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DECENTRALIZED TRADE MITIGATES THE LEMONS PROBLEM *

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Abstract

In markets with adverse selection, only low-quality units trade in the competitive equilibrium when the average quality of the good held by sellers is low. Under decentralized trade, however, both high and low-quality units trade, although with delay. Moreover, when frictions are small the surplus realized is greater than the (static) competitive surplus. Thus, decentralized trade mitigates the lemons problem. Remarkably, payoffs are competitive as frictions vanish, even though both high and low-quality units continue to trade and there is trade at several prices.

Keywords: Market for Lemons, Adverse Selection, Competitive Markets, Decentralized Trade.

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1 Introduction

Markets differ in the degree in which trade is centralized. Call markets, used to set opening prices on the NYSE, are highly centralized and all trade takes place at a single price (the market clearing price). In contrast, in housing, labor, or used car markets, trade is highly decentralized, and prices are determined by bilateral bargaining between buyers and sellers, and may differ between trades. The competitive model abstracts away from these institutional aspects, thus providing a model suitable, in principle, for the study of both centralized and decentralized markets. Our results suggest that decentralized markets with adverse selection may perform better than anticipated by the static-competitive model, and therefore that these institutional features cannot be ignored.

It is known that in markets for homogenous goods, decentralized trade tends to yield competitive outcomes when trading frictions are small. Since competitive equilibrium is efficient in these markets, this implies that decentralized trade generates nearly efficient outcomes. In markets with adverse selection, however, competitive equilibria need not be efficient, which raises the possibility that alternative market structures perform better.

In this paper we study a version of Akerlof's (1970) market for lemons in which trade is decentralized. Each period an equal measure of buyers and sellers enters the market; every seller is endowed with a unit of either high or low quality. At each period every agent in the market has a positive probability of meeting an agent of the opposite type. Once matched, a buyer, without observing the quality of the seller's unit, makes a take-it-or-leave-it price offer. If the seller accepts, then they trade at the offered price and both agents exit the market. If the seller rejects the offer, then both agents remain in the market at the next period. Discounting of future gains and the possible delay in matching with a trading partner constitute trading "frictions."

When the average quality of the good held by entering sellers is low, the market has a unique competitive equilibrium (CE) in which the price equals the buyers' value of a low-quality unit and only low-quality trades. We show that when trade is decentralized there is trade at several prices and both qualities trade (although with delay). When frictions are small, decentralized trade yields a surplus *greater* than the competitive surplus since the gains realized from trading high-quality units more than offsets the surplus lost due to trading frictions. As frictions vanish, however, each trader's payoff converges to his competitive equilibrium payoff, even though both high and low-quality units continue to trade.

When average quality is high, the market has an inefficient CE in which only low-quality trades, as well as efficient CE in which both qualities trade. We find that when trade is decentralized all trade is at a price equal to the cost of high-quality sellers, and both high and low-quality sellers trade as soon as they are matched. Thus, decentralized trade yields a surplus smaller than the surplus at an efficient CE merely due to the trading frictions. As frictions vanish, each trader's payoff converges to his payoff in the efficient CE in which the price is the cost of a high-quality unit.

Key to understanding these results is recognizing that the proportion of sellers in the market with a high-quality unit need not be the same as the proportion of sellers entering the market with a high-quality unit. Consider the case where the average quality of entering sellers is low. We show that in equilibrium buyers mix over price offers equal to the cost of high quality (such offers are accepted by both high and low-quality sellers), the value of low-quality (accepted only by low-quality sellers), and lower prices which are rejected by both types of sellers.[1] High-quality sellers therefore trade at a slower rate than low-quality sellers, and are thus present in the market in a higher proportion than they enter the market. The mixture over price offers is such that (i) the expected value of a random unit is equal to the cost of high quality, and (ii) the reservation price of low-quality sellers equals the value of low-quality.

Thus, buyers obtain a payoff of zero with each type of price offer, high-quality sellers also obtain a payoff of zero and, just as in the static competitive equilibrium, only low-quality sellers capture any surplus. In fact, low-quality sellers obtain more than their competitive surplus: because a low-quality seller is indifferent between accepting and rejecting an offer equal to the value of low quality (his reservation price), his *discounted* expected utility is the value of low quality minus the cost of low quality, i.e., it is his competitive surplus. His undiscounted expected utility therefore

[1] It is easy to see that equilibrium involves buyers mixing. If all price offers were equal to the cost of high quality, for example, then both types of sellers trade at the same rate, and therefore the proportion of high-quality sellers in the market equals the proportion of high-quality sellers entering the market. Since average quality is low, this offer yields a negative payoff and hence is not optimal.

exceeds his static-competitive surplus, but as the discount factor approaches one they coincide.

It is remarkable that in markets with adverse selection decentralized trade yields more surplus than anticipated by the competitive model when average quality is low. This suggests that decentralized trade mitigates the lemons problem.[2] It is also interesting to observe that the competitive model does not accurately describe outcomes in decentralized markets with adverse selection when average quality is low even if frictions are small: whereas the competitive model predicts that only low quality trades and that all trade is at one price, with decentralized trade both qualities trade and there is trade at several prices. Nevertheless, traders obtain competitive payoffs as frictions vanish whether average quality is high or low; hence the competitive model correctly predicts payoffs.

These results raise the question of whether the static competitive model provides an appropriate benchmark for competitive outcomes in a dynamic market with adverse selection. We discuss this issue in Section 6.

Related Literature

Results establishing that decentralized trade generates competitive outcomes in markets for homogenous goods have been obtained by, e.g., Gale (1987) and Binmore and Herrero (1988) when bargaining is under complete information, and by Serrano and Yosha (1996) and Moreno and Wooders (1999) when bargaining is under incomplete information. There are, however, important exceptions to this conclusion – see Rubinstein and Wolinsky (1985 and 1990). Except for introducing adverse selection, our model of decentralized trade is standard – in Rubinstein and Wolinsky (1985) traders engage in an alternating offer bargaining game, while in Gale (1986) one agent in a match is randomly selected to make a take-it-or-leave-it price offer; to avoid signalling issues, we have the uniformed party (the buyer) make price offers. The efficiency of decentralized markets with non-negligible frictions has been studied by Jackson and Palfrey (1999).

The first paper to consider a matching model with adverse selection is Williamson

[2]Equilibrium in a decentralized market with one-time entry (rather than a constant flow of entrants, as considered here) also yields more than the competitive surplus when frictions are small and average quality is low. See Moreno and Wooders (2001).

and Wright (1994), who show that fiat money can increase welfare. Also Velde, Weber and Wright (1999) investigate Gresham's Law in a matching model with adverse selection. Neither of these papers studies the efficiency properties of decentralized markets in comparison with other market structures. Inderst and Müller (2002) show that the adverse selection problem may be mitigated if sellers can sort themselves into different submarkets.

In a paper concurrent to ours, Blouin (2001) studies a decentralized market for lemons in a model which differs from ours in that the probability of matching is set to one and, more significantly, trade may occur at only one of three exogenously given prices. In this three-price set up, introduced by Wolinsky (1990), Blouin obtains results quite different from ours; for example, each type of trader obtains a positive (non-competitive) payoff even as frictions vanish. This result, which is at odds with our finding that payoffs are competitive as frictions vanish, seems to be driven by the exogeneity of prices. (In our model, prices are determined endogenously without prior constraints.) In addition, the comparison of the surplus generated in this setting to the competitive equilibrium surplus depends upon these exogenous prices; since these prices do not seemingly relate to economic primitives, this comparison is inconclusive.

The paper is organized as follows. In section 2 we describe the market. In section 3 we introduce our model of decentralized trade. In section 4 we present and discuss our results. In Section 5 we present an example, and we conclude in Section 6 with a discussion on the appropriate competitive benchmark – see also Appendix B for a formal treatment of this issue. The proofs are presented in Appendix A.

2 A market for Lemons

Consider a market in which there is a continuum of buyers and sellers who trade an indivisible commodity which can be of either high or low quality. Buyers and sellers are present in equal measures, which we normalize to one. A measure $q^H \in (0,1)$ of the sellers are endowed with a unit of high-quality, and a measure of $q^L = 1 - q^H$ of sellers are endowed with a unit of low-quality. A seller knows the quality of his good, but quality is unobservable to buyers. The cost to a seller of a high (low) quality unit of the good is c^H (c^L). The value to a buyer of a high (low) quality unit of the good is u^H (u^L). Each type of good is valued more highly by buyers than by sellers (i.e., $u^H > c^H$ and $u^L > c^L$), and both buyers and sellers value high quality more than low quality (i.e., $u^H > u^L$ and $c^H > c^L$). Also we assume that $c^H > u^L$, since otherwise the lemon's problem does not arise. Thus, $u^H > c^H > u^L > c^L$. Buyers and sellers are risk neutral. Hence the expected utility to a buyer of a randomly selected unit of the good is

$$u(q^H) = q^H u^H + (1 - q^H)u^L.$$

In this market the properties of competitive equilibria (CE) depend on whether *average quality is high* (relative to values and costs), i.e., $u(q^H) - c^H \geq 0$, or *average quality is low*, i.e., $u(q^H) - c^H < 0$. The supply and demand schedules for each case are described by figures 1a and 1b, respectively.

Figure 1: Competitive Equilibria

When average quality is low there is a unique CE. In this equilibrium only low-quality units trade (at the price u^L), and the competitive surplus is $q^L(u^L - c^L)$.

When average quality is high there are multiple CE: for every $p \in [c^H, u(q^H)]$ there is an equilibrium in which all units of both qualities trade at the price p; there is also an equilibrium in which all low-quality units and some high-quality units trade at the price $p = c^H$ (represented by the "dot" in Figure 1a); and there is an equilibrium in which only low-quality units trade at the price u^L. The competitive surplus ranges from $q^H(u^H - c^H) + q^L(u^L - c^L)$ for the efficient CE to $q^L(u^L - c^L)$ for the least efficient CE.

