Price Caps with Capacity Precommitment*

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Abstract

We examine the effectiveness of price caps to regulate imperfectly competitive markets in which the demand is uncertain. To that effect, we study a monopoly that makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon observing the realization of demand. We show that the optimal price cap must trade off the incentives for capacity investment and capacity withholding, and is above the unit cost of capacity. Moreover, while a price cap provides incentives for capacity investment and mitigates market power, it cannot eliminate inefficiencies. Capacity payments provide a useful complementary instrument.

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1 Introduction

Since Littlechild (1983)'s report, price cap regulation is regarded as an effective instrument to mitigate market power, foster cost minimization, and ultimately enhance surplus: When precise information about cost and demand is available, the introduction of a binding price cap raises firms' marginal revenue near the equilibrium output and leads to an increase of the equilibrium output and surplus, and to a decrease of the market price. Moreover, under broad regularity conditions on the demand and cost functions, for any price cap above marginal cost both output and surplus decrease, and the market price increases with the price cap. Further, in the most favorable conditions (e.g., when firms produce the good with constant returns to scale), a price cap equal to marginal cost is able to eliminate inefficiencies. (In contrast, rate-of-return regulation, used for most of the 20th century to regulate public utilities, distorts incentives for cost minimization – see, e.g., Joskow (1972) – or cost reduction – see, e.g., Cabral and Riordan (1989).)

We study the effectiveness of price cap regulation under demand uncertainty and capacity precommitment and withholding. Demand uncertainty may be interpreted also as variations of demand over time – see Green and Newbery (1992) for a discussion of this interpretation in electricity markets. Capacity withholding is common in markets such as sport events, hotel accommodation, agricultural products, or electricity. In markets for agricultural products, farmer associations sometimes destroy part of the output. In electricity markets firms may declare some of their generators to be unavailable – data for the California electricity market during the time period May 2000-December 2001 show that at the price cap some generators did not supply all of their uncommitted capacity – see Cramton (2003) and Joskow and Kahn (2002).

It is easy to show that in the absence of capacity precommitment, e.g., when the good can be produced instantly upon the realization of demand or there is slack capacity, the effectiveness of price caps and their comparative static properties with respect to the expected output, expected price, and expected surplus remain the same as when the demand is deterministic. The only effect of uncertainty is smoothing the non-differentiability at the lowest non-binding price cap arising when the demand is deterministic. In particular, a price cap equal to marginal cost maximizes the expected surplus. The intuition of these results is analogous to that of the deterministic demand case – see Lemus and Moreno (2015). The analysis of this case is relevant for, e.g., the Spanish or California electricity markets, in which firms have excess capacity (at least in recent times), and their bids are short lived (firms compete to serve the demand for only hourly or half hourly periods). Of course, price cap regulation has an impact on firms' capacity investments, which are long run decisions made prior to the realization of demand. Thus, endogenizing firms' capacity investment decisions seems a natural next step to take.

In order to tackle this issue, we consider a setting in which a monopoly makes irreversible capacity investments ex-ante, and then chooses its output up to capacity upon observing the realization of demand. Thus, the monopoly may withhold capacity if it is beneficial to do so. In this setting, inefficiencies arise both because the monopoly installs a low level of capacity in order to precommit to high prices, and because the monopoly withholds capacity for low demand realizations in order to keep prices from falling too much.

Focusing on the monopolistic case allows us to avoid some potential conundrums that arise in oligopolistic settings, which are distractions from the issue under scrutiny – the impact of price cap regulation. For example, it is unclear what is the appropriate model of competition to consider at the ex-post stage. Moreover, when demand is uncertain there are well known difficulties therein to guarantee existence, uniqueness and symmetry of equilibrium – see, e.g., Reynolds and Wilson (2000), Gabszewicz and Poddar (1997).

The effects of price cap regulation with demand uncertainty and capacity precommitment and withholding are subtle. We show that, much as in the absence of capacity precommitment, the introduction of a sufficiently large binding price cap raises the firms' marginal return to capacity investment near the equilibrium capacity and leads to an increase of the equilibrium capacity, the expected output and the expected total surplus, and to a decrease of the expected market price. However, price caps near the unit cost of capacity are suboptimal because they reduce the return to capacity investment below its cost, and lead the monopoly to install no capacity.

The optimal price cap (i.e., the price cap that maximizes surplus) must trade off the incentives for capacity investment (a dynamic efficiency effect) and capacity withholding (a static efficiency effect), and tends to be well above the unit cost of capacity. When the unit cost of capacity is high, the dynamic effect on capacity investment is a first order effect, while the static effect on capacity withholding is a second order effect. Thus, in this case the optimal price cap maximizes capacity investment. When the unit cost of capacity is low, however, near the price cap that maximizes capacity investment the dynamic effect on capacity investment is a second order effect, while the static effect on capacity withholding is a first order effect. Thus, in this case reducing the price cap below the level that maximizes capacity investment increases expected surplus, and therefore the optimal price cap does not maximize capacity investment.

The comparative static properties of price caps are complex: the effect of a change of the price cap on expected output and expected surplus depend on the magnitudes of the static effect (on withholding) and the dynamic effect (on capacity investment), which may have opposite signs. Under standard regularity assumptions on the demand distribution, capacity investment is a single peaked function of the price cap: for low price caps capacity investment increases with the price cap until it reaches a maximum at some binding price cap r^*, and then decreases with the price cap above r^*. When the unit cost of capacity is large the signs of the effects of changes in the price cap on expected output, expected surplus and capacity investment coincide. Interestingly, when the unit cost of capacity is small, expected output and expected surplus decrease with the price cap above and around r^*, and thus the optimal price cap is below r^*. Price caps affect the market price directly, but also indirectly via their impact on the level of capacity. Thus, an increase of the price cap increases the expected price above and around r^*, but has an ambiguous effect below r^*.

Introducing a sufficiently large binding price cap enhances the incentives for capacity investment and discourages capacity withholding. Nonetheless, a price cap alone is unable to provide the appropriate incentives for capacity investment and simultaneously eliminate the inefficiencies arising from capacity withholding: the optimal price cap induces a low level of capacity, and does not prevent capacity withholding. Hence, with demand uncertainty and capacity precommitment an optimal regulatory policy may require using other instruments.

While a full analysis of complementary instruments available to reduce inefficiencies is outside the scope of the present paper, we study the impact of capacity payments, which have been used in, e.g., electricity markets. We show that when the cost of capacity is large, introducing a small capacity payment, and accommodating accordingly the optimal price cap, increases the surplus. (Signing the effect of capacity payments when the cost of capacity is small seems difficult.) The effect of capacity payments is further illustrated in the examples in section 6, in which we evaluate the impact on equilibrium of a small capacity payment combined with an optimal reduction the price cap: relative to the equilibrium arising with only an optimal price cap, in this equilibrium there is more capacity investment and less withholding and, consequently, the expected output and surplus are larger – see figures 5 and 6.

Our assumption that demand is linear and subject to an additive shock is restrictive, although it is common in the literature. However, our main conclusions seem to hold more generally. For example, we obtain analogous results with a multiplicative, uniformly distributed demand shock.

Earle et al. (2007) studies an oligopolistic model in which firms make output decisions ex-ante, i.e., firms choose their output before the realization of demand and supply it inelastically and unconditionally. In this setting, they show that for price caps near marginal cost the output is suboptimally low and may increase with the price cap. Moreover, they establish that the comparative static properties of price caps that hold when the demand is deterministic fail for a generic stochastic demand schedule. (The source of this result is not demand uncertainty *per se*, but quantity precommitment, which is assumed in the model.) Grimm and Zoettl (2010) establish that under standard regularity assumptions the comparative static properties of price caps are recovered. (Also, Grimm and Zoettl (2010) consider a setting in which firms may withhold capacity, but do not study the trade-offs of capacity investment and withholding, and mistakenly conclude that maximizing the expected surplus amounts to maximizing capacity.) In a similar setting, Reynolds and Rietzke (2012) study the impact of price caps in oligopolistic markets with endogenous entry, and identify conditions under which a price cap improves welfare.

Other authors have studied the dynamic effects of price cap regulation. Dixit (1991) studies a competitive market in which the demand is uncertain and firms make ex-ante irreversible investments, and shows that price caps delay investments and lead to higher prices over time. Biglaiser and Riordan (2000) show that in the presence of exogenous technological progress price caps provide better incentives for capacity investment and replacement than rate-of-return regulation. Dobbs (2004) studies the intertemporal effect of an optimal price cap on the size and timing of the investments of a monopoly that faces an uncertain demand, and shows that it leads to under investment and quantity rationing – Roques and Savva (2009) obtain similar conclusions in an oligopolistic extension of this setting. Also, consistent with our results, Dobbs (2004) shows that a price cap is an effective instrument when the unit cost of capacity is small relative to the consumers' willingness to pay (or the demand rate or growth), than when it is large. As in our setting, these models assume constant return to scale. However, they do not allow for capacity withholding.

The paper is organized as follows: In Section 2 we describe our model and derive preliminary results. In Section 3 we study the comparative static properties of price caps. In Section 4 we study optimal price caps. In Section 5 we discuss the usefulness of capacity payments. In Section 6 we apply our analysis to a simple example, which provides clear illustration of our findings. The Appendix contains the proofs.

2 The Model

Consider a monopoly facing an uncertain demand given for $p \in \mathbb{R}_+$ by $D(X, p) = \max\{X - p, 0\}$, where X is a continuous random variable with *c.d.f.* and *p.d.f.* denoted by F and f, respectively. The monopoly must decide how much capacity to install, $k \in \mathbb{R}_+$, before the demand is realized. The cost of installing a unit of capacity is a positive constant $c > 0$. Once capacity is installed the good can be produced with constant returns to scale up to capacity. We assume without loss of generality that the production cost is zero. The monopoly decides its output upon observing the realization of demand, and may withhold capacity if doing so is beneficial.

