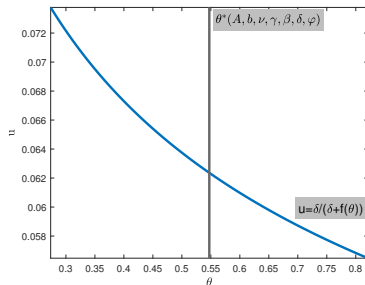
$$u^* = \frac{\delta}{\delta + f(\theta)}.$$

An increase in the job destruction rate shifts the unemployment rate for a given labor market tightness.

- I normalize labor productivity to  $A = 1$ .
- I set unemployment benefits to  $b = 0.4$  which is more than average non-employment benefits to represent the value of leisure.
- The frequency is monthly and, thus,  $\beta = 0.96^{1/12}$ .
- The average monthly job separation rate in the data is  $\delta = 0.029$ .
- Regressing the log job filling rate on log labor market tightness yields  $\alpha = 0.74$ .
- I set the bargaining power  $\gamma = \alpha$  which implies that the economy is efficient.

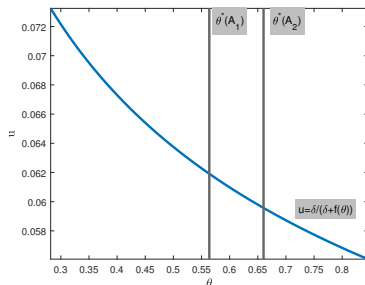
- I calibrate the matching efficiency to an average unemployment rate of 0.062:  $\varphi = 0.51$ .
- I calibrate the vacancy posting costs to an average labor market tightness of 0.55:  $\nu = 0.35$ .

# Equilibrium



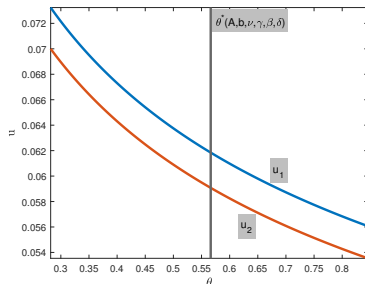
- Labor demand determines equilibrium labor market tightness.
- Equilibrium unemployment results from the resulting balancing of in- and out-flows.
- The wage is  $w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$ .

# Comparative statics: An increase in $A$



- Increasing productivity increases labor demand.
- The result is a higher labor market tightness.
- The higher job finding rate reduces equilibrium unemployment.
- Wages increase:  $w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$ .

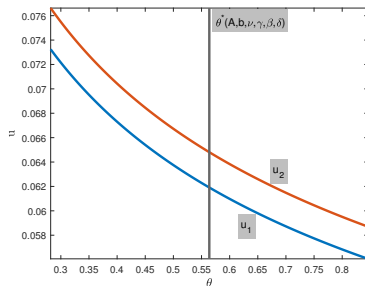
# Comparative statics: An increase in $\varphi$



- Increasing matching efficiency increases the job finding rate for any labor market tightness.
- The result is a decrease in the unemployment rate.
- The equilibrium tightness is almost unchanged (not shown).
- Wages are almost unchanged as  $\theta$  is almost unchanged.

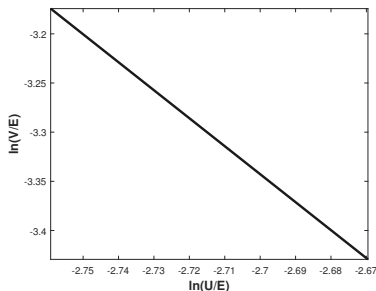


# Comparative statics: An increase in $\delta$



- Increasing the job destruction rate leads to a higher unemployment rate for any labor market tightness.
- The equilibrium tightness is almost unchanged (not shown).
- Wages are almost unchanged as  $\theta$  is almost unchanged.

# Beveridge curve

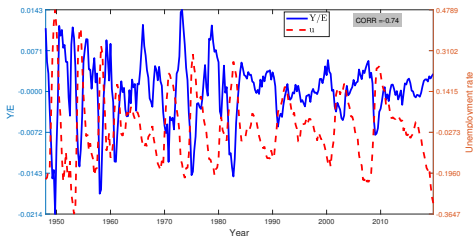


- Solving the model for different  $A$  gives us the Beveridge curve.
- Higher productivity increases vacancy creation and, thereby, lowers the unemployment rate.
- Hence, as in the data, we obtain a negative relationship between the log vacancy rate and the log unemployment to employment ratio.