3 A Decentralized Market for Lemons

Consider the market for lemons described in Section 2, but assume now that the market operates for infinitely many consecutive periods. Each period t a measure $q^H \in (0, 1)$ of high-quality sellers, a measure $q^L = 1 - q^H$ of low-quality sellers, and a measure one of buyers enter the market. As in Section 2 we say that average quality is high when $u(q^H) - c^H \geq 0$, and that average quality is low when $u(q^H) - c^H < 0$.

Every buyer (seller) in the market meets a randomly selected seller (buyer) with probability $\alpha \in (0, 1)$. A matched buyer proposes a price at which to trade. If the proposed price is accepted by the seller, then the agents trade at that price and both leave the market. If the proposed price is rejected by the seller, then the agents remain in the market at the next period. An agent who is unmatched in the current period also remains in the market at the next period. An agent observes only the outcomes of his own matches.

If a buyer and a seller trade at the price p, then the instantaneous utility of the buyer is $u - p$ and that of the seller is $p - c$, where $u = u^H$ and $c = c^H$ if the unit traded is of high quality, and $u = u^L$ and $c = c^L$ if it is of low quality. Agents discount utility at a common rate $\delta \in (0, 1)$.

In this market, a buyer must be ready to make a price offer at each date. Thus, a *pure strategy for a buyer* is a sequence $\{p_t\}$, where $p_t \in \mathbb{R}_+$ is the price she offers at date t.[3] Likewise, a seller must be ready to respond to a price offer at each date. Thus, a *pure strategy for a seller* is a sequence $\{r_t\}$, where $r_t \in \mathbb{R}_+$ is his reservation price (i.e., the smallest price he accepts) at date t.

Traders' strategies in the market are described by a probability distribution over price offers made by buyers, and a probability distribution over reservation prices employed by each type of seller. Denote by λ_t the *c.d.f.* of price offers at date t; i.e., a matched seller is offered p or less with probability $\lambda_t(p)$. Since in a market equilibrium traders of the same type must obtain the same expected utility, the reservation prices of sellers of the same type are identical. Thus, in equilibrium the distribution of reservation prices employed by each type of seller at each date is degenerate. Therefore without loss of generality we focus attention on strategy distributions $\{\lambda_t, r_t^H, r_t^L\}$, where $r_t^\tau \in \mathbb{R}_+$ is the reservation price used by all sellers of type $\tau \in \{H, L\}$ at date t.

Market Dynamics

[3] Price offers are "unconditional" since a buyer doesn't know whether he is matched with a high or a low quality seller. Also, we consider only strategies in which a trader does not condition his actions in the current match on the history of his prior matches, but this restriction is inconsequential. Since a trader only observes the outcomes of his own matches, his decision problem is the same regardless of his history in prior matches – see Osborne and Rubinstein (1990), pp. 154-162.

Let $\{\lambda_t, r_t^H, r_t^L\}$ be a strategy distribution. The evolution of the market over time is described by the stock of sellers of each type and the expected utilities of the traders in the market at each date t, denoted by (K_t^H, K_t^L) and (V_t^H, V_t^L, V_t^B), respectively. (We do not keep track of the stock of buyers, assuming implicitly that it is equal to the stock of sellers at each date. Since the measures of buyers and sellers entering the market each period are identical, this assumption seems natural.) The laws of motion for these variables are as follows:

For $\tau \in \{H, L\}$ denote by λ_t^τ the probability that a matched τ-quality seller trades at date t, i.e., the probability that he is offered a price greater than or equal to r_t^τ. This probability is given by

$$\lambda_t^\tau = \int_0^\infty I(p, r_t^\tau) d\lambda_t(p),$$

where $I(p, r_t^\tau)$ is an indicator function, taking the value 1 if $p \geq r_t^\tau$, and taking the value zero otherwise. Since a fraction $\alpha\lambda_t^\tau$ of the stock of τ-quality sellers trade (and leave the market) at date t, the stock of τ-quality sellers at date $t+1$ is

$$K_{t+1}^\tau = (1 - \alpha\lambda_t^\tau) K_t^\tau + q^\tau. \tag{1}$$

The payoff of a matched seller of quality $\tau \in \{H, L\}$ at date t who is offered a price $p \in \mathbb{R}_+$ is $p - c^\tau$ if $p \geq r_t^\tau$ (i.e., if $I(p, r_t^\tau) = 1$) and it is δV_{t+1}^τ if $p < r_t^\tau$ (i.e., if $I(p, r_t^\tau) = 0$). The seller's expected utility at date t is therefore given by

$$V_t^\tau = \alpha \int_0^\infty \left(I(p, r_t^\tau)(p - c^\tau) + (1 - I(p, r_t^\tau))\delta V_{t+1}^\tau \right) d\lambda_t(p) + (1 - \alpha)\delta V_{t+1}^\tau. \tag{2}$$

Likewise, since a matched buyer meets a τ-quality seller with probability $K_t^\tau/(K_t^H + K_t^L)$, her payoff if she offers the price $p \in \mathbb{R}_+$, which we denote by $B_t(p)$, is given by

$$B_t(p) = \sum_{\tau \in \{H,L\}} \frac{K_t^\tau}{K_t^H + K_t^L} \left(I(p, r_t^\tau)(u^\tau - p) + (1 - I(p, r_t^\tau))\delta V_{t+1}^B \right).$$

The expected utility of a buyer at date t is then

$$V_t^B = \alpha \int_0^\infty B_t(p) d\lambda_t(p) + (1 - \alpha)\delta V_{t+1}^B. \tag{3}$$

Stationary Equilibrium

We study stationary market equilibria; that is, equilibria where the distributions describing the strategies of the traders, and the stocks and expected utilities of the traders are constant over time. A *steady state* is a list $[(\lambda, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$ satisfying the system (1)-(3), where the time subscript t is eliminated. The description of a steady state includes the distribution of price offers made by buyers, $\lambda \in \Delta\mathbb{R}_+$, the reservation prices of sellers, $r^H, r^L \in \mathbb{R}_+$, the stocks of sellers of each type, $K^H, K^L \in \mathbb{R}_+$, and the expected utilities of the traders $V^H, V^L, V^B \in \mathbb{R}_+$, at every period. Note that in a steady state the probability that a matched τ-quality seller trades, λ^τ, and the payoff to a matched buyer who offers the price $p \in \mathbb{R}_+$, $B(p)$, are also constant over time. Also we denote by μ^τ the proportion of τ-quality sellers in the stock of sellers, given for $\tau \in \{H, L\}$ by

$$\mu^\tau = \frac{K^\tau}{K^H + K^L}.$$

A stationary market equilibrium is a steady state where buyers and sellers behave optimally. Formally:

A *stationary market equilibrium* is a steady state $[(\lambda, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$ satisfying

$(ME.\tau)$ $r^\tau - c^\tau = \delta V^\tau$ for $\tau \in \{H, L\}$, and

$(ME.B)$ $\bar{p} \in \arg\max_{p \in \mathbb{R}_+} B(p)$ for every $\bar{p}$ in the support of λ.

Condition $ME.\tau$ ensures that the reservation price of each type τ seller makes him indifferent between accepting or rejecting an offer of his reservation price. Condition $ME.B$ ensures that the price offers made by (almost all) buyers are optimal.

Given a stationary equilibrium, the *(flow) surplus*, S^F, is the sum of the expected utilities of the flow of agents entering every period, i.e.,

$$S^F = V^B + q^H V^H + q^L V^L.$$

4 Results

The basic properties of stationary market equilibria are established in Proposition 1. Recall that λ^τ is the probability of a price offer of r^τ or greater.

Proposition 1. *In every stationary market equilibrium*:
$(P1.1)$ *The reservation price of high-quality sellers is equal to their cost, and is greater than the reservation price of low-quality sellers (i.e.,* $r^H = c^H > r^L$*).*

$(P1.2)$ *The only prices that may be offered with positive probability are* r^H *(which is accepted by all sellers),* r^L *(which is accepted only by low-quality sellers), and prices below* r^L *(which are rejected by all sellers).*

$(P1.3)$ *The expected utility of high-quality sellers is zero (i.e.,* $V^H = 0$*), the expected utility of low-quality sellers is positive (i.e.,* $V^L > 0$*), and the expected utility of buyers is zero whenever rejected offers are made with positive probability (i.e.,* $\lambda^L < 1$ *implies* $V^B = 0$*).*

$(P1.4)$ *High-quality sellers remain in the market longer than low-quality sellers (i.e.,* $\lambda^H \leq \lambda^L$*), and are present in the market in a proportion at least as great as the proportion in which they enter (i.e.,* $\mu^H \geq q^H$*).*

The intuition for these results is straightforward: it is easy to see buyers never offer a price above the cost of high-quality sellers, c^H.[4] Hence the expected utility of high-quality sellers is zero, and their reservation price is $r^H = c^H$. And since delay is costly, low-quality sellers accept price offers below c^H, i.e., $r^L < c^H = r^H$. This implies that the price offers that are accepted by high-quality sellers are also accepted by low-quality sellers, and therefore the probability that a matched high-quality seller trades, λ^H, is less than or equal to the probability that a matched low-quality seller trades, λ^L; hence high-quality sellers leave the market at a slower rate than low-quality sellers, and are therefore a larger fraction of the stock of sellers than of the flow of entrants, i.e., $\mu^H \geq q^H$.

Since buyers make price offers, they keep sellers at their reservation prices; that is, prices $p > r^H$, accepted by both types of buyers, or prices in the interval (r^L, r^H), accepted only by low-quality sellers, are suboptimal, and are therefore made with probability zero. Hence a buyer must decide whether: (i) to offer a high price, r^H, thus trading for sure and getting a unit which is of high quality with probability μ^H and of low quality with probability $\mu^L = 1 - \mu^H$; or (ii) to offer a low price, r^L, thus trading only if the seller in the match has a unit of low quality (which occurs with probability μ^L); or (iii) to offer a very low price ($p < r^L$), thus not trading for sure. Of course, if buyers make price offers that are rejected it is because delay has no cost, that is, because their continuation utility is zero.