In order to rule out trivial cases in which the monopoly installs no capacity we assume that $\mathbb{E}(X) > c$. Also we reduce notation by assuming that the support of X is the interval $[0, 1]$. Under this assumption the consumers' willingness to pay is always above the cost of production, which implies that the equilibrium price is a well-defined random variable. This facilitates presenting and interpreting our results, but entails a small loss of generality.

Suppose that a regulatory agency imposes a price cap $r \in [0, 1]$. In order to identify the monopoly's capacity choice $k^*(r)$, we proceed by backward induction to identify first the monopoly's output $Q(r, k, X)$, and the market price $P(r, k, X)$. Since the cost of capacity is sunk and the cost of production up to capacity is zero, then at the stage of output choice the monopoly maximizes revenue. We note that levels of capacity $k > \max\{1 - r, 1/2\}$ are suboptimal: If the monopoly was not capacity constrained, then its output for $x \in [0, 1]$ would be $x/2 \leq 1/2$ if $x/2 \leq r$, and it would be $x - r \leq 1 - r$ if $x/2 > r$. Hence if $k > \max\{1 - r, 1/2\}$, then the monopoly would maintain idling capacity, and therefore since $c > 0$ it would be able to increase its profit by installing less capacity. Thus, we restrict attention to price cap-capacity pairs $(r, k) \in [0, 1]^2$ such that $k \leq \max\{1 - r, 1/2\}$.

Figure 1 describes a partition of the set of relevant price cap-capacity pairs into three regions, $A = \{(r, k) \in [0, 1]^2 \mid r \leq k \leq 1 - r\}$, $B = \{(r, k) \in [0, 1]^2 \mid k < \min\{1 - r, r\}\}$, and $C = \{(r, k) \in [0, 1]^2 \mid 1 - r \leq k \leq 1/2\}$. We calculate the equilibrium price and output in regions A, B, and C for each realization of the demand parameter X. In all three regions the monopoly equilibrium emerges for low demand realizations $x \in [0, 2r)$. However, as we consider larger realizations of the demand parameter, while in region A the price cap becomes binding (i.e., the monopoly equilibrium price is above the price cap) before the capacity is fully utilized, in region B capacity binds before the price cap does. In region C the price

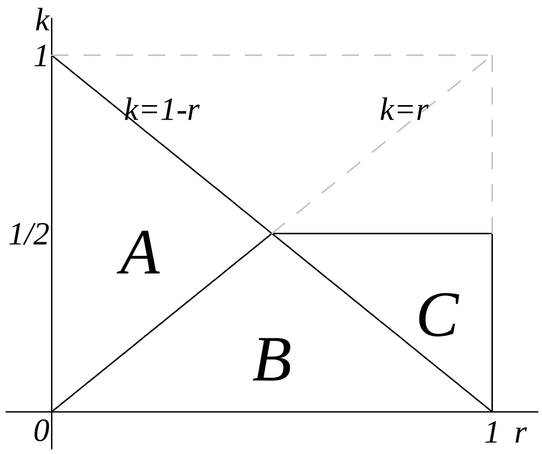

Figure 1: Relevant Price Cap-Capacity Pairs

cap never binds.

Table A describes the prices and outputs for $(r, k) \in A$.

X	$[0, 2r)$	$[2r, r + k)$	$[r + k, 1]$
$P(r, k, x)$	$x/2$	r	r
$Q(r, k, x)$	$x/2$	$x - r$	k

Table A: Equilibrium Output and Price for $(r, k) \in A$.

Figure 2 illustrates the results in Table A. For low demand realizations (such as $x_0 < 2r$ in Figure 2), the price cap is non-binding, and the unconstrained monopoly equilibrium arises. For intermediate demand realizations (such as $x_1 \in (2r, r + k)$ in Figure 2) the price cap binds, and the monopoly serves the demand at the price cap, $x_1 - r$. (Note that marginal revenue becomes negative for output levels greater than $x_1 - r < k$.) Thus, for intermediate demand realizations a marginal decrease of the price cap leads to an increase of output. For high demand realizations (such as $x_2 > r + k$ in Figure 2) the marginal revenue remains equal to $r > 0$ even if the monopoly serves its entire capacity. Hence the monopoly serves its entire capacity k, and the demand $x_2 - r > k$ is rationed.

Note the main features of equilibrium for price cap-capacity pairs in region A: the monopoly withholds capacity except for high demand realizations, the demand is rationed only for high demand realizations, and the market price $P(r, k, x)$ is independent of the level of installed capacity k. Increasing capacity affects the revenue only for high demand realizations $x > r + k$ for which the monopoly supplies its entire capacity. For these demand realizations the price cap r is binding. Thus, the expected

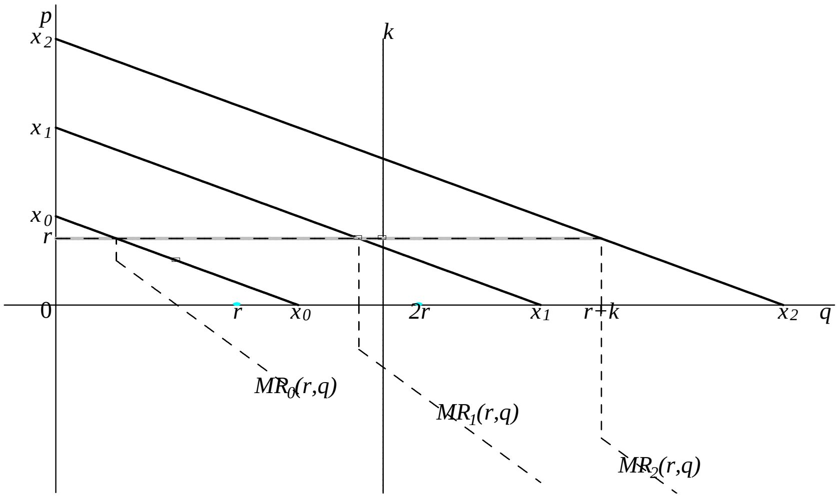

Figure 2: The Effect of a Price Cap when $(r, k) \in A$

revenue increases by r times the probability that the additional marginal unit of capacity is supplied, i.e., the expected marginal revenue to capacity is $r[1 - F(r + k)]$ – see the proof of Lemma 1 in the appendix.

Table B describes the prices and outputs for $(r, k) \in B$.

X	$[0, 2k)$	$[2k, r + k)$	$[r + k, 1]$
$P(r, k, x)$	$x/2$	$x - k$	r
$Q(r, k, x)$	$x/2$	k	k

Table B: Equilibrium Output and Price for $(r, k) \in B$.

Figure 3 illustrates the results in Table B. For low demand realizations (such as $x_0 < 2k$ in Figure 3) the price cap is not binding, and the unconstrained monopoly equilibrium arises. For intermediate demand realizations (such as $x_1 \in (2k, r + k)$ in Figure 3), marginal revenue remains positive even when the monopoly serves its full capacity, and the price that clears the market when the monopoly serves its full capacity is below the price cap. Thus, the monopoly serves its full capacity and the price cap is non-binding. For high demand realizations (such as $x_3 > r + k$ in Figure 3) marginal revenue remains positive even when the monopoly serves its full capacity, but the price that clears the market is above the price cap. Thus, the monopoly serves its full capacity and the price-cap is binding.

Note the main features of equilibrium for price cap-capacity pairs in region B: the monopoly withholds capacity only for low demand realizations, the demand is rationed only for high demand realizations, and the market price $P(r, k, x)$ depends

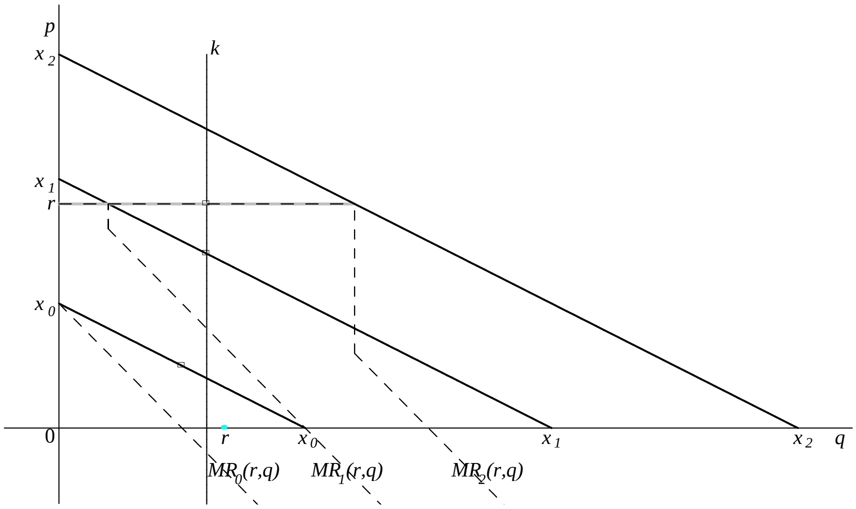

Figure 3: The Effect of a Price Cap when $(r, k) \in B$

on the level of capacity. Changes in the price cap affect the output for intermediate and high demand realizations, and the market price for high demand realizations.

Table C describes the prices and output for $(r, k) \in C$.

X	$[0, 2k)$	$[2k, 1]$
$P(r, k, x)$	$x/2$	$x - k$
$Q(r, k, x)$	$x/2$	k

Table C: Equilibrium Output and Price for $(r, k) \in C$.

