Universidad Carlos III de Madrid
<u>ECONOMETRICS I</u>
Academic year 2006/07
FINAL EXAM
September 22, 2007

Exam's type: 1

- TIME: 2 HOURS 30 MINUTES

Directions:

- BEFORE YOU START TO ANSWER THE EXAM:
  - Fill in your personal information in the optical reading form, which will be the only valid answering document. Remember that you must complete all your identifiying data (name and surname(s), and **NIU**, which has 9 digits and always begins by 1000) both in letters and in the corresponding optical reading boxes.
  - Fill in, both in letters and in the corresponding optical reading boxes, the course code (10188) and your group (65 or 75).
- AT THE END OF THE EXAM, YOU MUST HAND OUT THE THE OPTICAL READING FORM, TOGETHER WITH THE QUESTIONNAIRE AND THE PROBLEM SET.
- **Check that this document contains 40 questions sequentially numbered.**
- Check that the number of exam type that appears in the questionnaire matches the number indicated in the optical reading form.
- Read the questions carefully.
  Whenever a question is referred to a Problem included in the enclosed document, the question will include within parentheses at the beginning of the question the corresponding problem number.
  It is advised to read carefully the text of the problem before answering its corresponding questions.
- For each row regarding the number of each question, fill the box which corresponds with your chosen option in the optical reading form .
- **Each question, that must be answered filling the box which corresponds to the chosen option, only has one correct answer (A, B, C or D).**
  Any question in which more than one answer is selected will be considered incorrect and its score will be zero.
- To obtain a grade of 5 over 10 you must correctly answer **22 questions**.
- If you wish, you may use the answer table as a draft, although such table does not have any official validity.
- You can use the back side of the problem text as a draft (no additional sheets will be handed out).
- The relevant **statistical tables** are attached at the end of this document.

- **Any student who were found talking or sharing any sort of material during the exam will be expelled out inmediately and his/her overall score will be zero, independently of any other measure that could be undertaken.**

- **Date of grades publication**: Tuesday, September, 25th.
- **Date of exam revision**: Thursday, September, 27th, at 15:00 h, classrooms 15.0.04, 15.0.5 and 15.0.06.
- **Rules for exam revision**:
  - Its only purpose will be to check that the number of correct answers is right.
  - To be entitled for revision, the student should bring a *printed copy* of the exam solutions, which will be available in Aula Global from the date of grades publication.
  - Any complaint about hypothetical errors in the exam contents with respect to the official solutions must be done in writing. The complaint sheet must be hand in at the moment of the exam revision, indicating name and surname, NIU, and university e-mail address. After the exam revision, no complaint will be accepted in any case. If, in a five days time, the complaint were not answered and/or the grade were not amended in Aula Global, it must be understood that the complaint has been disregarded, what would end up the complaint procedure to the course professor.

Draft of ANSWERS									
ANSWER	**(a)**	**(b)**	**(c)**	**(d)**	**ANSWER**	**(a)**	**(b)**	**(c)**	**(d)**
1.					**21.**				
2.					**22.**				
3.					**23.**				
4.					**24.**				
5.					**25.**				
6.					**26.**				
7.					**27.**				
8.					**28.**				
9.					**29.**				
10.					**30.**				
11.					**31.**				
12.					**32.**				
13.					**33.**				
14.					**34.**				
15.					**35.**				
16.					**36.**				
17.					**37.**				
18.					**38.**				
19.					**39.**				
20.					**40.**				

1. **(Problem 2)** Exploiting the fact that in OUTPUT 1 the only explanatory variable is binary, the proportion in the sample of accused black individuals who are condemned to death is approximately equal to:

   (a) We do not have enough information to calculate.

   (b) 23%.

   (c) 28%.

   (d) 8%.

2. **(Problem 2)** Exploiting the fact that in OUTPUT 1 the only explanatory variable is binary, if the specified model in OUTPUT 1 had been estimated by a linear probability model instead of using the logit model, the estimate of the constant would approximately be:

   (a) -2.08.

