Econometrics II - EXAM

Answer each question in separate sheets in three hours

1. Consider the panel data model

$$\begin{aligned} y_t &= \mathbf{z}_t'\boldsymbol{\beta} + u_t, \qquad t = 1, \ldots, T \\ \mathbb{E}\left[u_t | \mathbf{z}_t, u_{t-1}, \mathbf{z}_{t-1}, \ldots\right] &= 0 \qquad t = 1, \ldots, T \\ \mathbb{E}\left[u_t^2 | \mathbf{z}_t\right] &= \mathbb{E}\left[u_t^2\right] = \sigma_t^2, \quad t = 1, \ldots, T, \end{aligned}$$

where σ_t^2 may vary in each time period.

(a) Show that $\boldsymbol{\Omega} := \mathbb{E}\left[\mathbf{u}\mathbf{u}'\right]$ is diagonal, $\mathbf{u} = (u_1, \ldots, u_T)'$.

(b) Write down the GLS estimator when $\boldsymbol{\Omega}$ is known.

(c) Show that

$$\mathbb{E}\left[\mathbf{Z} \otimes \mathbf{u}\right] = \mathbf{0}.$$

does not necessarily hold under the assumptions made.

(d) Show that the GLS estimator of (b) is consistent for $\boldsymbol{\beta}$. Is

$$\mathbb{E}\left[\mathbf{Z}\mathbf{u}\right] = \mathbf{0},$$

$\mathbf{Z} = (\mathbf{z}_1, \ldots, \mathbf{z}_T)$, necessary or sufficient for consistency of GLS?

(e) Explain how to estimate consistently each σ_t^2 $(n \to \infty)$.

(f) Under the previous assumptions, is valid the inference based on usual OLS diagnostics for Pooled OLS estimates? Propose a solution if the answer is negative.

(g) How would you test for $\sigma_t^2 = \sigma^2$ for all $t = 1, \ldots, T$? Discuss the asymptotic properties of FGLS if this is true.

2. Suppose y_t is generated by the Autoregressive Moving Average model of order (1,1), ARMA$(1,1)$,

$$y_t = \theta_0 y_{t-1} + e_t + \phi_0 e_{t-1} \tag{1}$$

where: (i) $|\theta_0| < 1$, (ii) $|\phi_0| < 1$; (iii) $\theta_0 \neq \phi_0$; (iv) $e_t \sim \text{iid}(0, \sigma_e^2)$.

(a) Show that y_t has the following infinite order moving average representation

$$y_t = e_t + \sum_{i=1}^{\infty} (\phi_0 \theta_0^{j-1} + \theta_0^j) e_{t-i}.$$

(b) Define

$$\gamma(k) = \text{Cov}[y_t, y_{t-k}] \quad \text{for } k = 0, 1, 2....$$

Show that

$$\gamma(k) = \theta_0 \gamma(k-1) + E[e_t y_{t-k}] + \phi_0 E[e_{t-1} y_{t-k}]$$

and hence that the autocovariance function of y_t is given by:

$$\begin{aligned} \gamma(0) &= \frac{\sigma_e^2 (1 + \phi_0^2 + 2\phi_0 \theta_0)}{1 - \theta_0^2} \\ \gamma(1) &= \frac{\sigma_e^2 (1 + \phi_0 \theta_0)(\phi_0 + \theta_0)}{1 - \theta_0^2}. \\ \gamma(k) &= \theta_0 \gamma(k-1), \quad \text{for } k > 1. \end{aligned}$$

(c) Is y_t covariance stationary? Is y_t strictly stationary? Explain your answers briefly.

(d) Now suppose that a researcher is only interested in the autoregressive parameter θ_0. A first solution is to run a least squares regression

$$y_t = \theta y_{t-1} + error_t.$$

What is the probability limit of the OLS estimate $\hat{\theta}$? When is this consistent?

(e) Define $z_t(k) = [y_t, y_{t-1}, ..., y_{t-k}]'$ for some finite positive integer k. Show that

$$E[z_t(k)(y_t - \theta_0 y_{t-1})] \neq 0$$

but

$$E[z_{t-i}(k)(y_t - \theta_0 y_{t-1})] = 0 \tag{2}$$

for $i > 1$.

(f) Deduce conditions on θ_0 and ϕ_0 under which θ_0 is identified by (2).

(g) Now suppose that the data are generated by (1) but the researcher overestimates the AR order to be two. Denote the (autoregressive) parameter vector of this model by the 2×1 vector ψ. Note that the true value of ψ is $\psi_0 = (\theta_0, 0)'$. Suppose that the researcher proposes estimating ψ_0 via GMM based on

$$E[z_{t-i}(k)(y_t - \psi_0' x_t)] = 0 \tag{3}$$

where $x_t = (y_{t-1}, y_{t-2})'$. Show that if $i = 2$ then ψ_0 is identified by (3) but that if $i > 2$ then ψ_0 is under-identified by (3).

3. Consider the model:

$$\begin{aligned} y_t - y_{t-1} &= \beta_0(y^* - y_{t-1}) + \alpha_0 x_t + u_t \\ u_t &= \rho_0 u_{t-1} + e_t \end{aligned}$$

where y^* represents the desired level of the process y_t, x_t is a scalar, and e_t is an i.i.d. process with mean zero. Let $\theta = (\beta, \rho, y^*, \alpha)'$, and suppose it is desired to estimate θ_0 based on the population moment condition

$$E[z_t e_t(\theta_0)] = 0$$

where

$$e_t(\theta) = y_t - \beta(1-\rho)y^* - (1+\rho-\beta)y_{t-1} - (\beta-1)\rho y_{t-2} - \alpha x_t + \rho\alpha x_{t-1}$$

and z_t is a vector of instruments. Note that this model is nonlinear in θ.

(a) Derive the condition for local identification in this model, and isolate conditions on the matrix of cross-moments of the vector $\tilde{x}_t = (1, y_{t-1}, y_{t-2}, x_t, x_{t-1})'$ with z_t, and on the parameters $\theta = (\beta, \rho, y^*, \alpha)'$.

(b) In particular, consider the identification under these two situations: (i) $\rho = 0$; (ii) $\alpha = 0$; (iii) $\beta = 0$.

(c) If θ_0 is identified, describe the asymptotic properties of the corresponding GMM estimate, stating any additional conditions you may require.

(d) Verify that θ_0 is globally identified if the condition in part (a) is satisfied by using the reduced form version of the model,

$$y_t = \mu_0 + \mu_1 y_{t-1} + \mu_2 y_{t-2} + \mu_3 x_t + \mu_4 x_{t-1} + e_t, \tag{4}$$

which is linear in the parameters $\tilde{\mu} = (\mu_0, \mu_1, \mu_2, \mu_3, \mu_4)'$, assuming that x_t and x_{t-1} are exogeneous.

(e) How is the identification affected if $\alpha = 0$? Explain briefly.

(f) Discuss how would you make inference on θ from OLS estimation of equation (4) .