In region C the price cap is never binding. The monopoly withholds capacity only for low demand realizations, $x \in [0, 2k)$, and supplies its entire capacity otherwise. Demand is never rationed. The market price $P(r, k, x)$ depends on capacity.

We study the monopoly's capacity choice. Given (r, k) the monopoly revenue is

$$R(r, k, X) = P(k, r, X)Q(r, k, X),$$

and its expected profit is

$$\bar{\Pi}(r, k) = \mathbb{E}\left(R(r, k, X)\right) - ck.$$

Clearly $\bar{\Pi}$ is continuous on $A \cup B \cup C$.

Using the results described in tables A, B and C we readily calculate the monopoly's expected profit, and verify that the expected marginal revenue to capacity, $\overline{MR}(r, k) = \partial \mathbb{E}\left(R(r, k, X)\right)/\partial k$, is decreasing in k, and is differentiable on $A \cup B \cup C$ – see the proof of Lemma 1 in the Appendix. Note that while in region A a marginal increase of capacity increases revenue only for high demand realizations (i.e., for $x > r + k$), in regions B and C a marginal increase of capacity increases revenue for high and intermediate demand realizations (i.e., for $x > 2k$).

In equilibrium, the monopoly's capacity $k^*(r)$ maximizes $\bar{\Pi}(r, \cdot)$. Thus, since $\bar{\Pi}(r, \cdot)$ is a twice differentiable and strictly concave function for all $r \in [0, 1]$, $k^*(r) = 0$ if $\overline{MR}(r, 0) < c$, and otherwise $k^*(r)$ is the unique solution of the equation

$$\overline{MR}(r, k) = c. \tag{1}$$

Moreover, the Maximum Theorem implies that k^* is a continuous function. We summarize these results in Lemma 1.

Lemma 1. *For all $r \in [0, 1]$, $\bar{\Pi}(r, \cdot)$ is a twice differentiable and strictly concave function, and hence the equilibrium capacity $k^*(r)$ is a continuous function.*

Calculating the equilibrium capacity is somewhat involved. Obviously, the equilibrium capacity is zero for price caps below the unit cost of capacity c. Hence, price caps near the unit cost of capacity are suboptimal. Moreover, it is easy to see that the equilibrium capacity is also zero for price caps r above but near the unit cost of capacity: because the probability of demand realizations $x < c$ is positive, for r above but near c the expected marginal revenue is below c even for $k = 0$. Therefore installing capacity entails losses. Thus, the equilibrium capacity is zero unless the price cap is sufficiently high that expected marginal revenue for levels of capacity near zero is greater than c, i.e., $r \geq \underline{r}(c)$, where $\underline{r}(c)$ is the unique solution to the equation $\overline{MR}(r, 0) = c$. (If the lower bound of the support of X is $\alpha > c$, instead of zero as we have assumed, then for $r = c$ the expected marginal revenue is c and profits are zero for $k \in [0, \alpha - c]$, whereas profits are negative for $k > \alpha - c$. Hence the equilibrium capacity may be positive, and may increase or decrease with r near the unit cost of capacity depending of the distribution of demand.)

Obviously, sufficiently large price caps are non-binding. Specifically, the largest binding price cap $\bar{r}(c)$ is the unique solution to the equation $c = \overline{MR}(r, 1 - r)$. For $r \geq \bar{r}(c)$ the price cap is not binding, and $(r, k^*(r))$ is in region C. We denote by k_C the equilibrium capacity, i.e., solution to the equation (1), when the price cap is not binding.

Intermediate price caps $r \in [\underline{r}(c), \bar{r}(c))$ affect the equilibrium capacity in more complex ways. The equilibrium capacity as a function of the price cap, $k^*(r)$, differs depending on whether $(r, k^*(r))$ is in region A or B. The solution to equation (1) in region A is $k_A(r) = F^{-1}(1 - c/r) - r$. The solution to equation (1) in region B, $k_B(r)$, cannot be obtained in closed form. We show that $k^*(r) = k_A(r)$ for price caps such that $\overline{MR}(r, r) \geq c$, and that $k^*(r) = k_B(r)$ otherwise.

When the hazard rate of X is increasing, then for $c \leq M^* := \max_{r \in [0,1/2]} \overline{MR}(r, r)$, $\overline{MR}(r, r) \geq c$ on an *interval* $[r_-(c), r_+(c)]$, where $r_-(c)$ and $r_+(c)$ are the smaller and larger solutions to the equation $\overline{MR}(r, r) = c$, and satisfy $\underline{r}(c) < r_-(c) < r_+(c) < \bar{r}(c)$. Hence under this assumption the equilibrium capacity is zero below $\underline{r}(c)$, $k_B(r)$ on $[\underline{r}(c), r_-(c))$, $k_A(r)$ on $[r_-(c), r_+(c)]$, $k_B(r)$ on $(r_+(c), \bar{r}(c)]$, and k_C above $\bar{r}(c)$. Lemma 2 states these results precisely.

Lemma 2. *The equilibrium capacity is* $k^*(r) = 0$ *if* $r \in [0, \underline{r}(c))$, *and it is* $k^*(r) = k_C$ *if* $r \in [\bar{r}(c), 1]$. *For* $r \in [\underline{r}(c), \bar{r}(c))$, $k^*(r) = k_A(r)$ *if* $\overline{MR}(r, r) \geq c$, *and* $k^*(r) = k_B(r)$ *otherwise; moreover, if the hazard rate of* X *is increasing, then* $\overline{MR}(r, r) \geq c$ *holds on a subinterval of* $(\underline{r}(c), \bar{r}(c))$ *when* $c \leq M^*$, *and does not hold otherwise.*

Using the results in tables A, B and C, and the description on the equilibrium capacity given in Lemma 2 , we can calculate the expected output and market price as well as the expected surplus, thus providing a complete description of the monopoly equilibrium. In Section 6 we solve an example in which X in uniformly distributed.

3 Comparative Statics

In this section we study the comparative static properties of price caps when the hazard rate of X is increasing and its *p.d.f.* f is differentiable. We first show that under these regularity assumptions on the distribution of demand the equilibrium capacity k^* is a *single peaked* function of the price cap r on $(\underline{r}(c), \bar{r}(c))$. Thus, the comparative static properties of the equilibrium capacity are analogous to those price caps have on expected output when capacity has no precommitment value.

It is easy to see that in our setting when capacity lacks precommitment value (i.e., when it can be built instantly), regardless of whether or not the hazard rate of X is increasing and/or f is differentiable, the expected output (which is equal to capacity) is zero when the price cap r is below c, has an upward discontinuity at $r = c$, at which point reaches its maximum value, and decreases smoothly with r above c – see Lemus and Moreno (2015). We show that with capacity precommitment capacity is zero for $r \in [0, \underline{r}(c))$, where $\underline{r}(c) > c$, then increases with r until it reaches its maximum value on $(\underline{r}(c), \bar{r}(c)))$, and then decreases with r until the price cap becomes non-binding at $\bar{r}(c)$, remaining constant above $\bar{r}(c)$. These general features are illustrated in figures

5 and 6 of Section 6 in which we offer graphs of these functions for an example in which X is uniformly distributed. We shall see that expected output and surplus behave analogously.

We state these results in Proposition 1. The proofs of these results, which are standard, involve implicitly differentiating the first order condition for profit maximization – see the Appendix.

Proposition 1. *Assume that the hazard rate of X is increasing and its p.d.f. is differentiable. Then $k^*(\cdot)$ is a differentiable single peaked function on $(\underline{r}(c), \bar{r}(c))$; i.e., $k^*(\cdot)$ has a maximum at some $r^*(c) \in (\underline{r}(c), \bar{r}(c))$, and $dk^*(r)/dr > 0$ on $(\underline{r}(c), r^*(c))$ whereas $dk^*(r)/dr < 0$ on $(r^*(c), \bar{r}(c))$.*

Next, we discuss the effects of changes in the price cap on the expected output and the expected price. The expected output is readily calculated using the results described in tables A, B and C. In region A, the monopoly maintains idling capacity for intermediate demand realizations in which the price cap is binding. Thus, in region A the expected output strictly decreases with the price cap given the level of capacity. Since for price caps $r \in [r_-(c), r_+(c)]$ the equilibrium capacity k^* satisfies $(r, k^*(r)) \in A$, then the expected output decreases on $[r_-(c), r_+(c)]$ provided the equilibrium capacity does not decrease, i.e.,

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} < 0.$$

Hence when the price cap that maximizes capacity $r^*(c)$ is in the interval $[r_-(c), r_+(c)]$, the expected output decreases with the price cap on $[r^*(c), r_+(c)]$. Therefore the price cap that maximizes output is below $r^*(c)$ since, as the proof of Proposition 2 given in the Appendix shows, near $r^*(c)$ a decrease of the price cap has only a second order effect on capacity, while it has a first order effect on demand via price reduction.

