Online Appendix to "Endogenous Hours and the Wealth of Entrepreneurs"

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May 31, 2020

A Solving entrepreneurs' problem

Here we focus on the first order conditions of an entrepreneurs' problem, taking as given the productivity level z, the disutility of work v, wealth a, and also her choice on asset holdings a' and own labor supply, h.

Case 1: Employers. We start with the solution in case of employing workers. For this, it is convenient to define the following term for aggregate labor:

$$X \equiv \left[h^\rho + L^\rho\right]^{\frac{1}{\rho}}.$$

The first order conditions, given the corresponding variable is positive, are:

$$(K) \qquad \eta\alpha\frac{Y}{K} = (r+\delta)(1+C)$$
$$(L) \qquad \eta(1-\alpha)\frac{Y}{X}X^{1-\rho}L^{\rho-1} = \omega(1+C)$$

Then, if financial constraints do not bind:

$$Y = \left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta} X^{(1-\alpha)\eta}\right]^{\frac{1}{1-\alpha\eta}}$$

This equation gives Y as a function of X. The second equation to solve these two unknowns is:

$$X = \left[h^\rho + \left(\frac{(1-\lambda)(1-\alpha)\eta Y X^{-\rho}}{\omega}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}.$$

Putting two together, we get:

$$X^\rho = h^\rho + \left(\frac{(1-\alpha)\eta\left[z\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta}\right]^{\frac{1}{1-\alpha\eta}} X^{\frac{(1-\alpha)\eta}{1-\alpha\eta}-\rho}}{\omega}\right)^{\frac{\rho}{1-\rho}}$$

Which is one equation for one unknown X. Once we solve this, we get Y from the first equation. Then, we recover L and K from:

$$L = \left[\frac{(1-\alpha)\eta Y X^{-\rho}}{\omega}\right]^{\frac{1}{1-\rho}}, \quad K = \frac{\alpha\eta Y}{r+\delta}.$$

If the financial constraints bind we use the following non-linear equation to find L, and then K:

$$L = \frac{\theta a}{\omega\left[1 + \frac{\alpha}{1-\alpha}\left(\lambda\left(\frac{h}{L}\right)^{\rho} + 1\right)\right]}, \quad K = \frac{\alpha}{1-\alpha}\frac{\omega}{r+\delta}L\left[\left(\frac{h}{L}\right)^{\rho} + 1\right]$$

Case 2: Non-employers. The FOC for capital rental is:

$$\alpha\eta\frac{Y}{K} = (r+\delta)(1+C)$$

If the financial constraints do not bind:

$$Y = \left[z\kappa\left(\frac{\alpha\eta}{r+\delta}\right)^{\alpha\eta} h^{\lambda\eta}\right]^{\frac{1}{1-\alpha\eta}}, \quad \Pi = Y(1-\alpha\eta), \quad K = \alpha\eta\frac{Y}{r+\delta}$$

If the financial constraints bind, then:

$$Y = z\kappa\left(\frac{\theta a}{r+\delta}\right)^{\alpha\eta} h^{\lambda\eta}, \quad \Pi = Y - \theta a, \quad K = \frac{\theta a}{r+\delta}.$$

Iterating the prices using the non-entrepreneurs' problem We compute the aggregate supply levels not absorbed by the entrepreneurial sector of each factor, $K_j^{s,n}$, $L_j^{s,n}$ and use the first order conditions from the non-entrepreneurial sector to compute an update for equilibrium prices:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\frac{\omega}{r+\delta}, \quad \alpha^{\alpha}(1-\alpha)^{1-\alpha} = (r+\delta)^{\alpha}\omega^{1-\alpha}$$

Using these, we update the prices as follows:

$$r_{j+1} = \epsilon r_j + (1-\epsilon)\left\{\frac{\alpha}{1-\alpha}\omega_j\frac{L_j^{s,n}}{K_j^{s,n}} - \delta\right\}, \quad \omega_{j+1} = \left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_{j+1}+\delta)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$$

where ϵ is a relaxation parameter to make the updates smoother.