In view of Proposition 1, in order to complete the description of the market equi-

[4] This is the Diamond Paradox – see Diamond (1971).

libria that may arise we need to determine the probabilities with which the three types of prices, r^H, r^L or prices less than r^L, are offered. (The expected utility and reservation price of low-quality sellers, V^L and r^L, and the expected utility of buyers, V^B, can be readily calculated once these probabilities are determined.) Since the probability of offering a price greater than r^H is zero by $P1.2$, then λ^H is the probability of an offer of exactly r^H. And since prices in the interval (r^L, r^H) are offered with probability zero by $P1.2$, then $\lambda^L - \lambda^H$ is the probability of an offer of exactly r^L. The probability of an offer below r^L is $1-\lambda^L$. Hence in a stationary market equilibrium the probabilities λ^H and λ^L determine the distribution of equilibrium transaction prices. Ignoring the distribution of rejected price offers, which is inconsequential, we describe a stationary market equilibrium by a list $[(\lambda^H, \lambda^L, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$, where $0 \leq \lambda^H \leq \lambda^L \leq 1$.

Proposition 2 below establishes that when frictions are small the values of λ^H and λ^L depend on whether average quality is high or low. When average quality is high all price offers are r^H (i.e., $\lambda^H = \lambda^L = 1$). In this case, the equilibrium surplus is *smaller* than the surplus in the efficient competitive equilibrium (but greater than the surplus in the least efficient competitive equilibrium). When average quality is low all three types of price offers r^H, r^L and $p < r^L$ are made with positive probability (i.e., $\lambda^H < \lambda^L < 1$). The precise values of λ^H and λ^L are provided in the proof of Proposition 2 – see Appendix A. In this case, the equilibrium surplus is *greater* than the surplus in the unique competitive equilibrium: the gains realized from trading some high-quality units more than off-sets the surplus lost due to low-quality units trading with probability less than one, yielding a net gain in surplus over the competitive equilibrium surplus in spite of trading frictions.

Proposition 2 establishes also that in either case as frictions vanish each trader obtains a competitive equilibrium payoff. Specifically, when average quality is high payoffs converge to the payoffs at the competitive equilibrium in which all units trade at the price c^H; when average quality is low payoffs converge to the payoffs at the unique competitive equilibrium (with price u^L). This is remarkable since in the competitive equilibrium only low-quality units trade, while in the stationary market equilibrium high-quality units also trade.

Proposition 2. *Assume that δ is near one.*

($P2.1$) *If average quality is high, then there is a stationary market equilibrium in which buyers offer* r^H *with probability one (i.e.,* $\lambda^H = \lambda^L = 1$*), and all matched agents trade. In this equilibrium, if frictions are small but non-negligible, the surplus is below the surplus at the efficient competitive equilibrium.*
($P2.2$) *If average quality is low, then there is a stationary market equilibrium in which all three types of prices offers (*r^H*,* r^L*, and prices less than* r^L*) are made with positive probability (i.e.,* $0 < \lambda^H < \lambda^L < 1$*). In this equilibrium, if frictions are small but non-negligible, the surplus is above the competitive equilibrium surplus.*
($P2.3$) *In these equilibria as* δ *approaches one the traders' expected utilities approach their expected utilities at a competitive equilibrium.*

The intuition for P2.1 is clear. If all buyers offer $r^H = c^H$, then the proportion of high-quality sellers in the market is the same as the proportion in which they enter, i.e., $\mu^H = q^H$, and therefore an offer of c^H yields a payoff of $u(\mu^H) - c^H = u(q^H) - c^H \geq 0$. Since a seller is eventually matched and gets an offer of c^H, for δ is sufficiently near one the reservation price of low-quality sellers is near c^H, and therefore above u^L; hence an offer of r^L yields a negative payoff of $u^L - r^L$ when it is accepted. Also an offer $p < r^L$ yields a payoff of zero. Therefore it is optimal for buyers to offer r^H. Note that this equilibrium generates the same payoffs (up to frictions) as the competitive equilibrium where the price is c^H; i.e., decentralized trade selects an efficient equilibrium outcome – recall that when average quality is high there are multiple competitive equilibria.

When average quality is low, the proof of P2.2 establishes that buyers "mix," making offers of c^H ($= r^H$) which are accepted by both types of sellers, offers of u^L ($= r^L$) which are accepted by only low quality-sellers, and very low price offers which are rejected by both types of sellers. High-quality sellers leave the market slower than low-quality sellers and are, therefore, present in the market in a proportional greater than they enter the market (i.e., $\mu^H > q^H$). In equilibrium, the proportion of high-quality sellers in the market is such that the payoff to offering c^H is zero, i.e., $u(\mu^H) = c^H$. The equilibrium mixture over price offers is such that a low-quality seller has a reservation price exactly equal to u^L. Since offers of u^L are accepted only by low-quality sellers, the payoff to offering u^L is also zero. Hence all three types of price offers yield a payoff to zero and each type of offer is optimal.

Buyers and high-quality sellers obtain their competitive surplus (of zero). Low-quality sellers are indifferent between accepting or rejecting a price offer of their reservation price, i.e., $r^L - c^L = \delta V^L$. As noted above, in equilibrium $r^L = u^L$ and hence the expected surplus of low-quality sellers is

$$V^L = \frac{u^L - c^L}{\delta},$$

which is greater than their static-competitive surplus of $u^L - c^L$. The surplus obtained by low-quality sellers from occasionally trading at c^H more than offsets the surplus lost due to possibly trading with delay.

As for $P2.3$, when average quality is high it is easy to see why payoffs are competitive as frictions vanish: since all price offers are c^H, as δ approaches one the traders' expected utilities approach their expected utilities at the CE in which the price is c^H. When average quality is low, the surplus of low-quality sellers, $V^L = (u^L - c^L)/\delta$, decreases and approaches $u^L - c^L$ (their competitive surplus) as δ approaches one. In this case, both λ^H and λ^L are decreasing in δ. Although low-quality sellers become more patient as δ increases, delay also increases and in equilibrium the later effect on their payoffs dominates.

Our last proposition establishes that when the gain to trading a high-quality unit is greater than the gain to trading a low-quality unit, the equilibrium described in Proposition 2 is unique.

Proposition 3. *If δ is near one and $u^H - c^H > u^L - c^L$, then there is a unique stationary equilibrium.*

The key result in establishing Proposition 3 is that when δ is close to one, in equilibrium either (i) buyers offer $r^H = c^H$ with probability one (i.e., $\lambda^H = 1$), or (ii) buyers offer with positive probability prices which are rejected (i.e., $\lambda^L < 1$) – see Lemma 2 in Appendix A. To see why, suppose to the contrary that the only prices offered with positive probability are r^H and r^L, i.e., $0 < \lambda^H < \lambda^L = 1$. An offer of r^L, which is accepted only by low-quality sellers, may be optimal only if $r^L \leq u^L$. For δ close to one $r^L \leq u^L$ holds only if λ^H, the probability of a price offer of $r^H = c^H$, is small. But if λ^H is small then high-quality sellers exit the market at a slower rate ($\alpha\lambda^H$) than low-quality sellers (who leave the market at the rate α); this implies that the proportion of high-quality sellers in the stock of sellers, $\mu^H = q^H/(q^H + q^L\lambda^H)$, is near one. Hence offering c^H yields a payoff near $u^H - c^H$, whereas offering r^L yields at most $u^L - c^L$. Thus, the "single crossing condition," $u^H - c^H > u^L - c^L$, implies that offering r^L is not optimal, and therefore that such a distribution of price offers is not part of an equilibrium.

When average quality is high the implications of this result are immediate: since in equilibrium high-quality sellers are present in the market in a proportion at least as great as the proportion in which they enter by $P1.4$, i.e., $\mu^H \geq q^H$, then a price offer of $r^H = c^H$ yields a positive payoff, which means that it is not optimal to offer a price which will be rejected (i.e., $\lambda^L = 1$). Hence the result above implies that all buyers offer r^H (i.e., $\lambda^H = 1$).

When average quality is low, since high-quality sellers must exit the market, price offers of r^H must be made with positive probability, and therefore must be optimal, i.e., $u(\mu^H) - c^H \geq 0$. Hence $\mu^H > q^H$, and therefore price offers of r^L are made with positive probability as well, i.e., $\lambda^L - \lambda^H > 0$ –otherwise both types of sellers exit the market at the same rate, and $\mu^H = q^H$. Hence the result above implies that price offers which are rejected are also made with positive probability (i.e., $\lambda^L < 1$). Therefore $0 < \lambda^H < \lambda^L < 1$. (For r^H and r^L and $p < r^L$ to be optimal price offers, payoffs must be zero, i.e., $u(\mu^H) - c^H = 0$ and $r^L = u^L$. These equations uniquely determine the probabilities λ^H and λ^L –see the proof of Proposition 3 in Appendix A.)

5 An Example

Consider a market where $u^H = 1$, $c^H = 3/5$, $u^L = 2/5$, $c^L = 1/5$, $q^H = 1/5$, $\alpha = 2/3$, and $\delta > 3/4$. Since $u(q^H) - c^H < 0$, then average quality is low. The following is a stationary market equilibrium: $\lambda^H = 3(1-\delta)/(2\delta)$, $\lambda^L = 2\lambda^H$, $r^H = 3/5$, $r^L = 2/5$, $K^H = \delta/[5(1-\delta)]$, $K^L = 2K^H$, $V^H = 0$, $V^L = 1/(5\delta)$, and $V^B = 0$. (The condition $\delta > 3/4$ is needed for the values of λ^H and λ^L to be in $(0,1)$.)