   (b) We do not have enough information to calculate.

   (c) 0.11.

   (d) 0.28.

3. **(Problem 2)** Exploiting the fact that in OUTPUT 1 the only explanatory variable is binary, if the specified model in OUTPUT 1 had been estimated by a linear probability model instead of using the logit model, the estimate of the parameter for RAZA_ACUSADO would approximately be:

   (a) 0.08.

   (b) -0.03.

   (c) We do not have enough information to calculate.

   (d) -0.39.

4. **(Problem 2)** Taking into account that in OUTPUT 1 the only explanatory variable is binary, the estimate of $E($ CONDENA $\mid$ RAZA_ACUSADO $= 1$, RAZA_VICTIMA $= 0$ $)$ is approximately equal to:

   (a) It cannot be calculated because the estimated models do not characterize conditional expectation models.

   (b) 0.08.

   (c) 0.83.

   (d) 0.23.

5. **(Problem 2)** Exploiting the fact that in OUTPUT 1 the only explanatory variable is binary, if the specified model in OUTPUT 1 had been estimated by a linear probability model instead of using a logit model:
   (i) The error term would present heteroscedasticity (conditional on the explanatory variables).
   (ii) The magnitude of the estimated effect of the explanatory variable on the probability of being condemned to death would be the same.
   (iii) The predicted probabilities could be larger than 1 or smaller than 0.

   (a) The three statements are true.

   (b) Only (i) and (ii) are true.

   (c) Only (i) and (iii) are true.

   (d) Only (ii) and (iii) are true.

6. **(Problem 2)** The predicted probability that an accused black person is condemned when the victim is white is approximately equal to:

   (a) 0.23.
   (b) 0.96.
   (c) 0.08.
   (d) 0.83.

7. **(Problem 2)** If the specified model in the OUTPUT 2 had been estimated by a linear probability model instead of using the logit model:
   (i) The error term would present heteroscedasticity (conditional on the explanatory variables).
   (ii) The magnitude of the estimated effects of the explanatory variables on the probability of being condemned to death would be the same.
   (iii) The predicted probabilities could be larger than 1 or smaller than 0.

   (a) The three statements are true.
   (b) Only (i) and (ii) are true.
   (c) Only (i) and (iii) are true.
   (d) Only (ii) and (iii) are true.

8. **(Problem 2)** According to the estimations of OUTPUT 2, the average of the estimated effect of being an accused black person on the probability of being condemned to death is approximately equal to:

   (a) 0.22.
   (b) We do not have enough information to calculate this effect.
   (c) 0.09.
   (d) 0.11.

9. **(Problem 2)** According to the estimations of OUTPUT 2, the average of the estimated effect that the victim is black on the probability of being condemned to death is approximately equal to:

   (a) -0.13.
   (b) We do not have enough information to calculate this effect.
   (c) -0.19.
   (d) 0.20.

10. **(Problem 2)** Given the results, we can assert that:

   (a) If the victim is white, it is more likely that the accused is condemned.
   (b) The race of the victim only has an influence if the accused is black.
   (c) The race of the victim does not influence the probability of being condemned.
   (d) If the victim is black, it is more likely that the accused is condemned.

11. **(Problem 2)** Given the model in OUTPUT 2:
(i) If we keep the assumption of a logistic distribution, the estimations of the parameters would be identical if we estimate by Maximum Likelihood as well as if we apply non-linear least squares.
(ii) If we estimated by Maximum Likelihood assuming a normal distribution instead of a logistic one (and assuming that there are not too many extreme values in the sample) the magnitudes of the estimated coefficients would be bigger in the case of the normal distribution.
(iii) If, keeping the assumption of a logistic distribution, we estimate by non-linear least squares instead of by Maximum Likelihood, the corresponding heteroskedasticity-robust standard errors of the coefficients will be smaller than or equal to the conventional standard errors of such coefficients.