# Business cycle dynamics

- We now use the model to study business cycle dynamics.
- We focus on the dynamics of labor market variables and ignore capital.
- Hence, instead of studying TFP, we study fluctuations in labor productivity  $Y/L$  which may also result from shocks to the capital stock.
- For simplicity, we assume that shocks to labor productivity are the only driver of business cycle fluctuations.

# Fluctuations in labor productivity and unemployment



- The data shows a strong negative correlation between the unemployment rate and labor productivity.
- Note, in the labor literature, it is conventional to also take the logs of rates when calculating business cycle fluctuations.

# The value of a vacancy with stochastic productivity

Labor productivity evolves as an  $AR(1)$  process in logs:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_t. \quad (28)$$

Hence, the value of a vacancy depends on the expected productivity in the next period:

$$I(A) = -\nu + \beta \mathbb{E} \left\{ (1 - q(\theta)) I(A') + q(\theta) J(A') \right\}. \quad (29)$$

Every period, there is free entry into the market for vacancies after observing the productivity realization and, hence,  $I(A') = 0$ :

$$I(A) = -\nu + \beta q(\theta) \mathbb{E} J(A') = 0. \quad (30)$$

# The value of a filled vacancy

The value of a filled vacancy becomes:

$$J(A) = A - w + \beta \mathbb{E} \left\{ (1 - \delta)J(A') + \delta I(A') \right\} \quad (31)$$

$$J(A) = A - w + \beta(1 - \delta)\mathbb{E}J(A'). \quad (32)$$

Writing the equation forward yields:

$$J(A) = \mathbb{E} \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [A_s - w_s] \quad (33)$$

The value of a filled vacancy are the discounted future flow profits. Discounting takes into account the time discount factor and the survival probability.

# The worker problem

An employed worker receives a wage  $w(A)$  per period:

$$W(A) = w(A) + \beta \mathbb{E} \left\{ \delta U(A') + (1 - \delta) W(A') \right\}.$$

An unemployed worker receives a flow benefit  $b$  per period:

$$U(A) = b + \beta \mathbb{E} \left\{ (1 - f(\theta)) U(A') + f(\theta) W(A') \right\}.$$

We will assume  $b < w(A)$ , i.e., the worker accepts all job offers.



The match surplus becomes:

$$S(A) = A - b + v + \beta \mathbb{E} \left\{ W(A') - U(A') \right\} [1 - \delta - f(\theta)] + \beta \mathbb{E} J(A') [(1 - \delta) - q(\theta)]. \quad (34)$$

Nash-bargaining solves:

$$\max_w \left\{ J^{1-\gamma} (W(A) - U(A))^\gamma \right\} \quad (35)$$

$$\max_w \left\{ (1 - \gamma) \ln(J(A)) + \gamma \ln(W - U) \right\}. \quad (36)$$

# Solution to Nash-bargaining

This solves for

$$(1 - \gamma)[W(A) - U(A)] = \gamma J(A). \quad (37)$$

Or in terms of the match surplus:

$$W(A) - U(A) = \gamma S(A) \quad (38)$$

$$J(A) = (1 - \gamma)S(A). \quad (39)$$

# Solution to Nash-bargaining II

$$(1 - \gamma) \left[ w - b + \beta \mathbb{E} \left\{ W(A') - U(A') \right\} [1 - \delta - f(\theta)] \right] = \\ \gamma \left[ A - w + v + \beta \mathbb{E} J(A') [(1 - \delta) - q(\theta)] \right]. \quad (40)$$

Rearranging yields:

$$(1 - \gamma)(w - b) - \gamma(A - w + v) = \\ \gamma \beta \mathbb{E} J(A') [(1 - \delta) - q(\theta)] - (1 - \gamma) \beta \mathbb{E} \left\{ W(A') - U(A') \right\} [1 - \delta - f(\theta)]. \quad (41)$$