In region B, however, the output does not dependent directly on the price cap, but only indirectly via its impact on the equilibrium level of capacity. Thus, for price caps $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, for which $(r, k^*(r)) \in B$, the signs of the effects of changes in the price cap on expected output and capacity are the same, i.e.,

$$\frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk^*}{dr} \gtreqless 0.$$

Let us discuss the effect of changes in the price cap on the expected price. In region A the market price is independent of k, and therefore a change in the price cap only has a direct (positive) effect on P. Hence the expected market price increases with the price cap regardless of its impact on capacity. Since $(r, k^*(r)) \in A$ for $r \in [r_-(c), r_+(c)]$, then

$$\frac{d\mathbb{E}(P(r, k^*(r), X)}{dr} > 0$$

on $[r_-(c), r_+(c)]$. In region B, however, the market price depends on k, and therefore a change in the price cap has an indirect effect on the market price via its impact on the level of capacity, as well as a direct (positive) effect. When this indirect effect is also positive, i.e., when $dk^*/dr < 0$, then the total effect is positive, but when the indirect effect is negative, the sign of the total effect is ambiguous. Since $(r, k^*(r)) \in B$ for $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{d\mathbb{E}(P(r, k^*(r), X)}{dr} > 0.$$

Therefore

$$\frac{d\mathbb{E}(P(r, k^*(r), X)}{dr} > 0$$

on $[\underline{r}(c), r^*(c))$. However, the sign of this derivative on $(r^*(c), \bar{r}(c)]$ is ambiguous. Obviously, changes in the price cap on $[0, \underline{r}(c)) \cup (\bar{r}(c), 1]$ have no effect on the expected price. We summarize these results in Proposition 2.

Proposition 2. *Assume that the hazard rate of X is increasing and its p.d.f. is differentiable. If $r^*(c) \in (r_-(c), r_+(c))$, then the expected output decreases with the price cap above and around $r^*(c)$; otherwise the expected output increases with the price cap on $(\underline{r}(c), r^*(c))$ and decreases on $(r^*(c), \bar{r}(c))$. Moreover, the expected price increases with the price cap on $[r_-(c), r_+(c)] \cup [r^*(c), \bar{r}(c))$.*

Proposition 2 reveals that with demand uncertainty and capacity precommitment the comparative static properties of price caps are somewhat complex. In particular, when c is small the capacity maximizing price cap $r^*(c)$ does not maximize the expected output: decreasing the price cap below $r^*(c)$ leads to an increase of the expected output even though installed capacity decreases. Of course, this fact has direct implications on the price cap that maximizes the expected surplus, as we shall see in the next section.

4 Optimal Price Caps

A regulator who wants to maximize the expected surplus using a price cap as its single instrument, and cannot force the monopoly to serve its full capacity, must trade off the incentives for capacity investment and capacity withholding, and must account for the cost of installing capacity (some of which may be seldom utilized). Thus, the optimal price cap may differ from the price cap that maximizes capacity investment $r^*(c)$. Indeed, we show that when the unit cost of capacity is small the optimal price cap is below $r^*(c)$: For price caps near $r^*(c)$ reducing the price cap has a first order positive effect on surplus by discouraging capacity withholding, and only a second order negative effect on surplus by diminishing the incentives for capacity investment. When the unit cost of capacity is high, however, the price cap affects surplus only via its impact on capacity investment, and thus the optimal price cap is $r^*(c)$. (Hence when capacity cannot be withheld, as in the model of Earle et al. (2007) and Grimm and Zoettl (2010), maximizing the expected surplus simply amounts to maximizing capacity.) Obviously a price cap affects the distribution of surplus also. A regulator who wants to maximize the consumer surplus, for example, would choose as well a price cap below $r^*(c)$ when the cost of capacity is low.

Denote by $S(r, k, X)$ the equilibrium gross surplus (i.e., the surplus ignoring the cost of capacity) as a function of the price cap, capacity, and demand realization. Following the literature, we simplify somewhat the problem by assuming efficient rationing, i.e., when demand is rationed the consumers with the largest willingness to pay receive priority to buy the good. See tables 3A and 3BC in the proof of Proposition 3 in the Appendix. The expected surplus is

$$\bar{S}(r, k) := \mathbb{E}(S(r, k, X)) - ck.$$

An optimal price cap maximizes $\bar{S}(r, k^*(r))$ on $[0, 1]$.

As Table A above shows, for $(r, k) \in A$ the monopoly withholds capacity for demand realizations in the interval $[0, r + k)$, and therefore the expected gross surplus depends directly on the price cap, as well as indirectly through its effect on capacity. When $(r, k) \in B \cup C$, however, the price cap has no direct effect on the expected gross surplus, but only has an indirect effect via its influence on capacity – see tables B and C. These observations are made precise by differentiating $\bar{S}$, to obtain

$$\frac{d\bar{S}(r, k^*(r))}{dr} = s(r) I_{[r_-(c), r_+(c)]}(r) + \frac{dk^*(r)}{dr} \left(\int_{r+k^*(r)}^{1} (x - k^*(r)) f(x) dx - c \right), \quad (2)$$

where I is the indicator function, and $s(r) = -r[F(r + k^*(r)) - F(2r)]$. (See the proof of Proposition 3 in the Appendix.)

For $r \in [r_-(c), r_+(c)]$ the two terms in the expression (2) identify the direct and indirect effects on surplus, respectively, of changes in r. Since $(r, k^*(r)) \in A$, then $k^*(r) = k_A(r) > r$, and therefore the sing of the direct effect is negative, i.e., $s(r) < 0$. Moreover, if $r \in [r_-(c), r^*(c)]$, then $dk^*(r)/dr \leq 0$, i.e., the indirect effect is also negative, and therefore the total effect is negative, i.e., $d\bar{S}(r, k^*(r))/dr < 0$. Hence the expected surplus decreases with the price cap at $r^*(c)$: Even though decreasing the price cap below $r^*(c)$ decreases capacity, it discourages capacity withholding and increases surplus. Thus, the optimal price cap is below $r^*(c)$.

For $r \in [0,1]\backslash[r_-(c), r_+(c)]$ the first term in (2) is zero: Changes in the price cap have only an indirect effect on surplus via their impact on capacity investment, and the sign of $d\bar{S}(r, k^*(r))/dr$ is that of $dk^*(r)/dr$. Thus, when $r^*(c) \in (\underline{r}(c), \bar{r}(c))\backslash[r_-(c), r_+(c)]$ the optimal price cap is $r^*(c)$ – see the proof of Proposition 3 in the Appendix.

Proposition 3 summarizes these results.

Proposition 3. *Assume that hazard rate of X is increasing and its p.d.f. is differentiable. If $r^*(c) \in [r_-(c), r_+(c)]$ then the expected surplus decreases with the price cap above and around $r^*(c)$, whereas if $r^*(c) \in (\underline{r}(c), \bar{r}(c))\backslash[r_-(c), r_+(c)]$, then $r^*(c)$ maximizes the expected surplus.*

5 Capacity Payments

In the absence of capacity precommitment a price cap equal to the unit cost of capacity eliminates all inefficiencies. With capacity precommitment, however, the optimal price cap has to trade off the incentives for capacity investment and capacity withholding, and cannot eliminate inefficiencies: as we show, capacity investment is inadequately low and underused.

In order to see this, we calculate the surplus realized when $k \in [0,1]$ units of capacity are installed and supplied unconditionally, denoted by S^*, which is given by

$$S^*(k) = \frac{1}{2}\int_0^k x^2 f(x)dx + \frac{1}{2}\int_k^1 (2x-k)kf(x)dx - ck.$$

Differentiating S^* yields $d^2S^*(k)/dk^2 = -[1-F(k)] < 0$. Hence S^* is a concave function, and since it is increasing near $k = 0$, the socially optimal capacity, denoted by k^W, solves the equation $dS^*(k)/dk = 0$.

Proposition 4 establishes that an optimal price cap alone fails to provide incentives to install the optimal level of capacity; that is, $k^W > k^*(r^*(c))$. In addition, price caps alleviate, but do not eliminate the inefficiencies arising from capacity withholding. Of course, taking control of the firm, and then installing and supplying unconditionally k^W units would eliminate inefficiencies. However, such intervention likely involves a large subsidy with a prohibitively large opportunity cost, be it in terms of the distortions created in raising such revenue, or in terms of the benefits of its alternative use.

Investment adequacy is a traditional theme of the literature on market regulation. This literature regards capacity payments as a useful instrument to restore investment adequacy. In the electricity industry, for example, capacity markets have been introduced in the US, Central and South America and, more recently, the United Kingdom. Also, Sweden and Finland incentivize strategic reserves, and Spain, Portugal, Italy and Ireland provide capacity subsidies – Joskow (2007) and Briggs and Kleit (2013) study of the impact of capacity subsidies in competitive electricity markets.

Let us then examine the impact on the social surplus of a marginal capacity payment combined with a price cap set up to maximize the surplus. A capacity payment z amounts to reducing the cost of capacity to the monopoly from c to $c-z$. Let us denote by $\tilde{k}^*(r,z)$ the monopoly's capacity choice with a price cap $r \in [0,1]$ and a capacity payment $z \in [0,c]$, and by $\tilde{r}^*(z)$ the price cap that maximizes the expected surplus, $\tilde{S}(r,z)$, which is given by

$$\tilde{S}(r,z) = \mathbb{E}(S(r,\tilde{k}^*(r,z),X)) - c\tilde{k}^*(r,z).$$

In Proposition 4 we show that when the cost of capacity is large introducing a small capacity payment increases surplus. We establish this result by evaluating $d\tilde{S}(\tilde{r}^*(z),z)/dz$ and showing that is positive near $z=0$ – see the Appendix. The sign of this derivative is unclear when the cost of capacity is small. In the example discussed in the next section, however, a small capacity payment has a positive impact on surplus both when the cost of capacity is large and when it is small.

Proposition 4. *Assume that hazard rate of X is increasing and its p.d.f. is differentiable. Then the equilibrium capacity with an optimal price cap alone is below the optimal level of capacity k^W. Moreover, if the cost of capacity is large, i.e., the optimal price cap maximizes capacity, then a marginal capacity payment increases the net surplus and the installed capacity.*

6 An Example

Assume that X is uniformly distributed. Hence its *p.d.f.*, which is given by $f(x)=1$, is differentiable, and its hazard rate, $h(x)=(1-x)^{-1}$, is increasing. Since $\mathbb{E}(X) = 1/2$, we consider values of the unit costs of capacity $c \in (0,1/2)$.