It is easy to check that these values form a steady state. Also, since $c^H + \delta V^H = 3/5 = r^H$ and $c^L + \delta V^L = 2/5 = r^L$, sellers are setting their reserve prices correctly (i.e., $ME.H$ and $ME.L$ hold). Hence in order to check that this is an equilibrium we need to check that buyers are behaving optimally; i.e., that all three price offers made with positive probability are optimal so that $ME.B$ holds. To see this note that the proportion of high-quality sellers in the market is $\mu^H = K^H/(K^H + K^L) = 1/3$. Thus offering $r^H = c^H = 3/5$ (which is accepted by both types of sellers) yields

$$B(r^H) = \frac{1}{3}(1 - \frac{3}{5}) + \frac{2}{3}(\frac{2}{5} - \frac{3}{5}) = 0.$$

Likewise, offering of $r^L = u^L$ (which is accepted only by low-quality sellers) yields

$$B(r^L) = \frac{2}{3}(\frac{2}{5} - \frac{2}{5}) + \frac{1}{3}\delta V^B = 0.$$

Also offering a price $p < r^L$ (which is rejected by both types of sellers) yields $B(p) = \delta V^B = 0$. Therefore all three price offers are optimal.

Note that

$$u^H - c^H = 2/5 > 1/5 = u^L - c^L;$$

and therefore by Proposition 3 the equilibrium described is the unique stationary market equilibrium.

In this market the equilibrium surplus is

$$S^F = q^L V^L = \frac{4}{5}\left(\frac{1}{5\delta}\right),$$

which is $1/\delta$ times the competitive surplus, given by

$$q^L \left(u^L - c^L\right) = \frac{4}{5}\left(\frac{1}{5}\right).$$

When $\delta = .9$, the surplus is around 11% greater than the competitive surplus.

Both the surplus and the payoff to low-quality sellers, V^L, approach (from above) their values at the competitive equilibrium as δ approaches one. The total surplus falls as δ approaches one. As frictions vanish, the probability of an offer of r^H or r^L also falls, and although low-quality sellers become more patient, the result is a lower surplus. All these features are not peculiar to the example, but hold generally – see the proof of Proposition 2.2 in Appendix A.

As illustrated by the example, when average quality is low and frictions are small, equilibrium is characterized by delay. In fact, delay increases as δ approaches one. In the example, if $\delta = .9$ then the probability that a high-quality seller trades when matched (λ^H) is only $\frac{1}{6}$, and the probability that a low-quality seller trades when matched (λ^L) is $\frac{1}{3}$. Hence the majority of matches end without trade.

Even though delay may be unavoidable due to the presence of adverse selection, the delay that traders experience in a decentralized market is inefficiently large. Consider a mechanism that in each match asks the seller to report whether he has a high or low-quality unit. If the seller reports that he has a high-quality unit, then the buyer and seller trade with probability $Z^H = \frac{3}{10}$ at the price $\frac{3}{5}$, and if he reports that he has a low-quality unit, then they trade with probability $Z^L = 1$ at the price $\frac{1}{2}$. It is easy to see that for the stocks $K^H = 1$ and $K^L = 6/5$, this mechanism leaves the market in a steady state: the flow of high-quality sellers leaving the market is

$$\alpha Z^H K^H = (\frac{2}{3})(\frac{3}{10})(1) = \frac{1}{5} = q^H,$$

and the flow of low-quality sellers leaving the market is

$$\alpha Z^L K^L = (\frac{2}{3})(1)(\frac{6}{5}) = \frac{4}{5} = q^L.$$

In this mechanism, the expected utility of a high-quality seller is

$$V^H = \alpha Z^H(\frac{3}{5} - c^H) + (1 - \alpha Z^H)\delta V^H = 0,$$

the expected utility of a low-quality seller is

$$V^L = \alpha(\frac{1}{2} - c^L) + (1 - \alpha)\delta V^L = \frac{2}{7},$$

and the expected utility of a buyer is

$$V^B = \alpha[\mu^H Z^H(u^H - \frac{3}{5}) + \mu^L Z^L(u^L - \frac{1}{2})] + (1 - \alpha[\mu^H Z^H + \mu^L Z^L])\delta V^B = 0.$$

(Note that $\mu^H = \frac{5}{11}$.) Thus, the mechanism is individually rational. The mechanism is also incentive compatible. A matched low-quality seller who reports his type truthfully obtains

$$\frac{1}{2} - c^L = \frac{3}{10},$$

and he also obtains

$$Z^H(\frac{3}{5} - c^L) + (1 - Z^H)\delta V^L = \frac{3}{10},$$

if he reports that he is high-quality. Clearly incentive compatibility holds for a high-quality seller since he obtains zero by reporting his type truthfully, but obtains a negative payoff by reporting he is low-quality. The mechanism's flow surplus is

$$V^B + q^H V^H + q^L V^L = \frac{4}{5}(\frac{2}{7}) = \frac{8}{35}.$$

In contrast, as we saw earlier, the flow surplus obtained under decentralized trade is only

$$\frac{4}{5}\left(\frac{1}{5\delta}\right)=\frac{4}{5}\left(\frac{1}{5(\frac{9}{10})}\right)=\frac{8}{45}.$$

Hence equilibrium in a market with decentralized trade and adverse selection is inefficient.

6 Conclusion

We have shown that in markets with adverse selection, decentralized trade leads to competitive payoffs as frictions vanish. When average quality is high, there are several competitive equilibria; decentralized trade uniquely selects an efficient competitive equilibrium. When average quality is low, equilibrium under decentralized trade has several counter intuitive properties. In particular, while payoffs are competitive as frictions vanish, transaction prices and the patterns of trade (i.e., which qualities trade) are not. In addition, if frictions are small but non-negligible, the surplus generated under decentralized trade is *greater* than the competitive surplus, and it *decreases* as frictions become smaller.

This last result, that the decentralized surplus is greater than the competitive surplus, raises the question of whether the *static* competitive model provides an appropriate benchmark for competitive outcomes in a dynamic market. For markets with stationary flows of agents entering it has been shown that the unique stationary *dynamic competitive equilibrium* (DCE) is the repetition of the static competitive equilibrium — see Wooders (1998) for markets for homogeneous goods, and Janssen and Roy (2004) for markets with adverse selection and a continuum of qualities. One obtains the same result adapting these definitions of DCE to our setting.[5] The stationary DCE thus provides the same benchmark as the static competitive model.

However, when average quality is low there is a rich set of non-stationary DCE which exhibit cycles. In these cycles there is an initial phase in which only low-quality sellers trade (at price u^L), while high-quality sellers accumulate in the market; there is an intermediate phase in which there is no trade; and there is a final phase in which

[5] See Appendix B, for a formal definition of DCE, and for a proof that in our setting, when average quality is low, the unique stationary DCE is the repetition of the (static) competitive equilibrium.

both qualities trade (at price c^H). These non-stationary DCE generate more surplus than the stationary DCE, and some generate more surplus than the statationary equilibria of a decentralized market as well.

Table 1 below describes a DCE of this kind for the market in the example of Section 5 when $\delta = .9$.

t	p_t	m_t^H	m_t^L	m_t^B	u_t	K_t^H	K_t^L	K_t^B	V_t^L
1 – low	.4	0	.8	.8	.4	.2	.8	1.0	.20
2 – low	⋮	⋮	⋮	⋮	⋮	.4	⋮	1.2	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7 – low	⋮	⋮	.8	.8	⋮	1.4	.8	2.2	.20
8 – no trade	⋮	⋮	0	0	⋮	1.6	1.6	3.2	.21
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
13 – no trade	.4	0	0	0	.4	2.6	4.8	7.4	.36
14 – high	.6	2.8	5.6	8.4	.6	2.8	5.6	8.4	.40
15 – low	.4	0	.8	.8	.4	.2	.8	1	.20
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1: A non-stationary DCE with interesting properties

The table describes the evolution over the cycle of the market price (column p_t), the measures of trading agents of each type (m_t^H, m_t^L and m_t^B), the expected value to a buyer of a unit supplied (u_t), the stocks of agents of each type (K_t^H, K_t^L and K_t^B), and the expected utility of a low-quality seller (V_t^L).

In the first 7 periods of this DCE only low-quality sellers trade (at price $u^L = .4$); in the following 6 periods there is no trade at all (the price remains at u^L); finally there is a single period in which both qualities trade (at price $c^H = .6$). Low-quality sellers entering in periods 1 through 7 optimally trade in the period they enter rather than in period 14. (A low-quality seller entering in period 7, for example, obtains $.4 - c^L = .2$ trading in period 7 but obtains only $\delta^7(.6 - c^L) = .191$ trading in period 14.) In contrast, low-quality sellers entering in periods 8 through 13 obtain a payoff of at least $\delta^6(.6 - c^L) = .212$ if they trade in period 14 at price .6, while their payoff is only .2 if they trade in the period they enter at price .4. At period 14 the measures of high and low-quality sellers accumulated in the market are 2.8 ($= .2 \times 14$) and 5.6 ($= .8 \times 7$), respectively. All of these units are supplied at period 14 and hence the expected value to a buyer of a unit is .6. The buyers' trading decisions are therefore optimal since each buyer obtains a payoff of zero regardless of the period in which he trades.[6]

The DCE in Table 1 generates more surplus than the stationary equilibrium of a decentralized market. To see this, note that since only low-quality sellers capture any surplus in both outcomes, only those two surpluses need to be compared. The present value of the surplus generated in the DCE described in Table 1 over the 14-period cycle is $\sum_{t=1}^{14} \delta^{t-1} q^L V_t^L = 1.404$. In contrast, the flow surplus under decentralized trade is $q^L V^L = q^L(u^L - c^L)/\delta$, and the present value of the surplus over 14 periods is $\sum_{t=1}^{14} \delta^{t-1} q^L V^L = 1.371$.