    (a) Only (ii) is true.
    (b) Only (i) and (ii) are true.
    (c) Only (i) and (iii) are true.
    (d) The three statements are false.

12. **(Problem 2)** Given the model in OUTPUT 1:
(i) We would have obtained **exactly** the same estimates of the effects of the explanatory variable (race of the accused) if we had assumed a normal distributions instead of a logistic one.
(ii) If this model would be correct (that is, if it would include all the relevant explanatory variables and the assumption of logistic distribution would be true), we would have also obtained consistent estimators of the coefficients if we had estimated by nonlinear least squares.
(iii) If, keeping the assumption of logistic distribution, we had estimated by nonlinear least squares, the conventional standard errors would be inappropriate because of the existence of conditional heteroskedasticity.

    (a) The three statements are true.
    (b) Only (ii) and (iii) are true.
    (c) Only (i) and (ii) are true.
    (d) Only (iii) is true.

13. **(Problem 2)** Suppose that the proportion of white victims is 47% and the proportion of black accused individuals is 55%. Given OUTPUT 2, how does change, on average, the probability of being condemned with the race of the accused?

    (a) 0.11.
    (b) We do not have enough information to calculate this effect.
    (c) 0.06.
    (d) −0.07.

14. **(Problem 2)** Suppose that the proportion of white victims is 47% and the proportion of black accused individuals is 55%. Given OUTPUT 2, what is the effect that the accused is black on the probability of being condemned when RAZA_VICTIMA takes its sample average value?

    (a) 0.01.
    (b) 0.05.
    (c) 0.04.
    (d) We do not have enough information to calculate this effect.

15. **(Problem 2)** What is the effect that the victim is white on the probability of being condemned for an accused whose race corresponds to its modal value?

    (a) We do not have enough information to calculate this effect.

    (b) -0.10.

    (c) -0.20.

    (d) 0.10.

16. **(Problem 2)** Given OUTPUT 2, consider the following statements:
    (i) If the victim is black, the probability of being condemned is very small as compared with the average probability of being condemned.
    (ii) If the victim is black, the race of the accused makes the probability of being condemned to change relatively more than if the victim was white.
    (iii) Suppose that the victim is white. Then, the probability of being condemned is larger than if the victim is black. Besides, the effect of being a black accused has a stronger effect on the probability of being condemned than if the victim is black.

    (a) Only (ii) is true.

    (b) Only (i) and (ii) are true.

    (c) Only (i) and (iii) are true.

    (d) The three statements are true.

17. **(Problem 2)** According the following statements:
    (i) Looking at the number of correctly predicted cases, we conclude that both models are valid to know the impact of the race of the accused on the probability of being condemned.
    (ii) Looking at number of correctly predicted cases, both models have the same goodness of fit.
    (iii) Looking at the value of the likelihood function, the model estimated in OUTPUT 2 fits better the data.

    (a) Only (i) is false.

    (b) Only (i) and (ii) are false.

    (c) Only (iii) is false.

    (d) Only (ii) false.

18. **(Problem 2)** In case we wanted to test if the race of the accused is relevant at the moment to define the probability of death penalty:
    (i) Under the evidence that correspondent t statistics is approximately 2.31, we reject at 1% confidence level the null hypothesis that this variable does not have a statistically significant impact.
    (ii) Under the evidence that correspondent likelihood ratio is approximately 20.36, we reject at 1% confidence level the null hypothesis that this variable does not have a statistically significant impact.
    (iii) Under the evidence that correspondent t statistics is approximately -1.26, we cannot reject at 1% confidence level the null hypothesis that this variable does not have a statistically significant impact.

    (a) Only (i) is true.

    (b) Only (i) and (ii) are true.

    (c) Only (iii) is true.

    (d) Only (ii) is true.

19. **(Problem 1)** In OUTPUT 1:
(i) The constant term in the regression is a estimated of the average velocity of money .
(ii) The estimated impact of the short–run interest rate on the velocity of money is constant and approximately equal to -7.
(iii) An increase in the short–run interest rate in 0.1 percentage points reduces the velocity of money in 0.7 percentage points.