We calculate the equilibrium capacity. By Proposition 1,

$$k_A(r) = F^{-1}(1-\frac{c}{r}) - r = 1 - \frac{c}{r} - r.$$

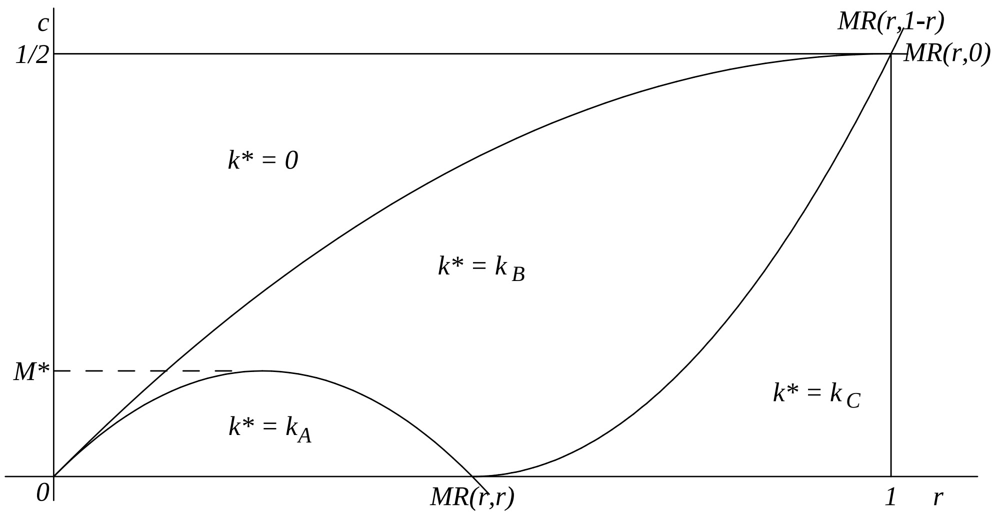

Figure 4: Equilibrium Capacity

The expected marginal revenue is $\overline{MR}(r,k) = (k^2 + 2(1-2k)r - r^2)/2$ in region B, and $\overline{MR}(r,k) = (1-2k)^2/2$ in region C – see equations (4) and (5) in the proof of Lemma 1 in the Appendix. Solving equation (1) yields

$$k_B(r) = 2r - \sqrt{2c - r(2-5r)}, \text{ and } k_C = \left(1 - \sqrt{2c}\right)/2.$$

The function $\underline{r}$, which is the solution to the equation $c = \overline{MR}(r,0) = r(2-r)/2$, is $\underline{r}(c) = 1 - \sqrt{1-2c}$. The functions r_- and r_+, which are the smaller and larger solutions to the equation $c = \overline{MR}(r,r) = r(1-2r)$, are readily calculated as $r_\pm(c) = (1 \pm \sqrt{1-8c})/4$. These functions are well defined for $c \in (0, M^*)$, where $M^* = 1/8$. (For $c \geq 1/8$ the equation has no solution on $[0,1]$, i.e., the interval $[r_-(c), r_+(c)]$ is empty.) The function $\bar{r}$, which is the solution to equation $c = \overline{MR}(r, 1-r) = (1-2r)^2/2$, is $\bar{r}(c) = (1+\sqrt{2c})/2$.

Figure 4 provides a description of the function $k^*(r)$ for $c \in (0, 1/2)$. For $c \leq 1/9$ the equilibrium capacity $k^*(r)$ reaches its maximum at the price cap $r_A^* = \sqrt{c} \in [r_-(c), r_+(c)]$. For $c > 1/9$, the equilibrium capacity $k^*(r)$ reaches its maximum at $r_B^* = (1 + 2\sqrt{10c-1})/5 \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Interestingly, for $c \in (1/9, 1/8)$ the equilibrium capacity $k^*(r)$ is increasing in the interval $(r_-(c), r_+(c)]$, and reaches its maximum at $r^*(c) \in (r_+(c), \bar{r}(c))$.

The expected surplus is $\bar{S}(r, k^*(r)) = 0$ on $[0, \underline{r}(c))$,

$$\bar{S}(r, k^*(r)) = \frac{r}{2}(4-9r) - c(1+2r) + \left(c + 2r - \frac{1}{2}\right)\sqrt{2c - r(2-5r)}.$$

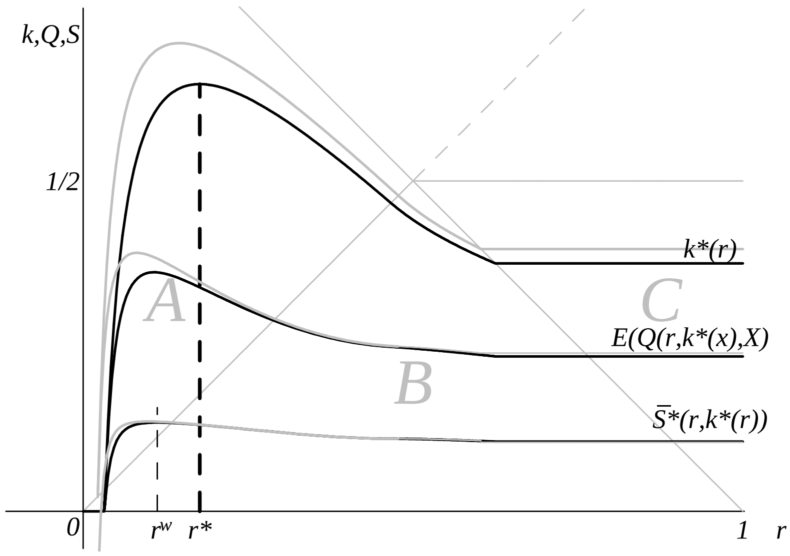

Figure 5: Capacity, Expected Output, and Surplus for $c = 1/32$

on $[\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$,

$$\bar{S}(r, k^*(r)) = \frac{r^3 (1 + 4r^3) + 3r^2 (c (c - 2r (1 - r)) - r^3) - c^3}{6r^3}.$$

on $[r_-(c), r_+(c)]$, and

$$\bar{S}(r, k^*(r)) = \frac{1 - 6c}{8} + \frac{\sqrt{2c^3}}{2}.$$

on $[\bar{r}(c), 1]$. When $c \geq 1/8$, the interval $[r_-(c), r_+(c)]$ is empty.

Figure 5 displays the equilibrium capacity, and the expected output and surplus as functions of the price cap when the unit cost of capacity is $c = 1/32$; note that maximum capacity is reached at a price cap-capacity pair in region A and, consistently with Proposition 3, the price cap that maximizes the expected surplus is below the price cap that maximizes capacity, i.e., $r^W \simeq .11 < r^* \simeq .17$. The grey curves in this figure provide graphs of these functions with a capacity payment $z = 1/100$; this small capacity payment has a positive impact on surplus even though in this example the cost of capacity is small.

Figure 6 shows the corresponding graphs for $c = 3/20$; the maximum capacity is reached at a price cap-capacity pair in region B, and consistently with Proposition 3, the expected surplus is maximal at this price cap. The grey curves in this figure provide graphs of these functions with a capacity payment $z = 1/100$; consistently with Proposition 4 a small capacity payments has a positive impact on the surplus.

It is interesting to observe that whether the cost of capacity is large or small,

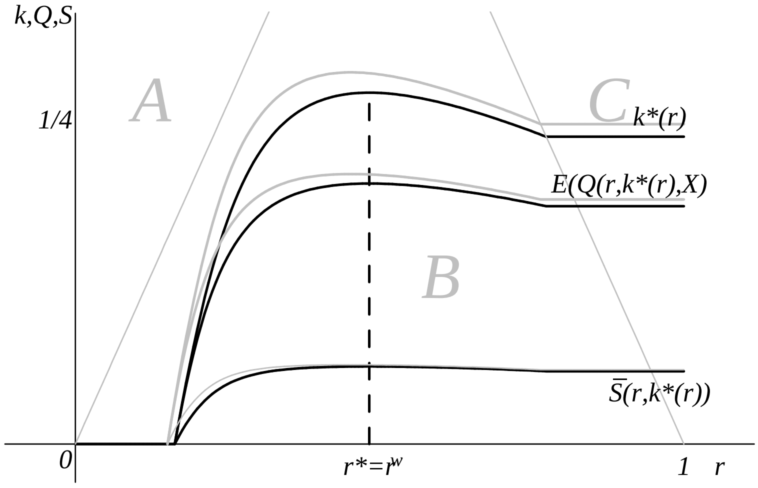

Figure 6: Capacity, Expected Output, and Surplus for $c = 3/20$.

introducing a small capacity payment reduces the optimal price cap: having a complementary instrument to provide incentives for capacity investments allows a more effective use of the price cap to discourage capacity withholding.

Figure 7 illustrates the effectiveness of price caps as measured by the ratio $\bar{S}^*(c)/S^*(k^W(c))$, where $\bar{S}^*(c) := \bar{S}(r^W(c), k^*(r^W(c)))$ is the expected surplus with an optimal price cap, and $S^*(k^W(c))$ is the maximum expected surplus that can be realized assuming that the socially optimal capacity $k^W(c)$ is installed and supplied unconditionally. A price cap is very effective when the unit cost of capacity is small, but its effectiveness decreases as the unit cost of capacity increases. The graph $k^*(r^W(c))/k^W(c)$ illustrates the effectiveness of a price cap to provide incentives for capacity investment. In the absence of a binding price cap the monopoly installs $k^W(c)/2$ units of capacity, and the expected surplus realized is $3S^*(k^W(c))/4$. Thus, the dashed lines at 1/2 and 3/4 in Figure 7 describe, respectively, the (constant) ratios of installed capacity to socially optimal capacity, and expected surplus realized to maximum expected surplus.