# Solution to Nash-Bargaining III

Using the fact that

$$\mathbb{E}\{W(A') - U(A')\} = \frac{\gamma}{1 - \gamma} \mathbb{E}J(A')$$

yields

$$w - (1 - \gamma)b - \gamma(A + v) = \gamma\beta\mathbb{E}J(A')[1 - \delta - q(\theta)] - \beta\gamma\mathbb{E}J(A')[1 - \delta - f(\theta)]. \quad (42)$$

Summarizing yields:

$$w = (1 - \gamma)b + \gamma(A + v) + \gamma\beta\mathbb{E}J(A')[f(\theta) - q(\theta)]. \quad (43)$$

# Solution to Nash-Bargaining IV

From the free entry condition for vacancies, we have

$$\mathbb{E}J(A') = \frac{\nu}{\beta q(\theta)} \quad (44)$$

Substituting in gives

$$w = (1 - \gamma)b + \gamma(A + \nu) + \gamma\nu \left[ \frac{f(\theta)}{q(\theta)} - 1 \right] \quad (45)$$

$$w = (1 - \gamma)b + \gamma A + \gamma\nu\theta, \quad (46)$$

which is the solution for the wage for a given labor market tightness  $\theta$ .

# The wage function

$$w = (1 - \gamma)b + \gamma A + \gamma\nu\theta$$

- Higher unemployment benefits increase a worker's outside option and, hence, her wage.
- Higher productivity increases match surplus of which the worker receives a share  $\gamma$ .
- Higher labor market tightness increases the worker's outside option and, hence, her wage.

For a given current productivity, we can compute the value of a filled vacancy:

$$J(A) = \mathbb{E} \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [A_s - w_s].$$

Given the value of a filled vacancy, the free entry condition determines labor market tightness  $\theta(A)$ :

$$q(\theta) = \frac{\nu}{\beta \mathbb{E} J(A')}.$$

Given the labor market tightness and current productivity, we have the wage:

$$w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$$

# Equilibrium II

Note that we only require to know  $\theta_t$  as a function of the states of the economy. Moreover, equilibrium tightness depends only on the current  $A_t$ , and not past realizations or the unemployment rate. That is, whenever  $A_t$  changes,  $\theta_t$  directly jumps to its new equilibrium level that solves the free entry condition. Despite  $\theta_t$  being a jump variable, unemployment adjusts only sluggishly:

$$u_{t+1} = (1 - f(\theta_t))u_t + \delta e_t.$$

$\theta_t$  being a jump variable is computationally convenient and depends crucially on a linear production function and a constant returns to scale matching function. Those assumptions assure that  $u_t$  is irrelevant for the value of the firm.



# Equilibrium system of equations

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_t$$

$$J(A_t) = A_t - w_t + \beta(1 - \delta)\mathbb{E}J(A_{t+1})$$

$$w_t = (1 - \gamma)b + \gamma A_t + \gamma \nu \theta_t$$

$$q(\theta_t) = \frac{\nu}{\beta \mathbb{E}J(A_{t+1})}$$

$$\theta_t = \frac{v_t}{u_t}$$

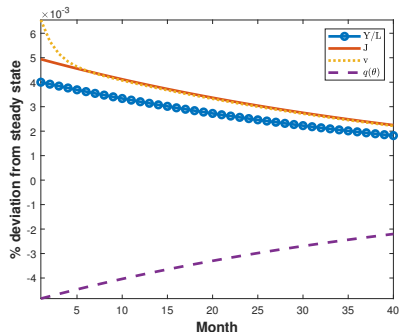
$$f_t = \varphi \theta_t^{1-\alpha}$$

$$q_t = \varphi \theta_t^{-\alpha}$$

$$u_{t+1} = (1 - f(\theta_t))u_t + \delta(1 - u_t).$$

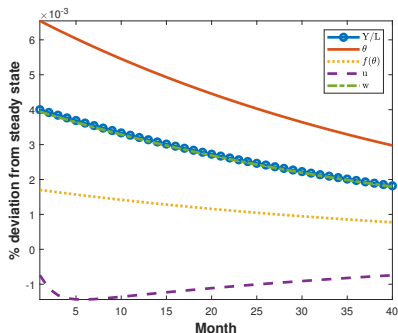
- We will use the same calibration as before.
- We use  $\rho = 0.98$  to match an autocorrelation of monthly labor productivity of 0.92.
- We use  $\sigma = 0.004$  to match a standard deviation of monthly labor productivity of 0.0085.