This example illustrates that the comparison of surplus under centralized and decentralized trade depends on what benchmark one adopts for the competitive surplus. An apples to apples comparison of the outcomes under centralized and decentralized trade requires a complete characterization of the set of non-stationary equilibria for both structures, which seems arduous as these sets are large. If, as is standard in the literature studying homogenous goods, one adopts the static-competitive benchmark, then the surplus under decentralized trade is greater when average quality is low and frictions are small. If one adopts the dynamic-competitive benchmark and restricts attention to comparing stationary equilibria, the result remains the same.

7 Appendix A: Proofs of Propositions 1 to 3

Proof of Proposition 1. Proposition 1 follows from the results established in Lemma 1 below. Specifically: $P1.1$ follows from $L1.3$ and $L1.7$; $P1.2$ is restated as $L1.4$; $P1.3$ is implied by $L1.5$ and $L1.6$; and $P1.4$ follows from $L1.3$, $L1.7$, and $L1.9$. Lemma 1 also establishes other auxiliary results that will be used in the proofs of propositions 2 and 3.

Lemma 1. *In a stationary market equilibrium:*

($L1.1$) $(I(p, r^H) - I(p, r^L))(p - c^H) \geq (I(p, r^H) - I(p, r^L))\delta V^H$ *for all* $p \in \mathbb{R}_+$;

($L1.2$) $V^L - V^H < c^H - c^L$;

($L1.3$) $r^H > r^L$ *and* $\lambda^H \leq \lambda^L$;

[6]Proposition B.2 in Appendix B characterizes this class of DCE.

($L1.4$) $\lambda(p) = \lambda(r^L)$ *for* $p \in (r^L, r^H)$, *and* $\lambda(r^H) = 1$;

($L1.5$) $\lambda^L < 1$ *implies* $V^B = 0$;

($L1.6$) $V^H = 0$ *and* $V^L = \alpha\lambda^H(c^H - c^L)/[1 - \delta(1 - \alpha\lambda^H)]$;

($L1.7$) $r^H = c^H$;

($L1.8$) $K^H = q^H/(\alpha\lambda^H)$ *and* $K^L = q^L/(\alpha\lambda^L)$.

($L1.9$) $\mu^H = q^H/[q^H + q^L(\lambda^H/\lambda^L)] \geq q^H$.

Proof. Let $[(\lambda, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$ be a stationary market equilibrium. We prove $L1.1$. This inequality trivially holds for p such that $I(p, r^H) - I(p, r^L) = 0$. If $I(p, r^H) - I(p, r^L) = 1$ then $p \geq r^H$. Hence $p - c^H \geq r^H - c^H = \delta V^H$ by $ME.H$, and the inequality holds. If $I(p, r^H) - I(p, r^L) = -1$ then $p < r^H$ and hence $p - c^H < r^H - c^H = \delta V^H$ and so $-(p - c^H) > -\delta V^H$.

We prove $L1.2$. Note that $L1.1$ implies

$$I(p, r^H)(p - c^H) + (1 - I(p, r^H))\delta V^H \geq I(p, r^L)(p - c^H) + (1 - I(p, r^L))\delta V^H.$$

Hence

$$\begin{aligned}
V^H &= \alpha \int_0^\infty \left(I(p, r^H)(p - c^H) + (1 - I(p, r^H))\delta V^H\right) d\lambda(p) + (1 - \alpha)\delta V^H \\
&\geq \alpha \int_0^\infty \left(I(p, r^L)(p - c^L + c^L - c^H) + (1 - I(p, r^L))\delta V^H\right) d\lambda(p) + (1 - \alpha)\delta V^H \\
&= \alpha \int_0^\infty I(p, r^L)(p - c^L) d\lambda(p) + \alpha\lambda^L(c^L - c^H) + (1 - \alpha\lambda^L)\delta V^H \\
&= \frac{\alpha}{1 - \delta(1 - \alpha\lambda^L)} \left(\lambda^L(c^L - c^H) + \int_0^\infty I(p, r^L)(p - c^L) d\lambda(p)\right),
\end{aligned}$$

Since

$$\begin{aligned}
V^L &= \alpha \int_0^\infty I(p, r^L)(p - c^L) d\lambda(p) + (1 - \alpha\lambda^L)\delta V^L \\
&= \frac{\alpha}{1 - \delta(1 - \alpha\lambda^L)} \int_0^\infty I(p, r^L)(p - c^L) d\lambda(p),
\end{aligned}$$

we have

$$V^H \geq \frac{\alpha\lambda^L(c^L - c^H)}{1 - \delta(1 - \alpha\lambda^L)} + V^L.$$

Since $\frac{\alpha\lambda^L}{1 - \delta(1 - \alpha\lambda^L)} < 1$ for $\delta < 1$, we have $V^L - V^H < c^H - c^L$.

Now $L1.2$ implies $c^H - c^L > \delta(V^L - V^H)$, and therefore $r^H = c^H + \delta V^H > c^L + \delta V^L = r^L$ by $ME.H$ and $ME.L$. And $r^H > r^L$ implies $\lambda^H \leq \lambda^L$. Hence $L1.3$ holds.

We prove $L1.4$. We show that any price offer in (r^L, r^H) yields a payoff less than a price offer of r^L; hence these prices offers are suboptimal, and therefore are not in the support of λ by $ME.B$. A price offer of r^L, which is accepted only by low-quality sellers (i.e., $I(r^L, r^L) = 1$ but $I(r^L, r^H) = 0$), yields a payoff

$$\begin{aligned} B(r^L) &= \sum_{\tau \in \{H,L\}} \mu^\tau \left(I(r^L, r^\tau)(u^\tau - r^L) + (1 - I(r^L, r^\tau))\delta V^B \right) \\ &= \mu^H \delta V^B + \mu^L (u^L - r^L). \end{aligned}$$

The payoff to offering $\bar{p} \in (r^L, r^H)$, which is also accepted only by low-quality sellers, is

$$\begin{aligned} B(\bar{p}) &= \sum_{\tau \in \{H,L\}} \mu^\tau \left(I(\bar{p}, r^\tau)(u^\tau - \bar{p}) + (1 - I(\bar{p}, r^\tau))\delta V^B \right) \\ &= \mu^H \delta V^B + \mu^L (u^L - \bar{p}) < \mu^H \delta V^B + \mu^L (u^L - r^L) = B(r^L). \end{aligned}$$

Analogously, it is easy to see that any price offer greater than r^H yields a payoff less than a price offer of r^H, and is therefore suboptimal.

We prove $L1.5$. Suppose that $\lambda^L < 1$. Hence there is a $\tilde{p} < r^L$ in the support of λ. The payoff to offering $\tilde{p}$ is

$$B(\tilde{p}) = \sum_{\tau \in \{H,L\}} \mu^\tau \left(I(\tilde{p}, r^\tau)(u^\tau - \tilde{p}) + (1 - I(\tilde{p}, r^\tau))\delta V^B \right) = \delta V^B.$$

Since $B(\tilde{p}) = B(p)$ for all p in the support of λ by $ME.B$, we have

$$V^B = \alpha \int_0^\infty B(p) d\lambda(p) + (1 - \alpha)\delta V^B = \alpha B(\tilde{p}) + (1 - \alpha)\delta V^B = \delta V^B.$$

Thus, $\delta < 1$ implies $V^B = 0$.

We prove $L1.6$. Since $r^H - c^H = \delta V^H$ and every price in the support of λ satisfies $p \le r^H$, we have

$$\begin{aligned} V^H &= \alpha \int_0^\infty \left(I(p, r^H)(p - c^H + r^H - r^H) + (1 - I(p, r^H))\delta V^H \right) d\lambda(p) + (1 - \alpha)\delta V^H \\ &= \delta V^H + \alpha \int_0^\infty I(p, r^H) \left(p - r^H \right) d\lambda(p) \\ &\le \delta V^H. \end{aligned}$$

Again $\delta < 1$ and $V^B \ge 0$ imply $V^H = 0$. By $L1.4$, buyers offer $r^H = c^H$ with probability λ^H, r^L with probability $\lambda^L - \lambda^H$ and a price below r^L with probability

$1-\lambda^L$. Hence the expected utility to a low-quality seller is

$$V^L = \alpha(\lambda^H(c^H - c^L) + (\lambda^L - \lambda^H)(r^L - c^L) + (1-\lambda^L)\delta V^L) + (1-\alpha)\delta V^L.$$

Since $r^L - c^L = \delta V^L$ by $ME.L$, we have

$$V^L = \alpha\lambda^H(c^H - c^L) + (1-\alpha\lambda^H)\delta V^L = \frac{\alpha\lambda^H(c^H - c^L)}{1-\delta(1-\alpha\lambda^H)}.$$

$L1.7$ is a direct implication of $L1.6$ and $ME.H$.

We prove $L1.8$. Since a τ-quality seller trades with probability λ^τ, for $\tau \in \{H, L\}$ we have

$$K^\tau = (1-\alpha\lambda^\tau) K^\tau + q^\tau = \frac{q^\tau}{\alpha\lambda^\tau}.$$

Finally, we prove $L1.9$. Because $\lambda^H \leq \lambda^L$ by $L1.3$, we have

$$\mu^H = \frac{K^H}{K^H + K^L} = \frac{q^H/\lambda^H}{q^H/\lambda^H + q^L/\lambda^L} \geq q^H. \ \square$$

Since the only price offer made with positive probability are r^H, r^L and prices less than r^L by $L1.4$, henceforth we ignore the distribution of rejected price offers, which as noted earlier is inconsequential, and describe a stationary market equilibrium by a list $[(\lambda^H, \lambda^L, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$.

Proof of Proposition 2.