This example also illustrates the differing effects of price caps in our setting and in the model studied by Earle et al. (2007) and Grimm and Zoettl (2010), in which the monopoly cannot withhold capacity. Simple calculations show that when the monopoly cannot withhold capacity the socially optimal price cap yields the level of capacity $\hat{k}^*(c) = k^w(c)/2$. Thus, a price cap is a poor instrument to provide incentives for capacity investment when the monopoly cannot withhold capacity and, consequently, the expected surplus realized with a socially optimal price cap $\hat{S}^*(c)$ is

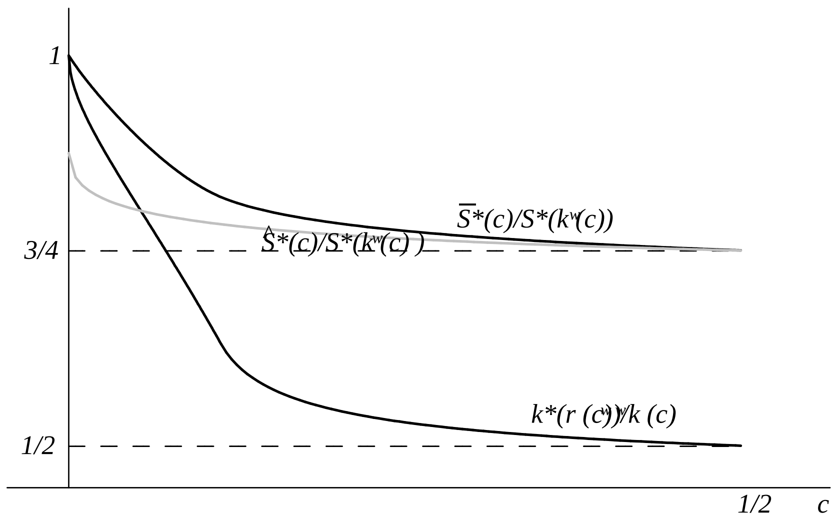

Figure 7: Price Cap Effectiveness with and without Capacity Withholding

well below the maximum expected surplus $S^*(k^w(c))$. Moreover, as Figure 7 shows the ratio $\hat{S}^*(c)/S^*(k^w(c))$ is uniformly below $\bar{S}^*(c)/S^*(k^w(c))$, and the difference between these ratios is considerably large for small values of the unit cost of capacity. (See Lemus and Moreno (2015), Appendix B, for a treatment of this model and example.) These conclusions suggest that disallowing capacity withholding may not be an advisable regulatory policy.

7 Appendix: Proofs

Proof of Lemma 1. Using the results described in tables A, B and C we readily calculate the monopoly's expected marginal revenue as

$$\overline{MR}(r,k) = \int_{r+k}^{1} rf(x)dx = r[1 - F(r+k)] \tag{3}$$

for $(r,k) \in A$,

$$\overline{MR}(r,k) = \int_{2k}^{r+k} (x-2k)\, f(x)dx + \int_{r+k}^{1} rf(x)dx \tag{4}$$

for $(r,k) \in B$, and

$$\overline{MR}(r,k) = \int_{2k}^{1} (x-2k)f(x)dx \tag{5}$$

for $(r,k) \in C$. Since (3) and (4) coincide for $k = r$, and (4) and (5) coincide for $r > 1/2$ and $k = 1 - r$, then $\overline{MR}$ is continuous on $A \cup B \cup C$.

Differentiating $\overline{MR}$ we get

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -rf(r+k) < 0 \tag{6}$$

for $(r,k) \in A$,

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -kf(r+k) - 2\left[F(r+k) - F(2k)\right] < 0 \tag{7}$$

for $(r,k) \in B$, and

$$\frac{\partial \overline{MR}(r,k)}{\partial k} = -2\left[1 - F(2k)\right] < 0 \tag{8}$$

for $(r,k) \in C$. Moreover, since (6) and (7) coincide for $k = r$, then $\overline{MR}$ is differentiable on $A \cup B \cup C$, except perhaps in the boundary of B. Hence the expected marginal revenue function $\overline{MR}$ is strictly decreasing, and therefore the monopoly's expected revenue is a strictly concave function on $A \cup B \cup C$. □

Proof of Lemma 2 . We calculate the equilibrium capacity $k^*(r)$. The expected marginal revenue when capacity is zero is

$$\overline{MR}(r,0) = \int_0^r xf(x)dx + r\left(1 - F(r)\right).$$

Hence $d\overline{MR}(r,0)/dr = 1 - F(r) > 0$ on $(0,1)$, and therefore the function $\overline{MR}(\cdot,0)$ has an inverse, which we denote by $\underline{r}$. For $r \in [0, \underline{r}(c))$ we have $\overline{MR}(r,0) < c$, and since $\overline{MR}(r,k)$ is decreasing in k, then $\overline{MR}(r,k) < c$ for all k. Hence $\bar{\Pi}(r,\cdot)$ is decreasing, and therefore $k^*(r) = 0$.

Let us consider price caps $r \in [\underline{r}(c), 1/2)$. Then $\bar{\Pi}(r,\cdot)$ takes values in regions A and B. Solving the equation (1) for $\overline{MR}$ given by (6) yields

$$k_A(r) = F^{-1}(1 - \frac{c}{r}) - r.$$

Hence

$$k_A(r) + r = F^{-1}(1 - \frac{c}{r}) < 1,$$

and therefore $k_A(r) < 1 - r$. In order for $(r, k_A(r)) \in A$, we must have $r \leq k_A(r)$. This inequality is equivalent to

$$c \leq \overline{MR}(r,r) = r\left(1 - F(2r)\right).$$

Denote by $k_B(r)$ the solution to equation (1) for $\overline{MR}$ given by (4). In order for $(r, k_B(r)) \in B$, the inequalities $0 < k_B(r) < r$ must hold. (Recall that we are identifying the monopoly capacity for $r < 1/2$, and therefore $k_B(r) < r$ implies

$k_B(r) < 1-r$.) The inequality $k_B(r) < r$ is equivalent to $c > \overline{MR}(r,r)$. The inequality $k_B(r) > 0$ is equivalent to $c < \overline{MR}(r,0)$,i.e., $r \geq \underline{r}(c)$.

Let us now consider price caps $r \in [1/2, 1]$. Then $\bar{\Pi}(r,\cdot)$ takes values in regions B and C. If $r < \underline{r}(c)$, then $k^*(r) = 0$ as shown above. For $r \geq \underline{r}(c)$, $\bar{\Pi}(r,\cdot)$ reaches its maximum in region B provided $k_B(r) < 1-r$. This inequality is equivalent to

$$\overline{MR}(r, 1-r) = \int_{2(1-r)}^{1} x f(x)dx - 2\,(1-r)\,[1 - F(2\,(1-r))] < c.$$

Note that

$$\frac{d\overline{MR}(r,1-r)}{dr} = 2(1 - F(2\,(1-r))) > 0.$$

Hence the function $\overline{MR}(r, 1-r)$ has an inverse on $(1/2, 1)$, which we denote by $\bar{r}(c)$, and therefore we may write the above inequality as $r < \bar{r}(c)$. Note that for $r = 1$ we have $\overline{MR}(r, 1-r) = \overline{MR}(1, 0) = \mathbb{E}(X)$. Hence, since $c < \mathbb{E}(X)$ by assumption, we have $\bar{r}(c) < 1$. For $r \in [\bar{r}(c), 1)$, $\bar{\Pi}(r,\cdot)$ increases with k in region B and reaches its maximum in region C. Denote by k_C the solution to the condition (1) for $\overline{MR}$ given by equation (5). Clearly k_C is independent of the price cap r. Also, since $\overline{MR}(r, 1/2) = 0$, then $k_C < 1/2$ for all $c \in (0, \mathbb{E}(X))$. Since the expected marginal revenue decreases with k, then $k_C > 1-r$ implies $c < \overline{MR}(r, 1-r)$. Moreover, since $r > 1/2$ and $\overline{MR}$ is decreasing, then $\overline{MR}(r, 1-r) < \overline{MR}(r,r)$. Hence k_C solves the monopoly problem if $r \geq \bar{r}(c)$.

Assume that the hazard rate of X, $h(\cdot) = f(\cdot)/[1 - F(\cdot)]$, is increasing. Differentiating yields

$$\frac{d\overline{MR}(r,r)}{dr} = (1 - F(2r)) - 2rf(2r) = (1 - F(2r))\,(1 - 2rh(2r))\,,$$

which is positive for values of r close to zero and negative for values of r close to $1/2$. Since h is increasing, then the function $\overline{MR}(r,r)$ is strictly concave and reaches its maximum value M^* on $(0, 1/2)$. If $c < M^*$, then the equation $\overline{MR}(r,r) = c$ has two solutions on $(0, 1/2)$, which we denote by $r_-(c)$ and $r_+(c)$ with $r_-(c) < r_+(c)$. Thus, for $r \in [r_-(c), r_+(c)]$, we have $c \leq \overline{MR}(r,r)$, and hence $k^*(r) = k_A^*(r)$. Since $c < \underline{r}(c)$ and $1/2 < \bar{r}(c) < 1$, then for $c < M^*$, $c < \underline{r}(c) < r_-(c) < r_+(c) < 1/2 < \bar{r}(c) < 1$. $\square$

The following lemma will be useful in the proof of Proposition 1.