# Impulse response functions



- An increase in labor productivity increases the value of a filled vacancy.
- Free entry results in more vacancy creation which pushes down the vacancy filling rate.

# Impulse response functions II



- The increase in vacancies increases labor market tightness.
- As a result the job finding rate increases.
- A higher finding rate decreases the unemployment rate.
- Higher productivity and tightness increase wages.

# Results I

Std. relative to $Y/L$						
	$u$	$v$	$\theta$	$f(\theta)$	$q(\theta)$	$w$
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	0.37	1.33	1.64	0.43	1.21	0.99

- The model successfully replicates the qualitative relationship between labor productivity and the endogenous variables.
- It fails quantitatively. Flow rates, unemployment, and vacancies do not nearly respond sufficiently to a productivity shock.
- This is known as the “Shimer puzzle” pointed out in Shimer (2005).
- In a sense, it is the same question as before: Given the little volatility in productivity over the business cycle, why do hours fluctuate this much?

# Understanding the Shimer puzzle

Let us start with the free entry condition and log-linearize it:

$$q(\theta) = \frac{\nu}{\beta \mathbb{E}J(A')} \quad (47)$$

$$\varphi(\theta^{ss})^{-\alpha} (1 - \alpha \hat{\theta}) = \nu \beta^{-1} (J(A)^{ss})^{-1} (1 - \mathbb{E}\hat{J}(A')) \quad (48)$$

$$\hat{\theta} = \frac{1}{\alpha} \mathbb{E}\hat{J}(A'). \quad (49)$$

Hence, to have large percentage changes in  $\theta$ , we require large percentage changes in the expected value of a filled job  $\mathbb{E}\hat{J}(A')$ .

# Understanding the Shimer puzzle II

The expected value of a filled job is its discounted profit stream:

$$\mathbb{E}J(A') = \mathbb{E} \sum_{s=1}^{\infty} \beta^s (1 - \delta)^s [A_{t+s} - w_{t+s}] \quad (50)$$

This, however, is not very volatile in our model. There are two ways to generate large volatility in the expected value of a filled job:

- Hagedorn and Manovskii (2008) suggest to make  $A_{t+s} - w_{t+s}$  very small in levels. Hence, small changes in  $A_{t+s}$  will lead to large changes in  $\mathbb{E}\hat{J}(A')$ .
- Hall (2005) suggest to make wages sticky, thus, creating more volatility in  $A_{t+s} - w_{t+s}$ .



# A small surplus calibration

- Hagedorn and Manovskii (2008) point out that the “standard” calibration has several shortcomings.
- Their first step is to compute the costs of an open vacancy in steady state.
- Their second step is to match the volatility of wages.
- Their last step is to match average labor market tightness.
- They show that this calibration implies  $A_{t+s} - w_{t+s}$  being small.
- Importantly, they keep the assumptions of wages being bargained every period.

# The costs of an open vacancy

- The model features only explicitly labor. However, in the real world, there is a production function employing capital and labor.
- We can think of the economy being an approximation to one using a fixed capital stock  $\bar{K}$ .
- Importantly, both filled and non-filled vacancies have some capital installed.
- Hence, part of the vacancy costs are those of the installed capital.

# The costs of an open vacancy II

In total, there are  $v + 1 - u$  jobs. Hence, the capital per job is:

$$\frac{\bar{K}}{v + 1 - u}, \quad (51)$$

and the amount of capital currently in operation is  $\frac{\bar{K}(1-u)}{v+1-u}$ . We assume TFP,  $E$ , is labor augmenting, hence, total efficient labor is  $E(1 - u)$  and output is produced by

$$Y = F \left( \frac{\bar{K}(1 - u)}{v + 1 - u}, E(1 - u) \right). \quad (52)$$

# The costs of an open vacancy III

Let  $\eta^K$  be the capital costs per vacancy. Capital is traded in a competitive market and, hence, its price is  $F_K$ :

$$\eta^K = \frac{F_K \bar{K}}{v + 1 - u} \quad (53)$$

$$= \underbrace{\frac{F_K \bar{K}}{F}}_{1/3} \frac{F}{1 - u} \underbrace{\frac{1 - u}{1 - u + v}}_{0.96}. \quad (54)$$

In steady state, labor productivity,  $A = F_L E$ , is one. Hence,

$$\frac{1 - u}{F} = \frac{F_L E (1 - u)}{F} \quad (55)$$

$$= 1 - \frac{F_K \frac{\bar{K}(1-u)}{v+1-u}}{F} = 0.68 \quad (56)$$

because with a CR production function  $\frac{F_L L}{F} = 1 - \frac{F_K K}{F}$ .