 (a) Only (i) and (ii) are true.
 (b) The three statements are false.
 (c) Only (i) and (iii) are true.
 (d) Only (ii) and (iii) are true.

20. **(Problem 1)** In OUTPUT 1, if RS were measured as percentage instead of being a value between 0 and 1:
(i) The estimated value of the coefficient associated to RS would be approximately -0.07005.
(ii) The t statistics of the coefficient associated to RS would be approximately -0.11041.
(iii) The standard error of the coefficient associated to RS would be approximately 0.006345.

 (a) Only (i) and (ii) are true.
 (b) The three statements are true.
 (c) Only (i) and (iii) are true.
 (d) Only (ii) and (iii) are true.

21. **(Problem 1)** Comparing models (1) and (2) in terms of the estimated coefficients of model (*):
(i) We get that $\alpha_3 = 0$ in model (1).
(ii) We get that $\alpha_2 = \delta_2$ in model (2).
(iii) We get that $\alpha_3 = \delta_3$ in model (2)

 (a) Only (i) is true.
 (b) The three statements are false.
 (c) Only (iii) is true.
 (d) Only (ii) is true.

22. **(Problem 1)** If we wanted to estimate the most general (flexible) model from the three models previously presented:
(i) We would use model (*).
(ii) We would use model (2).
(iii) Models (*) y (2) are different and not comparable between each other.

 (a) Only (i) is true.
 (b) Only (iii) is true.
 (c) Only (i) and (ii) are true.
 (d) Only (ii) is true.

23. **(Problem 1)** According OUTPUT 1 and assuming in model (1) all assumption in the classical regression are holding:
(i) The constant in the regression represents the velocity of money.
(ii) The average of the log of the velocity of money is approximately equal to 0.525.
(iii) An increase in the short–run interest rate in 0.1 percentage points increases the velocity of money in approximately 0.7%.

   (a) Only (i) and (ii) are true.
   (b) The three statements are false.
   (c) Only (i) and (iii) are true.
   (d) Only (ii) and (iii) are true.

24. **(Problem 1)** According OUTPUT 1 and assuming in model (1) all assumption in the classical regression are holding:
(i) The velocity of money increases when the short-run interest rate increases.
(ii) The average of the logarithm of the velocity of money is approximately -0.31.
(iii) A reduction in the short–run interest rate in 0.1 percentage points reduces the velocity of money in approximately 0.7%.

   (a) Only (i) and (ii) are true.
   (b) The three statements are false.
   (c) Only (i) and (iii) are true.
   (d) Only (ii) and (iii) are true.

25. **(Problem 1)** According OUTPUT 1 and assuming in model (1) all assumption in the classical regression are holding:
(i) We cannot reject at a 10% confidence level the null hypothesis that an increase of 0.1 percentage points in the short–run interest rate increases in average the velocity of money in approximately in 1%.
(ii) The statistic test for the null hypothesis $H_0 : \beta_2 = -10$ is approximately equal to 4.8.
(iii) We cannot reject at a 10% confidence level the null hypothesis that for each percentage point reduction in the short–run interest rate the velocity of money increases in average in a one percentage point.

   (a) Only (ii) is true.
   (b) The three statements are false.
   (c) Only (i) and (iii) are true.
   (d) The three statements are true.

26. **(Problem 1)** According OUTPUT 1 and assuming in model (1) all assumption in the classical regression are holding:
(i) The 56% of the variance of v is explained by the short–run interest rate.
(ii) The estimated coefficients for the constant and slope in model (1) are individually different from zero but they would not necessarily be jointly statistically different from zero.
(iii) The conditional variance of $-v$ is approximately equal to 0.012.

   (a) Only (i) and (ii) are true.
   (b) The three statements are true.
   (c) Only (i) and (iii) are true.
   (d) Only (ii) and (iii) are true.