Lemma 3. *Let g be a real valued function on $\mathbb{R}$, differentiable on some interval (a,b), and satisfying $g'(a) > 0 > g'(b)$, and $g''(y) < 0$ for all $y \in (a,b)$ such that $g'(y) = 0$. Then g has a unique global maximizer on $[a,b]$, $y^* \in (a,b)$, and g' is positive on (a, y^*) and negative on (y^*, b).*

Proof. Let $y^* = \sup\{y \in (a,b) \mid g'(y) > 0\}$ and $y^{**} = \inf\{y \in (a,b) \mid g'(y) < 0\}$. Since g' is continuous on (a,b), then $g'(y^*) = g'(y^{**}) = 0$, and therefore $a < y^{**} \leq y^* < b$. We show that $y^* = y^{**}$, which establishes the lemma. Suppose by way of contradiction that $y^{**} < y^*$. Since both $g''(y^*)$ and $g''(y^{**})$ are negative, then for $\varepsilon \in (0, y^* - y^{**})$ sufficiently small

$$g'(y^{**} + \varepsilon) < 0 < g'(y^* - \varepsilon).$$

Hence $g'(\bar{y}) = 0$ for some $\bar{y} \in (y^{**} - \varepsilon, y^* + \varepsilon)$, and g' is negative (positive) for y below (above) and near $\bar{y}$. Hence $g''(\bar{y}) > 0$, which is a contradiction. □

Proof of Proposition 1. Let $r \in (\underline{r}(c), \bar{r}(c))$. Since the expected marginal revenue $\overline{MR}(r,k)$ is differentiable in regions $A \cup B$, we can differentiate equation (1) to get

$$\frac{\partial \overline{MR}(r,k)}{\partial k} dk + \frac{\partial \overline{MR}(r,k)}{\partial r} dr = 0.$$

And since $\overline{MR}$ is decreasing, i.e., $\partial \overline{MR}(r,k)/\partial k < 0$, then

$$\frac{dk^*}{dr} = -\frac{\partial \overline{MR}(r,k)}{\partial r} \left(\frac{\partial \overline{MR}(r,k)}{\partial k} \right)^{-1},$$

and

$$\frac{dk^*}{dr} \gtreqless 0 \Leftrightarrow \frac{\partial \overline{MR}(r,k)}{\partial r} \gtreqless 0.$$

Since f is differentiable, then $\overline{MR}$ is twice differentiable, and

$$\begin{aligned} \frac{d^2 k^*}{dr^2} &= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k} \right)^{-1} \frac{d}{dr} \left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r} \right) \\ &\quad + \frac{\partial \overline{MR}(r,k)}{\partial r} \left(\frac{\partial \overline{MR}(r,k)}{\partial k} \right)^{-2} \frac{d}{dr} \left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k} \right) \\ &= -\left(\frac{\partial \overline{MR}(r,k)}{\partial k} \right)^{-1} \left(\frac{d}{dr} \left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r} \right) + \frac{dk^*}{dr} \frac{d}{dr} \left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial k} \right) \right). \end{aligned}$$

Hence, for r such that $dk^*/dr = 0$, we have

$$\frac{d^2 k^*}{dr^2} \gtreqless 0 \Leftrightarrow \frac{d}{dr} \left(\frac{\partial \overline{MR}(r,k^*(r))}{\partial r} \right) \gtreqless 0.$$

If $(r, k^*(r)) \in A$, then differentiating $\overline{MR}$ given in (3) yields

$$\frac{\partial \overline{MR}(r,k)}{\partial r} = 1 - F(r+k) - rf(r+k) = (1 - F(r+k))(1 - rh(r+k)),$$

and

$$\begin{aligned}\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k)\left(1+\frac{dk_A}{dr}\right)(1-rh\,(r+k)) \\ &\quad -(1-F(r+k))\,(h\,(r+k)+rh'\,(r+k))\left(1+\frac{dk_A}{dr}\right).\end{aligned}$$

Assume that $dk_A/dr = 0$. Then $1 - rh\,(r + k^*(r)) = 0$, and

$$\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) = -\,(1-F(r+k^*(r)))\,(h\,(r+k^*(r))+rh'\,(r+k^*(r)))\,.$$

If the hazard rate is increasing (i.e., $h' > 0$), then we have $d^2k_A/dr^2 < 0$, and therefore every critical point of k_A is a local maximum.

If $(r, k_B(r)) \in B$, then differentiating $\overline{MR}$ given in (4) yields

$$\frac{\partial\overline{MR}(r,k)}{\partial r} = 1-F(r+k)-kf(r+k) = (1-F(r+k))\,(1-kh(r+k))\,,$$

and

$$\begin{aligned}\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) &= -f(r+k^*(r))\,(1-k^*(r)h(r+k^*(r)))\left(1+\frac{dk_B}{dr}\right) \\ &\quad -(1-F(r+k^*(r)))\,k^*(r)h'(r+k^*(r))\left(1+\frac{dk_B}{dr}\right) \\ &\quad -(1-F(r+k^*(r)))\,h(r+k^*(r))\frac{dk_B}{dr}.\end{aligned}$$

Assume that $dk_B/dr = 0$. Then $1 - k^*(r)h\,(r + k^*(r)) = 0$, and

$$\frac{d}{dr}\left(\frac{\partial\overline{MR}(r,k^*(r))}{\partial r}\right) = -\,(1-F(r+k^*(r)))\,k^*(r)h'(r+k^*(r)).$$

If the hazard rate is increasing (i.e., $h' > 0$) we have $d^2k_B/dr^2 < 0$, and therefore every critical point of k_B is a local maximum.

Thus, for $r \in (\underline{r}(c), \bar{r}(c))$, $d^2k^*(r)/dr^2 < 0$ whenever $dk^*(r)/dr = 0$. Moreover, since $k_B(\bar{r}(c)) = 1 - \bar{r}(c)$, and

$$\begin{aligned}\left.\frac{\partial\overline{MR}(r,1-r)}{\partial r}\right|_{r=\bar{r}(c)} &= 1-F\left(\bar{r}(c)+(1-\bar{r}(c))\right)-(1-\bar{r}(c))\,f\left(\bar{r}(c)+(1-\bar{r}(c))\right) \\ &= -\,(1-\bar{r}(c))\,f(1)<0,\end{aligned}$$

then $dk_B(\bar{r}(c))/dr < 0$. And since $k_B(\underline{r}(c)) = 0$, and

$$\left.\frac{\partial\overline{MR}(r,0)}{\partial r}\right|_{r=\underline{r}(c)} = 1-F(\underline{r}(c))>0,$$

then $dk_B(\underline{r}(c))/dr > 0$. Hence k^* has a global maximum at some $r^*(c) \in (\underline{r}(c), \bar{r}(c))$, and satisfies $dk^*/dr > 0$ on $(\underline{r}(c), r^*(c))$ and $dk^*/dr < 0$ on $(r^*(c), \bar{r}(c))$ by Lemma 3. Since k^* is continuous on $[0,1]$, is equal to zero on $[0, \underline{r}(c)))$ and is equal to k_C on $[\bar{r}(c), 1)$, this implies that k^* is quasi-concave, i.e., single peak, on $[0,1]$. □

Proof of Proposition 2. The expected output is

$$\mathbb{E}(Q(r, k^*(r), X) = \int_0^{2r} \frac{x}{2} f(x)dx + \int_{2r}^{r+k^*(r)} (x-r)f(x)dx + \int_{r+k^*(r)}^1 k^*(r)f(x)dx,$$

for $r \in [r_-(c), r_+(c)]$, and

$$\mathbb{E}(Q(r, k^*(r), X) = \int_0^{2k^*(r)} \frac{x}{2} f(x)dx + \int_{2k^*(r)}^1 k^*(r)f(x)dx$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Hence

$$\frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} = -[F(r + k^*(r)) - F(2r)] + \frac{dk^*}{dr}(1 - F(r + k^*(r)))$$

for $r \in [r_-(c), r_+(c)]$, and

$$\frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} = \frac{dk^*}{dr}(1 - F(2k^*(r)))$$

for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$. Thus,

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} < 0$$

for $r \in [r_-(c), r_+(c)]$, that is, the expected output decreases with the price cap beyond the price cap that maximizes capacity, and therefore the price cap that maximizes output is below $r^*(c)$. Moreover,

$$\frac{d\mathbb{E}(Q(r, k^*(r), X)}{dr} \gtreqless 0 \Leftrightarrow \frac{dk^*}{dr} \gtreqless 0.$$

for $r \in [\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, that is, the expected output increases with the price cap for $r \in (\underline{r}(c), r^*(c))$, and decreases for $r \in (r^*(c), \bar{r}(c))$.

Likewise for $r \in [r_-(c), r_+(c)]$ the expected price is

$$\mathbb{E}(P(r, k^*(r), X) = \int_0^{2r} \frac{x}{2} f(x)dx + \int_{2r}^1 rf(x)dx,$$

and for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$ it is

$$\mathbb{E}(P(r, k^*(r), X) = \int_0^{2k^*(r)} \frac{x}{2} f(x)dx + \int_{2k^*(r)}^{r+k^*(r)} (x - k^*(r))f(x)dx + \int_{r+k^*(r)}^1 rf(x)dx.$$

Hence, for $r \in [r_-(c), r_+(c)]$

$$\frac{d\mathbb{E}(P(r,k^*(r),X)}{dr} = 1 - F(2r) > 0.$$

Also, for $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$,

$$\frac{d\mathbb{E}(P(r,k^*(r),X)}{dr} = -\frac{dk^*}{dr}[F(r+k^*(r)) - F(2k^*(r))] + [1 - F(r+k^*(r))],$$

and therefore

$$\frac{dk^*}{dr} \leq 0 \Rightarrow \frac{d\mathbb{E}(P(r,k^*(r),X)}{dr} > 0. \square$$

Proof of Proposition 3. Table 3A describes the function S for (r,k) in region A.