B Computational details

We discretize the individual state into 245 assets, 21 entrepreneurial productivity levels, 5 worker productivity levels, and 6 disutility levels of work. We allow for off grid asset choices with a total of 2197 possible choices. Regarding entrepreneurial productivity, given a share of only 9% of entrepreneurs in the economy, we are particularly interested in its right tail. Thus, we choose a uneven grid with 67% of the grid points lying in the top 10% of the ergodic productivity distribution. For the discrete hours choices, we allow for 10 grid points.

The algorithm starts by guessing an interest rate and the wage. Given prices, we solve the value functions at the grid points. We use a Monte-Carlo simulation of 10,000 individuals to compute the stationary distribution of the economy. Given the stationary distribution,

we compute total asset supply, total labor supply, and the capital and labor demand of entrepreneurs. Given these quantities, we compute the aggregate supply levels not absorbed by the entrepreneurial sector of each factor, K_n^s and L_n^s and use the first order conditions from the non-entrepreneurial sector to compute an update for equilibrium prices.

C Sensitivity analysis

We dedicate this appendix section to show the sensitivity of our model to a few changes in the modeling and calibration strategy. In particular, we first consider changes to the model features discussed in Section 2: the separate production function for non-employers, the option to return to salaried work, and the heterogeneity in the disutility of work. In addition, we show the sensitivity of our results to having a higher substitutability between owner hours and hired labor. We modify the calibration strategy one by one for each of these, and recalibrate each time the model to match (abstracting from the modified parameter) the same moments as the baseline model.

Our benchmark model allows for different production functions for employers and non-employers. In Section 3, we show that the calibration implies stronger concavity for the labor input in the case of non-employers relative to employers. The column labeled $\lambda = 1 - \alpha$ in Table A1 shows the result when we restrict these to be the same, i.e., $\lambda = 1 - \alpha$. The first panel shows that the model implies a similar dispersion of owner wealth and a similar mean owner wealth as the baseline. Similarly, the second panel shows that owners are still over-represented at the top of the economy-wide wealth distribution, yet, a sizable fraction of owners are in the bottom half of the distribution. The third panel shows that the financial frictions are weaker in this alternative calibration, particularly that the number of constrained owners is smaller. Relative to the baseline, being a non-employer becomes relatively more valuable for individuals with long hours, i.e., the selection between employers and non-employers becomes weaker. In fact, the average weekly hours of non-employers rise from 42 in the baseline to 46. With more of the productive owners staying non-employers, the collateral constrained needs to become weaker to match the average firm size among employers. The last panel shows that resulting from the less severe financial frictions, the impact of changing the collateral constrained is even weaker than in the baseline model.

Next, we focus on the probability of owners to have an option to become salaried workers, $1 - \chi_E$. In our model, the parameter χ_E captures the commitment to entrepreneurship, and we calibrate this parameter to match the share of firms surviving until year 5 relative to those of new startups. This implies a χ_E of 0.65 in our baseline calibration. To test the sensibility of our model to this calibration target, we decrease this parameter to 0.5, i.e. increase the probability of opportunity to become a salaried worker from 0.35 to 0.50. This leads to a drop in the firm survival rate to age 5 from 0.44 to 0.35. The column labeled $\chi_E = 0.5$ shows that this alternative calibration provides a dispersion of owner wealth similar to the baseline and an average level that is, though higher than in the baseline, well below the level implied by the model without owner hours. The second panel shows that the shares of owners in the top 5% and bottom 50% of the economy's wealth distribution are almost unchanged. Having fewer young firms, with a constant (calibrated) average firm age, implies that the typical owner must be further away from the participation threshold than in the baseline model. As a consequence, the second

