Econometrics II - EXAM

Choose ONLY 3 questions

1. Consider the system of simultaneous equations

$$y_1 = \gamma_{12} y_2 + x_1 \delta_{11} + x_2 \delta_{12} + x_3 \delta_{13} + u_1 \quad (1)$$
$$y_2 = \gamma_{21} y_1 + x_1 \delta_{21} + x_2 \delta_{22} + x_3 \delta_{23} + u_2 \quad (2)$$

where all the x_g are orthogonal with the u_h.

(a) Study the identification of each equation under the following conditions and specify which estimation method would you prefer, equation by equation 2SLS or 3SLS:

i. $\delta_{12} = \delta_{23} = 0$.

ii. $\delta_{21} = \delta_{22} = 0$.

iii. $\gamma_{21} = 0$.

iv. $\delta_{13} = 0$, $\delta_{12} = \delta_{21}$.

v. $\delta_{11} = \delta_{12} = \delta_{22} = \delta_{13} = \delta_{23} = 0$, $\mathbb{E}[u_1 u_2] = 0$.

(b) Obtain the reduced form of the system in case i. How you would estimate it consistently?

(c) If we change the first equation (1) by

$$y_1 = \gamma_{12} y_2 + \gamma_{13} y_2^2 + x_1 \delta_{11} + x_2 \delta_{12} + x_3 \delta_{13} + u_1,$$

discuss the identification of the system when $\delta_{23} = 0$.

(d) When $\delta_{12} = \gamma_{21} = \delta_{23} = 0$ explain why 2SLS estimates of the equation (1) are not using all information available about (1)-(2). Will the 3SLS estimates of the first equation be the same as the 2SLS estimates? Explain in which sense 2SLS is more robust than 3SLS for estimating the parameters of the first equation.

2. For a causal $AR(p)$ model
$$X_t = \alpha_1 X_{t-1} + \cdots + \alpha_p X_{t-p} + e_t,$$
with MD innovations e_t,
   (a) State when $AR(p)$ is causal and interpret this concept.
   (b) Show $\mathbf{X}_{t-1}^{t-p} e_t$ is itself a MD, where $\mathbf{X}_{t-1}^{t-p} := (X_{t-1}, \ldots, X_{t-p})'$.
   (c) Obtain the asymptotic distribution of the OLS estimate $\hat{\rho}_T(1)$ in a simple regression of X_t on X_{t-1} when X_t is a causal $AR(1)$. Particularize and interpret the result when X_t is a White Noise process, so $\alpha_1 = 0$. State clearly the regularity conditions or results you use, specially those regarding the conditional variance of innovations.
   (d) Find the probability limit of $\hat{\rho}_T(1)$ computed in the same way but when X_t is a causal $AR(2)$. Obtain the Autocorrelation Function of X_t in this case.

3. Given a regression model
$$y_t = \mathbf{z}_t' \boldsymbol{\beta} + v_t$$
for time series data,
   (a) Discuss alternative tests for detecting the autocorrelation of v_t if this were observable. State clearly the assumptions you make on v_t.
   (b) Repeat part (a) when v_t is not observable, but there is available a consistent estimate $\tilde{\boldsymbol{\beta}}_T$ of $\boldsymbol{\beta}$ based on a sample of $t = 1, \ldots, T$ observations. State clearly the assumptions you make on v_t and $\mathbf{z}_t$.
   (c) In case you find that v_t is correlated, how asymptotic inference on the regression coefficient of $\boldsymbol{\beta}$ could take into account this fact to provide valid rules?

4. Consider the ARCH$(1,1)$ model
$$\begin{aligned} u_t &= \sigma_t \epsilon_t, \qquad \epsilon_t \sim IID(0,1) \\ \sigma_t^2 &= \gamma + \alpha u_{t-1}^2 + \delta \sigma_{t-1}^2, \qquad \gamma > 0. \end{aligned}$$
   (a) Show that u_t^2 follows an ARMA$(1,1)$ model with non-Gaussian white noise innovations. Specify the values of the parameters of the ARMA model, give covariance stationarity conditions for u_t^2 and find its autocovariance function.
   (b) Assuming that ϵ_t is Gaussian, find the log-likelihood conditional on u_0 and σ_0^2, given a sample of $t = 1, \ldots, T$ observations.
   (c) Calculate the score of the previous log-likelihood for the parameters (γ, α, δ) and interpret its form.
   (d) How would you test the presence of dynamic conditional heteroskedasticity on u_t?