X	$[0, 2r)$	$[2r, r+k)$	$[r+k, 1]$
$S(r,k,x)$	$\frac{3}{8}x^2$	$\frac{1}{2}(x^2 - r^2)$	$\frac{1}{2}(2x-k)k$

Table 3A: Gross Surplus in Region A.

Table 3BC below describes the gross surplus in region $B \cup C$.

X	$[0, 2k)$	$[2k, 1]$
$S(r,k,x)$	$\frac{3}{8}x^2$	$\frac{1}{2}(2x-k)k$

Table 3BC: Gross Surplus in Regions B and C.

The expected gross surplus is

$$\begin{aligned}\mathbb{E}(S(r,k,X)) &= \frac{3}{8}\int_0^{2r} x^2 f(x)dx + \frac{1}{2}\int_{2r}^{r+k}(x^2 - r^2)f(x)dx \\ &\quad + \frac{1}{2}\int_{r+k}^{1}(2x-k)kf(x)dx.\end{aligned} \tag{9}$$

for $(r,k) \in A$, and is

$$\mathbb{E}(S(r,k,X)) = \frac{3}{8}\int_0^{2k} x^2 f(x)dx + \frac{1}{2}\int_{2k}^{1}(2x-k)kf(x)dx. \tag{10}$$

for $(r,k) \in B \cup C$. For $r \in [0,1]$ the net surplus is $\bar{S}(r,k^*(r)) = \mathbb{E}(S(r,k^*(r),X)) - ck^*(r)$.

For price caps $r \in [r_-(c), r_+(c)]$ the price cap-equilibrium capacity pair $(r, k^*(r))$ is in region A. Differentiating $\bar{S}$ given in (9) yields

$$\frac{d\bar{S}(r,k^*(r))}{dr} = \frac{dk^*(r)}{dr}\left(\int_{r+k^*(r)}^{1}(x - k^*(r))f(x)dx - c\right) - r[F(r+k^*(r)) - F(2r)],$$

Recall that $r^*(c)$ is the capacity maximizing price cap identified in Proposition 1. If $r^*(c) \in [r_-(c), r_+(c)]$, then $dk^*(r^*(c))/dr = 0$ and $k^*(r^*(c)) = k_A(r^*(c)) > r^*(c)$ imply

$$\frac{d\bar{S}(r^*(c), k^*(r^*(c)))}{dr} = -r^*(c)[F(r^*(c) + k^*(r^*(c))) - F(2r^*(c))] < 0.$$

Hence the optimal price cap is below $r^*(c)$.

For $r \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$ we have $(r, k^*(r)) \in B \cup C$. Differentiating $\bar{S}$ given in (10) yields

$$\frac{d\bar{S}(r, k^*(r))}{dr} = \frac{dk^*(r)}{dr}\left(\int_{2k^*(r)}^1 (x - k^*(r))f(x)dx - c\right).$$

Since $(r, k^*(r)) \in B$, then $k^*(r) < r$, and

$$\overline{MR}(r, k^*(r)) = \int_{2k^*(r)}^{r+k^*(r)} (x - 2k^*(r))\, f(x)dx + \int_{r+k^*(r)}^1 rf(x)dx = c.$$

Hence

$$\int_{2k^*(r)}^1 (x - k^*(r))f(x)dx - c = \int_{2k^*(r)}^{r+k^*(r)} k^*(r)f(x)dx + \int_{r+k^*(r)}^1 (x - k^*(r) - r)f(x)dx > 0,$$

and therefore

$$\frac{d\bar{S}(r, k^*(r))}{dr} = 0 \Leftrightarrow \frac{dk^*(r)}{dr} = 0.$$

Differentiating $d\bar{S}(r, k^*(r))/dr$ we get

$$\begin{aligned} \frac{d^2\bar{S}(r, k^*(r))}{dr^2} &= \frac{d^2k^*(r)}{dr^2}\left(\int_{2k^*(r)}^1 (x - k^*(r))f(x)dx - c\right) \\ &\quad - \left(\frac{dk^*(r)}{dr}\right)^2 [1 - F(2k^*(r))] - 2k^*(r)f(2k^*(r)). \end{aligned}$$

If $d\bar{S}(r, k^*(r))/dr = 0$, then $dk^*(r)/dr = 0$, which as shown above implies $d^2k^*(r)/dr^2 < 0$. Hence $d^2\bar{S}(r, k^*(r))/dr^2 < 0$. Thus, by Lemma 3 if $r^*(c) \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $r^*(c)$ is the unique global maximizer of $\bar{S}(r, k^*(r))$ on $(\underline{r}(c), \bar{r}(c))$. □

Proof of Proposition 4. By the Envelope Theorem

$$\begin{aligned} \frac{d\tilde{S}(\tilde{r}^*(z), z)}{dz} &= \frac{\partial \mathbb{E}(S(r, k, X))}{\partial r}\frac{dr^*}{dz} + \left(\frac{\partial \mathbb{E}(S(r, k, X))}{\partial k} - c\right)\left(\frac{\partial \tilde{k}^*}{\partial r}\frac{dr^*}{dz} + \frac{\partial \tilde{k}^*}{\partial z}\right) \\ &= \left(\frac{\partial \mathbb{E}(S(r, k, X))}{\partial k} - c\right)\left(\frac{\partial \tilde{k}^*}{\partial r}\frac{dr^*}{dz} + \frac{\partial \tilde{k}^*}{\partial z}\right). \end{aligned}$$

Moreover, when the cost of capacity is sufficiently large that $(\tilde{r}^*(z), \tilde{k}^*(\tilde{r}^*(z), z)) \in B$, then $\tilde{r}^*(z)$ maximizes $\tilde{k}^*(r, z)$ as well, and therefore the Envelope Theorem also implies

$$\frac{d\tilde{S}(\tilde{r}^*(z), z)}{dz} = \left(\frac{\partial \mathbb{E}(S(r,k,X))}{\partial k} - c\right)\frac{\partial \tilde{k}^*}{\partial z}.$$

We show that $k^W > k^*(\tilde{r}^*(z)) \geq k^*(r^*(c))$ for all $r \in [0,1]$. Let us fix c and reduce notation by writing k^* and r^* for $k^*(r^*(c))$ and $r^*(c)$, respectively. Differentiating S^* we get

$$\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} = \int_{k^*}^{1} (x - k^*) f(x)dx - c.$$

If $r^* \in [r_-(c), r_+(c)]$, then k^* solves

$$\overline{MR}(r^*, k^*) = \int_{r^*+k^*}^{1} r^* f(x)dx = c,$$

and therefore

$$\begin{aligned}\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1} (x - k^*) f(x)dx - \int_{r^*+k^*}^{1} r^* f(x)dx \\ &= \int_{k^*}^{r^*+k^*} (x - k^*) f(x)dx + \int_{r^*+k^*}^{1} (x - r^* - k^*) f(x)dx > 0.\end{aligned}$$

If $r^* \in (\underline{r}(c), \bar{r}(c)) \backslash [r_-(c), r_+(c)]$, then $k^* \leq r^*$ solves

$$\overline{MR}(r^*, k^*) = \int_{2k^*}^{r^*+k^*} (x - 2k^*) f(x)dx + \int_{r^*+k^*}^{1} r^* f(x)dx = c,$$

and therefore

$$\begin{aligned}\left.\frac{dS^*(k)}{dk}\right|_{k=k^*} &= \int_{k^*}^{1} (x - k^*) f(x)dx - \left(\int_{2k^*}^{r^*+k^*} (x - 2k^*) f(x)dx + \int_{r^*+k^*}^{1} r f(x)dx\right) \\ &= \int_{k^*}^{2k^*} (x - k^*) f(x)dx \\ &\quad + \int_{2k^*}^{r^*+k^*} k^* f(x)dx + \int_{r^*+k^*}^{1} (x - r^* - k^*) f(x)dx > 0.\end{aligned}$$

Hence $k^* < k^W$ in either case.

Assume that $(\tilde{r}^*(0), \tilde{k}^*(\tilde{r}^*(0), 0)) \in B$. Differentiating the equation $\overline{MR}(r,k) = c - z$ and noticing equations (6) and (7) we get

$$\frac{\partial \tilde{k}^*}{\partial z} = -\frac{\partial \overline{MR}(r,k)}{\partial k} > 0.$$

Also

$$\begin{aligned}\frac{\partial \mathbb{E}(S(r,k,X))}{\partial k} &= \int_{2k}^{1}(x-k)f(x)dx \\ &> \int_{2k}^{r+k}(x-2k)f(x) + \int_{r+k}^{1} rf(x)dx \\ &= \overline{MR}(r,k)\end{aligned}$$

for $(r,k) \in B$. Since $\overline{MR}(\tilde{r}^*(z), \tilde{k}^*(\tilde{r}^*(z), z)) = c - z$, then

$$\left.\frac{\partial \mathbb{E}(S(r,k,X))}{\partial k}\right|_{(r,k)=(\tilde{r}^*(0),\tilde{k}^*(\tilde{r}^*(0),0))} > c.$$

Therefore

$$\left.\frac{d\tilde{S}(\tilde{r}^*(z), z)}{dz}\right|_{z=0} > 0.$$

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