We prove $P2.1$. Assume that average quality is high; i.e., $u(q^H) \geq c^H$. We show that $[(\lambda^H, \lambda^L, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$, given by $\lambda^H = \lambda^L = 1$, $r^H = c^H$, $r^L = c^L + \delta\alpha(c^H - c^L)/(1-\delta(1-\alpha))$, $K^H = q^H/\alpha$, $K^L = q^L/\alpha$, $V^H = 0$, $V^L = \alpha(c^H - c^L)/(1-\delta(1-\alpha))$, and $V^B = \alpha(u(q^H) - c^H)/\left(1-\delta(1-\alpha)\right)$, is a stationary market equilibrium. It is easy to check that equations (1) to (3) are satisfied, and therefore that the values defined form a steady state. Since $r^H - c^H = \delta V^H$ and $r^L - c^L = \delta V^L$, $ME.H$ and $ME.L$ are satisfied. We show that c^H, the unique price in the support of λ, is an optimal price offer, and hence that $ME.B$ is satisfied.

Since $c^H = r^H > r^L$ is accepted by both types of sellers, a buyer who offers c^H obtains a payoff of $B(c^H) = u(q^H) - c^H$. We show that $B(c^H) \geq B(p)$ for all $p \in \mathbb{R}_+$ which establishes $ME.B$. If $p \geq c^H$ then $I(p, r^H) = I(p, r^L) = 1$, and therefore

$$B(p) = u(q^H) - p \leq u(q^H) - c^H = B(c^H).$$

Now, assume that $\delta < 1$ is sufficiently near one that $u^L - r^L < 0$, i.e.,

$$u^L - c^L - \frac{\delta\alpha(c^H - c^L)}{1 - \delta(1-\alpha)} < 0.$$

(Recall that $c^H > u^L$.) For $p \in [r^L, c^H)$, we have $I(p, r^H) = 0$ and $I(p, r^L) = 1$, which implies

$$B(p) = q^L(u^L - p) + q^H\delta V^B \leq q^L(u^L - r^L) + q^H\delta V^B < q^H\delta V^B < \delta V^B.$$

Also for $p < r^L$, we have $B(p) = \delta V^B$. Therefore in either case (i.e., for all $p < c^H$) we have

$$B(p) \leq \delta V^B = \frac{\delta\alpha}{1-\delta(1-\alpha)}(u(q^H) - c^H) < u(q^H) - c^H = B(c^H).$$

In order to complete the proof of $P2.1$ we compute the flow surplus. We have

$$S^F = V^B + q^H V^H + q^L V^L = \frac{\alpha}{1-\delta(1-\alpha)}(q^H(u^H - c^H) + q^L(u^L - c^L)).$$

Since $\alpha/(1-\delta(1-\alpha)) < 1$ for $\alpha < 1$, S^F is less than the surplus at the efficient competitive equilibrium.

We prove $P2.2$. Assume now that average quality is low; i.e., $u(q^H) < c^H$. We show that $[(\lambda^H, \lambda^L, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$, given by

$$\lambda^H = \frac{(1-\delta)(u^L - c^L)}{\alpha\delta(c^H - u^L)},$$

and

$$\lambda^L = \lambda^H \frac{q^L(c^H - u^L)}{q^H(u^H - c^H)},$$

$r^H = c^H$, $r^L = u^L$, $K^H = q^H/(\alpha\lambda^H)$, $K^L = q^L/[\alpha(\lambda^H + \lambda^L)]$, $V^H = 0$, $V^L = (u^L - c^L)/\delta$, and $V^B = 0$ is a stationary market equilibrium. Note that $u(q^H) = q^H u^H + q^L u^L < c^H = q^H c^H + q^L c^H$ implies $q^H(u^H - c^H) < q^L(c^H - u^L)$, and therefore $\lambda^L > \lambda^H$. Moreover, for δ close to one λ^H is sufficiently small that $\lambda^L < 1$. It is easy to check that equations (1) to (3) are satisfied, and therefore that the values defined form a steady state. Since $r^H - c^H = \delta V^H$ and $r^L - c^L = \delta V^L$, $ME.H$ and $ME.L$ are satisfied. We prove that $ME.B$ is also satisfied.

The proportions of sellers of each type are

$$\mu^H = \frac{K^H}{K^H + K^L} = \frac{c^H - u^L}{u^H - u^L},$$

and $\mu^L = 1 - \mu^H$, and the expected utility of a random unit is

$$u(\mu^H) = \mu^H u^H + \mu^L u^L = c^H.$$

Hence a price offers of $c^H (= r^H > r^L = u^L)$, which is accepted by both types of buyers, yields

$$B(c^H) = u(\mu^H) - c^H = 0.$$

A price offer of $u^L (= r^L)$, which is accepted only by low-quality sellers, yields

$$B(u^L) = \mu^L \left(u^L - u^L\right) + \left(1 - \mu^L\right) \delta V^B = 0.$$

And a price offer p less than $u^L (= r^L < r^H)$, which is rejected, yields

$$B(p) = \delta V^B = 0.$$

Hence all three price offers made with positive probability yield a payoff of zero. In order to show that these price offers are optimal, we prove that any price offer yields a non-positive payoff. Let $p \in \mathbb{R}_+$. If $p \geq c^H$ then the offer is accepted by all sellers, and yields a payoff of

$$B(p) = u(\mu^H) - p \leq u(\mu^H) - c^H = 0.$$

If $p \in [u^L, c^H)$, then the offer is accepted by only low-quality sellers, and yields a payoff of

$$B(p) = \mu^L(u^L - p) + (1 - \mu^L)\delta V^B \leq \mu^L(u^L - u^L) + (1 - \mu^L)\delta V^B = 0.$$

Finally, if $p < u^L$, then the offer is rejected, and yields $B(p) = \delta V^B = 0$. Hence $M.E.B$ holds.

The equilibrium surplus is

$$S^F = V^B + q^H V^H + q^L V^L = \frac{q^L(u^L - c^L)}{\delta}.$$

Thus, for $\delta < 1$ the equilibrium surplus is above the competitive surplus.

We prove of $P2.3$. If $u(q^H) \geq c^H$, then $V^H = 0$, $V^L = \alpha(c^H - c^L)/(1 - \delta(1 - \alpha))$, and $V^B = \alpha(u(q^H) - c^H) / \left(1 - \delta(1 - \alpha)\right)$, and therefore $\lim_{\delta \to 1} V^H = 0$, $\lim_{\delta \to 1} V^L = c^H - c^L$, and $\lim_{\delta \to 1} V^B = u(q^H) - c^H$; i.e., the traders' expected utilities converge to their expected utilities at the competitive equilibrium with price c^H. If $u(q^H) < c^H$, then $V^H = 0$, $V^L = \alpha(c^H - c^L)/(1-\delta(1-\alpha))$, and $V^B = 0$, and therefore $\lim_{\delta\to 1} V^H = 0$, $\lim_{\delta\to 1} V^L = u^L - c^L$, and $\lim_{\delta\to 1} V^B = 0$; i.e., the traders' expected utilities converge to their expected utilities at the competitive equilibrium. □

We now establish Proposition 3 by showing that every stationary equilibrium has the features described in $P2.1$ and $P2.2$. We first prove an intermediate result.

Lemma 2. *Assume that $u^H - c^H > u^L - c^L$. There is a $\hat{\delta} < 1$ such that if $\delta \in (\hat{\delta}, 1)$, then every stationary market equilibrium satisfies either (i) $\lambda^H = 1$ or (ii) $\lambda^L < 1$.*

Proof: Suppose by way of contradiction that for every $\hat{\delta}$ there is a $\delta \in (\hat{\delta}, 1)$ and a stationary market equilibrium such that neither (i) nor (ii) hold, i.e., $\lambda^H < 1 = \lambda^L$. Since $r^L < r^H$ by $L1.4$, we have

$$B(r^L) = \mu^L(u^L - r^L) + (1-\mu^L)\delta V^B,$$

and since r^L is in the support of λ, we have

$$V^B = \alpha B(r^L) + (1-\alpha)\delta V^B = \frac{\alpha\mu^L(u^L - r^L)}{\delta(1-\alpha\mu^L)}.$$

Hence $V^B \geq 0$ implies $u^L \geq r^L$. Since

$$r^L = c^L + \delta V^L = c^L + \delta\frac{\alpha\lambda^H(c^H - c^L)}{1-\delta(1-\alpha\lambda^H)}$$

by $ME.L$ and and $L1.6$, then $u^L \geq r^L$ can be written as

$$\lambda^H \leq \frac{1-\delta}{\delta\alpha}\frac{u^L - c^L}{c^H - u^L}.$$

This bound on λ^H can be made arbitrarily small by choosing $\hat{\delta}$ sufficiently close to 1. Furthermore, since $\lambda^L = 1$ then

$$\mu^H = \frac{K^H}{K^H + K^L} = \frac{q^H/\lambda^H}{q^H/\lambda^H + q^L/\lambda^L} = \frac{q^H}{q^H + q^L\lambda^H},$$

and μ^H is arbitrarily close to 1 for $\hat{\delta}$ sufficiently close to 1. Hence $u^H - c^H > u^L - c^L$ implies that there is a $\delta < 1$ such that

$$u(\mu^H) - c^H = [\mu^H u^H + (1-\mu^H)u^L] - c^H > u^L - c^L.$$

Fix a δ with this property. Since $r^L \geq c^L$ then

$$u(\mu^H) - c^H > u^L - r^L. \tag{4}$$

A price offer of p less than $r^L (< r^H)$ is rejected, and yields $B(p) = \delta V^B$; since r^L is in the support of λ, $ME.B$ implies

$$B(r^L) = \mu^L (u^L - r^L) + \mu^H \delta V^B \geq \delta V^B.$$

Because $\mu^L = 1 - \mu^H$, this inequality can be written as

$$u^L - r^L \geq \delta V^B.$$

Also a price offer of c^H ($= r^H$ by $L1.7$) is accepted, and yields $B(c^H) = u(\mu^H) - c^H$; again since r^L is in the support of λ, $ME.B$ implies

$$B(r^L) = \mu^L (u^L - r^L) + \mu^H \delta V^B \geq B(c^H) = u(\mu^H) - c^H.$$

Hence $\mu^L + \mu^H = 1$ and $u^L - r^L \geq \delta V^B$ implies

$$u^L - r^L \geq \mu^L (u^L - r^L) + \mu^H \delta V^B \geq u(\mu^H) - c^H,$$

which contradicts (4). □

Proof of Proposition 3. Assume that $u^H - c^H > u^L - c^L$ and that δ is sufficiently close to one that the conclusion of Lemma 2 holds. Let us be given a a stationary equilibrium $[(\lambda^H, \lambda^L, r^H, r^L), (K^H, K^L), (V^H, V^L, V^B)]$.