# The costs of an open vacancy IV

- Next to the capital costs, firms also spend labor in hiring.
- Survey evidence suggests that employers spend 14.0 hour per offer and require 1.14 offers per hire.
- Assuming 166 monthly hours and that this work is done by supervisors, this yields about 13.5% of monthly hours.

# The costs of an open vacancy $V$

Wages are  $w = \frac{2}{3}F$  and, hence, labor costs are:

$$lc = 0.135 \frac{2}{3} F. \quad (57)$$

We have computed  $F = \frac{1-u}{0.68} = 1.38$ . In the model, the expected hiring costs due to labor are

$$\frac{\nu^L}{q(\theta)} \quad (58)$$

with  $q(\theta) = 0.8$ . Hence,

$$\nu^L = q(\theta)lc = 0.1. \quad (59)$$

Hence,  $\nu = \nu^K + \nu^L = 0.57$ .

# The volatility of wages

So far, wages are too volatile in the model. Note, this far, we have not used the wage equation in our calibration:

$$w = (1 - \gamma)b + \gamma A + \gamma\nu\theta$$

The equation highlights that the change in wages as response to productivity depend on the worker's bargaining weight:

$$\frac{\partial w}{\partial A} = \gamma + \gamma\nu \frac{\partial \theta}{\partial A}.$$

Choosing  $\gamma = 0.05$ , i.e., a very low bargaining power of workers, delivers the desired wage volatility.

# The value of non-market work

Finally, they calibrate the value of non-market work to match the average labor market tightness using the free entry condition. In steady state, this is:

$$q(\theta) = \frac{\nu}{\frac{\beta}{1-\beta(1-\delta)} \left[ (1-\gamma)[A-b] - \gamma\nu\theta \right]}.$$

Note, with  $\gamma$  being small,  $[A-b]$  needs to be small. Indeed, we require  $b = 0.96$ . Note that a high value implies that wages are close to productivity:

$$w = (1-\gamma)b + \gamma A + \gamma\nu\theta$$

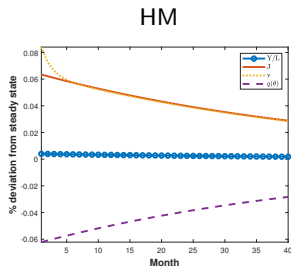
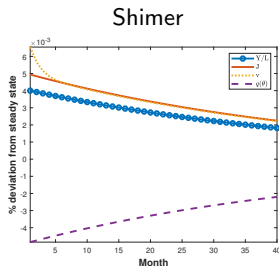
Hence, the expected firm value is small:

$$\mathbb{E}J(A') = \mathbb{E} \sum_{s=1}^{\infty} \beta^s (1-\delta)^s [A_{t+s} - w_{t+s}] \quad (60)$$

which implies that small changes in  $A_t$  will lead to large percentage changes in the firm value.

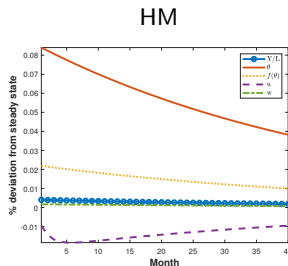
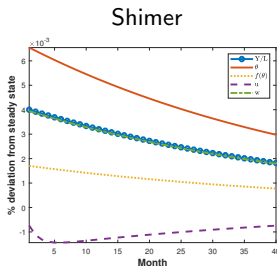


# Impulse response functions



- The same increase in labor productivity has a much larger effect on the percentage change in the value of the firm.
- As a result, vacancy creation responds much stronger.