	Benchmark	$\lambda = 1 - \alpha$	$\chi_E = 0.5$	$\frac{2}{3}\sigma_v$	$\rho = 0.7$
			Owner wealth		
Average	3.4	2.7	4.0	4.6	4.2
Sd (log)	1.6	1.6	1.6	1.7	1.6
P75/P25 ratio	8.1	8.4	8.2	9.6	8.8
			Share in wealth distribution (%)		
Top 5%	0.17	0.14	0.18	0.22	0.19
Bottom 50%	0.43	0.47	0.42	0.42	0.42
			Share of constrained owners (%)		
Before experiment	23.6	16.8	0.31	33.6	33.3
After experiment	13.6	9.39	0.19	21.3	22.8
			Change (rel. to corresponding baseline)		
Output (%)	49.5	36.5	60.5	63.2	84.3
Hired labor (%)	82.0	61.0	98.8	100.0	122.2
Wealth (%)	15.7	0.2	25.7	31.8	39.4
Wealth, P75/P25 ratio	0.9	1.0	1.9	2.9	2.1

Table A1: Sensitivity Analysis

Note: The table shows the main results of our sensitivity checks: (i) setting weight of owner hours in the non-employer production function to that of the employer function ($\lambda = 1-\alpha$), (ii) increasing the probability of having a salaried work option for entrepreneurs from 0.5 to 0.65 ($\chi_E = 0.65$), (iii) decreasing the volatility of disutility of working by one-third ($\frac{2}{3}\sigma_v$), and (iv) increasing the substitutability parameter between own hours and hired labor from 0.5 to 0.7 ($\rho = 0.7$). The first panel gives the average owner wealth relative to average worker wealth, and two measures of dispersion without entrepreneurs' wealth. The second panel gives the fraction of owners that are in the top 5% and bottom 50% of the economy-wide wealth distribution. The last two panels repeats the experiment that doubles owners' borrowing ability, θ, for each recalibration.

panel shows that financial frictions are somewhat stronger than in the baseline model, and the third panel shows that the effects of changing these frictions become stronger. Nevertheless, most owners are still unconstrained in this calibration and the effects of relaxing financial frictions are substantially weaker than in the model without owner hours.

For our benchmark model, we calibrate the variance parameter of the disutility of working shocks, σ_v targeting the standard deviation of owner hours. Here, we show the implications of reducing this parameter to a two-thirds of the benchmark calibrated value. As a consequence, the standard deviation of owner hours declines from 0.63 to 0.47. The calibration then implies a slight increased level, and an almost unchanged standard deviation of owners' wealth. The share of owners in the top 5% of the economy's wealth distribution rises to 22%, and the share of owners in the bottom 50% declines slightly relative to the baseline model. With fewer owners choosing entrepreneurship because of a high disutility of work, high productivity becomes more predominant as a motive for entrepreneurship. That is, the model becomes more similar to the model without owner hours where average firm productivity is higher, wealth still is highly dispersed, yet, around a counterfactually high level of owner wealth. Accordingly, the share of constrained owners increases to 33.6 percent, and the effects of relaxing financial frictions are larger than in the baseline model. Nevertheless, the impact of relaxing financial frictions remains much smaller than in the model without owner hours.

Finally, we study how does it affect our conclusions to increase the parameter guiding

the substitutability between owner hours and outside labor, ρ, from 0.5 to 0.7. This change naturally makes the owner hours less important for firm dynamics. Because it becomes easier to substitute owner hours, owners at intermediate and large firms work fewer hours. The average hours of owners with more than 5 employees declines from 50 to 44 hours. The column labeled $\rho = 0.7$ shows that the recalibrated model matches again the standard deviation of owner wealth and implies a somewhat higher mean owner wealth than in the baseline. Also, the shares of owners in the top 5% and bottom 50% of the economy's wealth distribution are almost unchanged. At the same time, making it easier to substitute owner hours implies that they have less of a role to play in mitigating financial frictions. Hence, the model implies that more owners are constrained than in the baseline model; the share increases to 33.3 percent. Consequently, the impact of relaxing financial frictions on the entrepreneurial sector becomes stronger. For example the change in output in the baseline model when relaxing the collateral constrained by one half is 49.5 percent and increases to 84.3 percent in the model with $\rho = 0.7$. Though the effect is substantially larger than in the baseline model, it is still only half of the effect in the model without owner hours.