Assume that $u(q^H) \geq c^H$. Then, since $\mu^H \geq q^H$ by $L1.9$, we have $u(\mu^H) > c^H$, and therefore offering c^H ($= r^H$ by $L1.7$) yields $B(c^H) = u(\mu^H) - c^H > 0$. Then by $ME.B$

$$V^B = \alpha \sum_{\tau \in \{H,L\}} \mu^\tau \int_0^\infty B(p) d\lambda(p) + (1-\alpha)\,\delta V^B \geq \alpha B(c^H) + (1-\alpha)\,\delta V^B > 0.$$

This implies that $\lambda^L = 1$ by $L1.5$, and therefore $\lambda^H = 1$ by Lemma 2. This in turn implies

$$V^B = \alpha B(c^H) + (1-\alpha)\,\delta V^B = \frac{\alpha(u(\mu^H) - c^H)}{1 - \delta\,(1-\alpha)}.$$

Also, replacing $\lambda^H = 1$ in the formula for V^L obtained in $L1.6$ we get

$$V^L = \frac{\alpha(c^H - c^L)}{1 - \delta(1-\alpha)}.$$

Now assume that $u(q^H) < c^H$. Since $K^H = q^H/(\alpha\lambda^H)$ by $L1.8$, then $\alpha\lambda^H K^H = q^H > 0$, and therefore $\lambda^H > 0$. Suppose that $0 < \lambda^H = \lambda^L$; then $\mu^H = q^H$ (see $L1.9$), and therefore offering c^H yields

$$B(c^H) = u(\mu^H) - c^H = u(q^H) - c^H < 0,$$

whereas offering $p < r^L$ yields a payoff $B(p) = \delta V^B \geq 0$. This contradicts $ME.B$. Hence $L1.3$ implies $\lambda^H < \lambda^L$, and therefore $\lambda^H < 1$, which in turn implies $\lambda^L < 1$ by Lemma 2. Thus $0 < \lambda^H < \lambda^L < 1$. Also $\lambda^L < 1$ implies $V^B = 0$ by $L1.5$. And $0 < \lambda^H < \lambda^L < 1$ imply by $ME.B$ that price offers of c^H, of r^L, and of less than r^L are optimal; i.e.,

$$u(\mu^H) - c^H = \mu^L(u^L - r^L) + \mu^H \delta V^B = \delta V^B = 0.$$

Since $\mu^L > 0$, this implies $r^L = u^L$. And since $r^L = c^L + \delta V^L$ by $ME.L$, we have

$$V^L = (u^L - c^L)/\delta.$$

Finally we show that the values of λ^H and λ^L are those specified in the proof of $P2.2$. We have

$$V^L = \alpha\lambda^H(r^H - c^L) + (1 - \alpha\lambda^H)\delta V^L = \frac{\alpha\lambda^H(r^H - c^L)}{1 - \delta(1 - \alpha\lambda^H)};$$

hence

$$\lambda^H = \frac{1-\delta}{\delta\alpha}\frac{u^L - c^L}{c^H - u^L}.$$

Furthermore $u(\mu^H) - c^H = 0$ implies

$$\frac{c^H - u^L}{u^H - u^L} = \mu^H = \frac{q^H}{q^H + q^L\frac{\lambda^H}{\lambda^L}},$$

and therefore

$$\lambda^L = \lambda^H \frac{q^L}{q^H}\frac{c^H - u^L}{u^H - c^H}. \square$$

8 Appendix B: Dynamic Competitive Equilibrium

A *dynamic competitive equilibrium* is defined as follows – see Wooders (1998) for a similar definition when goods are homogeneous, and Janssen and Roy (2003, 2004) for a definition when there is quality uncertainty. Assume that time runs from 1 to infinity. Let $p = \{p_t\}$ be a price sequence in $\mathbb{R}_+$ and let $K \in \mathbb{R}_+$. For each $\tau \in \{H, L\}$ and each $t \in \mathbb{Z}_+$ (the set of positive integers), define

$$s_t^\tau(p, K) = \begin{cases} K & \text{if } p_t - c^\tau > \max_{s \in \mathbb{Z}_+}\{0, \delta^s (p_{t+s} - c^\tau)\} \\ [0, K] & \text{if } p_t - c^\tau = \max_{s \in \mathbb{Z}_+}\{0, \delta^s (p_{t+s} - c^\tau)\} \\ 0 & \text{if } p_t - c^\tau < \max_{s \in \mathbb{Z}_+}\{0, \delta^s (p_{t+s} - c^\tau)\}, \end{cases}$$

to be the supply of the τ-quality good at date t if there is a measure K of τ-quality sellers in the market. A τ-quality seller supplies at date t only if the utility he obtains, i.e., $p_t - c^\tau$, at least as great as the utility $\delta^s (p_{t+s} - c^\tau)$ he would obtain by supplying at date $t + s$ or by never supplying.

Let $u = \{u_t\}$ be a sequence of utility values in $[u^L, u^H]$, where u_t is the expected value of a random unit drawn from the supply of units at date t.[7] For each t define

$$d_t(p, u, K) = \begin{cases} K & \text{if } u_t - p_t > \max_{s \in \mathbb{Z}_+}\{0, \delta^s (u_{t+s} - p_{t+s})\} \\ [0, K] & \text{if } u_t - p_t = \max_{s \in \mathbb{Z}_+}\{0, \delta^s (u_{t+s} - p_{t+s})\}. \\ 0 & \text{if } u_t - p_t < \max_{s \in \mathbb{Z}_+}\{0, \delta^s (u_{t+s} - p_{t+s})\}. \end{cases}$$

to be demand at date t if there is a measure K of buyers in the market, and the prices and expected utilities of a random unit are given by p and u, respectively. A buyer purchases a unit at date t only if the utility he obtains, $u_t - p_t$, is at least as great as the utility he would obtain by purchasing at any later date, or by not purchasing at all.

A *dynamic competitive equilibrium (DCE)* is a pair (p, m), where $p = \{p_t\}$ is a sequence in $\mathbb{R}_+$ indicating the market price at each date and $m = \{(m_t^H, m_t^L, m_t^B)\}$ is a sequence in $\mathbb{R}_+^3$ indicating the measures of agents of each type trading at each date, satisfying for all $t \geq 1$:

$(DCE.S)$ for $\tau \in \{H, L\}$: $m_t^\tau \in s_t^\tau(p, K_t^\tau)$, where $K_t^\tau \in \mathbb{R}_+$ satisfies $K_1^\tau = q^\tau$ and $K_t^\tau = K_{t-1}^\tau - m_{t-1}^\tau + q^\tau$ if $t > 1$.

$(DCE.B)$ $m_t^B \in d_t(p, u_t, K_t^B)$, where $u_t \in [u^L, u^H]$ satisfies $u^H \frac{m_t^H}{m_t^H + m_t^L} + u^L \frac{m_t^L}{m_t^H + m_t^L}$ whenever $m_t^H + m_t^L > 0$, and where $K_t^B \in \mathbb{R}_+$ satisfies $K_1^B = q^B$ and $K_t^B = K_{t-1}^B - m_{t-1}^B + q^B$ if $t > 1$;

[7] We assume $u_t \in [u^L, u^H]$ for each t in order to rule out trivial non-trading equilibria in which $p_t < c^L$ (and therefore no seller supplies at t) and $u_t < p_t$ (so no buyer demands a unit at date t).

$(DCE.M)$ $m_t^H + m_t^L = m_t^B$.

Condition $DCE.S$ guarantees that seller behavior is optimal. Condition $DCE.B$ requires that buyers behave optimally and that their expectations are correct. Finally, condition $DCE.M$ requires that markets clear at every date.

A *stationary dynamic competitive equilibrium (SDCE)* is a *DCE* for which the equilibrium price sequence is a constant sequence.

Even though this definition only requires that the sequence of prices be constant for a DCE to be stationary, our next proposition establishes that in the case of interest, i.e., when average quality is low, the unique SDCE generates a stationary flow of trade.