# Impulse response functions II

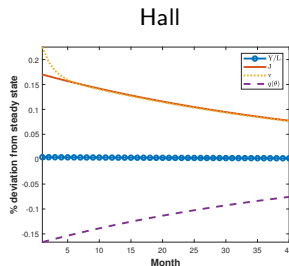
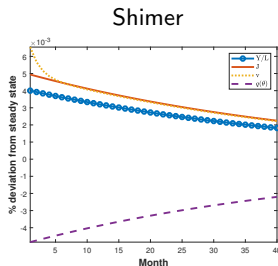


- As a result, labor market tightness and the job finding rate increase by more.
- The unemployment rate decreases by more.
- Wages rise now less than productivity.

Std. relative to $Y/L$						
	$u$	$v$	$\theta$	$f(\theta)$	$q(\theta)$	$w$
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	4.7	17.1	21	5.5	15.5	0.42

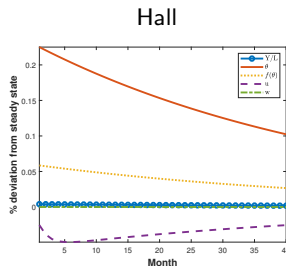
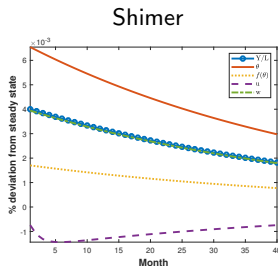
- Hall (2005) suggests to use fixed wages,  $\bar{w}$ , in the short-run.
- With fixed wages, we have to assure that  $A_t > \bar{w} \forall t$ .
- Hence, I will assume that  $\bar{w} = (1 - \gamma)b + \gamma(A^* - 2\sigma) + \gamma\nu\theta^*$ .

# Impulse response functions



- The same increase in labor productivity has a much larger effect on the change in the value of the firm.
- As a result, vacancy creation responds much stronger.

# Impulse response functions II



- As a result, labor market tightness and the job finding rate increase by more.
- The unemployment rate decreases by more.

Std. relative to $Y/L$						
	$u$	$v$	$\theta$	$f(\theta)$	$q(\theta)$	$w$
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	12.7	45.8	56.3	14.6	41.6	0

Remember that the solution is given by the following set of equations

$$\begin{aligned}q(\theta) &= \frac{\nu}{\beta \mathbb{E}J(A')} \\J(A) &= A - w + \beta(1 - \delta)\mathbb{E}J(A') \\w &= (1 - \gamma)b + \gamma A + \gamma \nu \theta\end{aligned}$$

I.e., we need to know  $J(A')$  to compute  $\theta$ . To know  $J(A')$ , we need to know  $w$ . But to know  $w$ , we need to know  $\theta$ .



# An algorithm to solve the model globally

① Guess  $\theta(A)$ .

② Solve for the wage:

$$w(A) = (1 - \gamma)b + \gamma A + \gamma \nu \theta$$

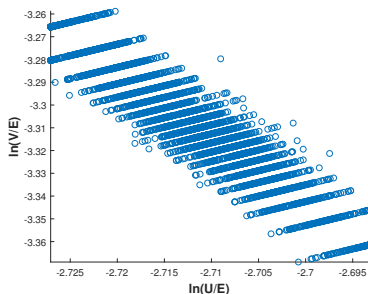
③ Solve for the firm value  $J(A)$  and its expected value  $\mathbb{E}J(A')$  by VFI:

$$J(A) = A - w + \beta(1 - \delta)\mathbb{E}J(A')$$

④ Solve for the implied  $\theta_{new}(A)$  by solving  $q(\theta) = \frac{\nu}{\beta \mathbb{E}J(A')}$ .

⑤ Check for convergence and update  $\theta = (1 - \lambda)\theta(A) + \lambda\theta_{new}(A)$ .

# Beveridge curve



- The model implies a downward-sloping Beveridge curve.
- Note, the curve is only perfect downward-sloping in steady state.
- Sluggish adjustment in unemployment and vacancies lead to small horizontal shifts.

- [1] Robert Shimer. “The cyclical behavior of equilibrium unemployment and vacancies”. In: *American economic review* 95.1 (2005), pp. 25–49.
- [2] Marcus Hagedorn and Jorgensen Manovskii. “The cyclical behavior of equilibrium unemployment and vacancies revisited”. In: *American Economic Review* 98.4 (2008), pp. 1692–1706.
- [3] Robert E Hall. “Employment fluctuations with equilibrium wage stickiness”. In: *American economic review* 95.1 (2005), pp. 50–65.