5. Let the scalar dependent variable y be a binary variable, $y \in \{0,1\}$ measuring for example whether a wife chooses or not to enter the labor force, and let $\mathbf{z}$ be some personal and family characteristics. The conditional probability of y given the vector of regressors $\mathbf{z}$ is given by

$$\begin{aligned}\Pr(y=1|\mathbf{z}) &= \Phi(\mathbf{z}'\boldsymbol{\theta}_0),\\ \Pr(y=0|\mathbf{z}) &= 1-\Phi(\mathbf{z}'\boldsymbol{\theta}_0),\end{aligned}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution and $\boldsymbol{\theta}_0$ is a vector of parameters.

(a) Verify that

$$\mathbb{E}\left(y|\mathbf{z}\right) = \Phi(\mathbf{z}'\boldsymbol{\theta}_0),$$

and find the value of $\mathbb{V}\left(y|\mathbf{z}\right)$. Write down the objective function for the Nonlinear Least Squares (NLS) estimates of $\boldsymbol{\theta}_0$ given a sample of size n of independent observations of $(y,\mathbf{z})$.

(b) Assuming consistency, deduce the asymptotic distribution of the NLS estimates $\hat{\boldsymbol{\theta}}_n$. Obtain the Gauss-Newton recursion to approximate them given an initial value. Propose a consistent estimate of the asymptotic variance of the NLE $\hat{\boldsymbol{\theta}}_n$. State any further assumption you use.

(c) Suppose that $\mathbf{z}$ contains the constant term, so $\mathbf{z}'\boldsymbol{\theta} = \boldsymbol{\theta}_1 + \mathbf{z}_2'\boldsymbol{\theta}_2$. Propose a LM test for deviations of the null hypothesis that the coefficients of all the other variables in $\mathbf{z}$ are zero, $\boldsymbol{\theta}_2 = 0$. Which will be your LS estimate of $\Pr(y=1|\mathbf{z})$ under the null?

(d) Consider alternatively the estimate obtained through this recursion,

$$\tilde{\boldsymbol{\theta}}_n = \hat{\boldsymbol{\theta}}_n - \mathbf{F}_n(\hat{\boldsymbol{\theta}}_n)^{-1}\mathbf{f}_n(\hat{\boldsymbol{\theta}}_n)$$

where $\hat{\boldsymbol{\theta}}_n$ is the NLS estimate and

$$\begin{aligned}\mathbf{f}_n(\boldsymbol{\theta}) :=& \quad \mathbb{E}_n\left[\frac{\phi\left(\mathbf{z}'\boldsymbol{\theta}\right)\left\{y-\Phi(\mathbf{z}'\boldsymbol{\theta})\right\}\mathbf{z}}{\Phi(\mathbf{z}'\boldsymbol{\theta})\left(1-\Phi(\mathbf{z}'\boldsymbol{\theta})\right)}\right]\\ \mathbf{F}_n(\boldsymbol{\theta}) :=& \quad \mathbb{E}_n\left[\frac{\left\{\phi\left(\mathbf{z}'\boldsymbol{\theta}\right)\right\}^2\mathbf{z}\mathbf{z}'}{\Phi(\mathbf{z}'\boldsymbol{\theta})\left(1-\Phi(\mathbf{z}'\boldsymbol{\theta})\right)}\right]\end{aligned}$$

with $\phi(x) = (d/dx)\Phi(x)$. Deduce the asymptotic distribution of $\tilde{\boldsymbol{\theta}}_n$ using that

$$n^{1/2}\left(\tilde{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_0\right) = -n^{1/2}\mathbf{F}_n(\hat{\boldsymbol{\theta}}_n)^{-1}\mathbf{f}_n(\boldsymbol{\theta}_0) + o_p(1).$$

Can you say anything about the efficiency of $\tilde{\boldsymbol{\theta}}_n$ and $\hat{\boldsymbol{\theta}}_n$?