Proposition B.1. *If average quality is low, then there is a unique stationary market equilibrium, (p, m), given by $p_t = u^L$ and $m_t = (0, q^L, q^L)$ for all t.*

Proof. It is easy to see that $p_t = u^L$, $m_t = (0, q^L, q^L)$ for all t, with $K_{t+1}^L = q^L$, $K_{t+1}^H = K_1^H + tq^H$, $K_{t+1}^B = K_1^B + tq^H$ and $u_t = u^L$ for all t, forms a *SDCE*. We prove that this equilibrium is unique. Let (p, u) with $p_t = \bar{p}$ for all t be a SDCE. We show that $\bar{p} \geq u^L$. Suppose that $\bar{p} < u^L$. We first show that $m_t^B = K_t^B$ for all t. Suppose to the contrary that $m_t^B < K_t^B$ for some t. A buyer in the market at date t, by delaying trade s periods, obtains at most

$$\delta^s(u_{t+s} - \bar{p}) \leq \delta^s(u^H - \bar{p}).$$

Hence a buyer delays trade by at most $\bar{s}$ periods, where $\bar{s}$ is the largest integer such that

$$\delta^{\bar{s}}(u^H - \bar{p}) \leq u_t - \bar{p}.$$

Therefore there is a $t' \in \{t+1, \ldots, t+\bar{s}\}$ such that $m_{t'}^B > 0$. Hence

$$\delta^{t'-t}(u_{t'} - \bar{p}) \geq u_t - \bar{p} \geq u^L - \bar{p},$$

where the first equality follows from $DCE.B$ and the second follows from $u_t \geq u^L$. Since $\delta < 1$ this implies $u_{t'} > u^L$. $DCE.M$ and $m_{t'}^B > 0$ imply $m_{t'}^H + m_{t'}^L > 0$ and hence $DCE.B$ implies

$$u_{t'} = u^H \frac{m_{t'}^H}{m_{t'}^H + m_{t'}^L} + u^L \frac{m_{t'}^L}{m_{t'}^H + m_{t'}^L}.$$

But $\bar{p} < u^L$ and $u^L < c^H$ implies $\bar{p} < c^H$, and hence $m^H_{t'} = 0$ by $DCE.S$. Thus $m^L_{t'} > 0 = m^H_{t'}$ implies $u_{t'} = u^L$, a contradiction. Hence $m^B_t = K^B_t$ $\forall t$. Since $\bar{p} < c^H$ then $m^H_t = 0$ for all t by $DCE.S$. By $DCE.M$ we have

$$m^L_t = m^B_t - m^H_t = m^B_t = K^B_t \geq q^B = 1.$$

Thus for each t we have

$$K^L_{t+1} = K^L_t - m^L_t + q^L \leq K^L_t - 1 + q^L = K^L_t - q^H.$$

This implies $K^L_t < 0$ for t sufficiently large, a contradiction. This establishes that $\bar{p} \geq u^L$.

We now show that $\bar{p} \leq u^L$. Suppose to the contrary that $\bar{p} > u^L$. Then $m^L_t = K^L_t \geq q^L > 0$ for all t by $DCE.S$. Condition $DCE.M$ implies $m^B_t > 0$ for all t, and then $DCE.B$ implies $u_t - \bar{p} \geq 0$. We have $m^B_t > 0$ and $m^B_t = m^L_t + m^H_t$ by $DCE.M$ and therefore u_t is given by $DCE.B$. Thus $u_t \geq \bar{p} > u^L$ implies $m^H_t > 0$ which, together with $DCE.S$, implies $\bar{p} \geq c^H$. Hence $DCE.B$ implies $u_t \geq c^H$.

As noted earlier, $m^L_t = K^L_t$ and hence $K^L_t = q^L$ for all t. Hence we have

$$u_t = u^H \frac{m^H_t}{m^H_t + q^L} + u^L \frac{q^L}{m^H_t + q^L} \geq c^H.$$

Let $\tilde{m}$ be such that

$$u^H \frac{\tilde{m}}{\tilde{m} + q^L} + u^L \frac{q^L}{\tilde{m} + q^L} = c^H.$$

Since $u_t \geq c^H$ then $m^H_t \geq \tilde{m}$, and since average quality is low then $\tilde{m} > q^H$. Thus for each t we have

$$K^H_{t+1} = K^H_t - m^H_t + q^H \leq K^H_t - \tilde{m} + q^H < K^H_t.$$

This implies $K^H_t < 0$ for t sufficiently large, a contradiction. This establishes that $\bar{p} \leq u^L$.

We have shown $\bar{p} \geq u^L$ and $\bar{p} \leq u^L$, and hence $\bar{p} = u^L$ in any SDCE. Since $c^L < u^L = \bar{p} < c^H$, then $DCE.S$ implies $m^H_t = s^H_t(p, K^H_t) = 0$ for all t and $m^L_t = s^L_t(p, K^L_t) = K^L_t$ for all t, and therefore $m^L_t = q^L$ for all t. □

Proposition B.2. *Assume that time runs from 1 to ∞ and δ is sufficiently large that $\delta(c^H - c^L) \geq u^L - c^L$. Let l, n and h be three positive integers satisfying*

$$\delta^n(c^H - c^L) \geq u^L - c^L \geq \delta^{n+1}(c^H - c^L) \qquad (*)$$

and

$$(l+n+h)q^H \geq (n+h)q^L \frac{c^H - u^L}{u^H - c^H}. \qquad (**)$$

(i) When average quality is low there are non-stationary DCE with cycles with three phases. The initial phase lasts l periods, the price is u^L and only low-quality units trade; the intermediate phase lasts n periods, the price is u^L, and there is no trade; the final phase lasts h periods, the price is c^H and both high and low quality units trade.

(ii) There are non-stationary three-phase DCE which generate more surplus than the stationary DCE.

Proof: Let $\tilde{m}$ be such that

$$u^H \frac{\tilde{m}}{\tilde{m}+q^L} + u^L \frac{q^L}{\tilde{m}+q^L} = c^H,$$

i.e., $\tilde{m} = q^L \frac{c^H - u^L}{u^H - c^H}$. Since average quality is low, then $\tilde{m} > q^H$. Let (p, m) be given for $t \in \{1, \ldots, l+n+h\}$ by

t	p_t	m_t^L	m_t^H	m_t^B
1	u^L	q^L	0	q^L
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
l	$\vdots$	q^L	$\vdots$	q^L
$l+1$	$\vdots$	0	$\vdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$l+n$	u^L	0	0	0
$l+n+1$	c^H	$(n+1)q^L$	$(n+1)\tilde{m}$	$(n+1)(q^L+\tilde{m})$
$l+n+2$	$\vdots$	q^L	$\tilde{m}$	$q^L+\tilde{m}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$l+n+h$	c^H	q^L	$\tilde{m}$	$q^L+\tilde{m}$

and for $t > l+n+h$, let $p_t = p_{t-(l+n+h)}$, and $m_t^\tau = m_{t-(l+n+h)}^\tau$ for $\tau \in \{H, L, B\}$.

We show that this pair, together with

$$K_{t+1}^\tau = K_t^\tau - m_t^\tau + q^\tau,$$

for all t, and $u_t = u^L$ for $t \in \{1, \ldots, l+n\}$, $u_t = c^H$ for $t \in \{l+n+1, ..., l+n+h\}$ and $u_t = u_{t-(l+n+h)}$ for $t > l+n+h$, forms a DCE

By construction $DCE.M$ is satisfied. Equation (**) implies that in period $l+n+h$ we have $K^H_{l+n+h} \geq m^H_{l+n+h}$. We show that $DCE.S$ holds. A low-quality seller in the market in period $t \in \{1, \ldots, l\}$ optimally trades at either t (obtaining utility $u^L - c^L$), or at period $l+n+1$ (obtaining $\delta^{l+n+1-t}(c^H - c^L)$). For each $t \in \{1, \ldots, l\}$ we have

$$u^L - c^L \geq \delta^{n+1}(c^H - c^L) \geq \delta^{l+n+1-t}(c^H - c^L),$$

where the first inequality holds by (*). Thus $m^L_t = K^L_t \in s^L_t(p, K^L_t)$ for $t \in \{1, \ldots, l\}$. A low-quality seller in the market at period $t \in \{l+1, \ldots, l+n\}$ optimally trades either at t or at period $l+n+1$. For each $t \in \{l+1, \ldots, l+n\}$ we have

$$u^L - c^L \leq \delta^{n}(c^H - c^L) \leq \delta^{l+n+1-t}(c^H - c^L),$$

where the first inequality holds by (*). Thus $m^L_t = 0 \in s^L_t(p, K^L_t)$ for $t \in \{l+1, \ldots, l+n\}$. Since $c^H > u^L > c^L$, clearly $m^L_t = K^L_t = s^L_t(p, K^L_t)$ for $t \in \{l+n+1, \ldots, l+n+h\}$.

For high-quality sellers we have

$$s^H_t(p, K^H_t) = \begin{cases} 0 & \text{if } t \in \{1, \ldots, l+n\} \\ [0, K^H_t] & \text{if } t \in \{l+n+1, \ldots, l+n+h\}. \end{cases}$$

Clearly $m^H_t = 0 \in s^H_t(p, K^H_t)$ for $t \in \{1, \ldots, l+n\}$. We show $m^H_t = (n+1)\,\tilde{m} \in s^H_t(p, K^H_t)$ for $t = l+n+1$. We have

$$\begin{aligned} K^H_{l+n+1} &= (l+n+1)q^H \\ &\geq (l+n+1)q^H + (h-1)\left(q^H - \tilde{m}\right) \\ &= (l+n+h)q^H - (h-1)\,\tilde{m} \\ &\geq (n+1)\,\tilde{m} \end{aligned}$$

where the first inequality follow from $\tilde{m} > q^H$ the last inequality follows from (**). For $s \in \{2, \ldots, h\}$ we have

$$\begin{aligned} K^H_{l+n+s} &= (l+n+s)q^H - (n+s-1)\tilde{m} \\ &\geq (l+n+s)q^H - (n+s-1)\tilde{m} + (h-s)(q^H - \tilde{m}) \\ &= (l+n+h)q^H - (n+h-1)\,\tilde{m} \\ &\geq \tilde{m}, \end{aligned}$$

where the last inequality follows from (**). Hence $m^H_t = \tilde{m} \in s^H_t(p, K^H_t)$ for $t \in \{l+n+2, \ldots, l+n+h\}$. Therefore $DCE.S$ holds.

In order to complete the proof we show that $DCE.B$ holds. Since $p_t = u_t$ for all t, we have

$$d_t(p, u, K_t^B) = [0, K_t^B]$$

for all t. We have already established that $m_t^H \in [0, K_t^H]$ and $m_t^L \in [0, K_t^L]$ for all t. Since $K_t^B = K_t^H + K_t^L$, then $m_t^B = m_t^H + m_t^L \in [0, K_t^H + K_t^L] = d_t(p, u, K_t^B)$. Hence $DCE.B$ holds.

Table 1 in Section 6 describes an example of a DCE in this class (in which $h = 1$) that generates more surplus than the stationary DCE. $\square$

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