27. **(Problem 1)** According OUTPUT 1 and assuming in model (1) all assumption in the classical regression are holding:
(i) The 56% of the variance in v is explained by the short–run interest rate.
(ii) The variance of v, conditional on RS, is approximately equal to $-(0.109142)^2$.
(iii) The conditional variance of $-v$ is approximately equal to 0.012.

(a) Only (i) and (ii) are true.

(b) The three statements are true.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

28. **(Problem 1)** Using the specification of model (1) and under the evidence of the results:
(i) RS is clearly endogenous, since the estimated coefficient varies from -7 in OUTPUT 1 to -6.5 in OUTPUT 2:.
(ii) Not only the estimation in OUTPUT 1 but also in OUTPUT 2 are consistent.
(iii) The estimates in OUTPUT 1 are more efficient than the ones in OUTPUT 2.

(a) Only (i) and (ii) are true.

(b) The three statements are false.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

29. **(Problem 1)** Using the specification of model (1) and under the evidence provide by the results of several outputs, we can conclude that:
(i) RS is endogenous.
(ii) Only the estimates based on OUTPUT 1 are correct.
(iii) We do not have good instruments for RS that correlated with this variable.

(a) Only (i) and (ii) are true.

(b) The three statements are false.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

30. **(Problem 1)** Using the specification of model (1) and under the evidence provide by the results of several outputs, we can conclude that:
(i) If RS was endogenous, RL would also be endogenous, since this last variable is correlated with RS.
(ii) $RS(-2)$ can not be a good instrument for RS, because it is correlated with the error term in OUTPUT 2: given the fact that RS explicative variable.
(iii) RL and $RS(-2)$ satisfy one condition to be instruments for RS, since they are jointly and individually correlated with RS.

(a) Only (i) is true.

(b) The three statements are true.

(c) Only (iii) is true.

(d) Only (ii) is true.

31. **(Problem 1)** Using the specification of model (1) and under the evidence provide by the results of several outputs, we find evidence that:
(i) There is first order autocorrelation.
(ii) There is second order autocorrelation.
(iii) There is not autocorrelation of an order higher than 2, under the evidence provide by the Durbin-Watson statistics in OUTPUT 1.

   (a) Only (i) and (ii) are true.
   (b) The three statements are false.
   (c) Only (i) is true.
   (d) Only (iii) is true.

32. **(Problem 1)** Using the specification of model (1) and under the evidence provide by the results of several outputs:
(i) The estimated coefficients in OUTPUTS 1 and 6 should be equal between each other.
(ii) Although there is residual autocorrelation in the specification (1), we can ensure that RS is statistically significant.
(iii) The statistics for the null hypothesis $H_0 : \beta_2 = -1$ is approximately equal to 0.29.

   (a) Only (i) and (ii) are true.
   (b) The three statements are false.
   (c) Only (i) and (iii) are true.
   (d) Only (ii) and (iii) are true.

33. **(Problem 1)** The specification in model (1)
(i) Constraints the impact of the short-run interest rate on the velocity of money to be constant.
(ii) Constraints the money demand elasticity to be constant in respect to the income.
(iii) Constraints the money demand elasticity to be one in respect to the income.

   (a) Only (i) and (ii) are true.
   (b) The three statements are true.
   (c) Only (ii) and (iii) are true.
   (d) Only (i) and (iii) are true.

34. **(Problem 1)** The specification in model (2)
(i) Constraints the impact of the short-run interest rate on the velocity of money to be constant.
(ii) Constraints the money demand elasticity to be constant in respect to the income.
(iii) Constraints the money demand elasticity to be one in respect to the income.

   (a) Only (i) is true.
   (b) The three statements are true.
   (c) Only (ii) is true.
   (d) Only (i) and (ii) are true.

35. **(Problem 1)** The specification in model (2) in relationship to the results that have been presented:
(i) There is evidence of conditional autocorrelation.
(ii) There is evidence of conditional heteroskedasticity.
(iii) We should follow our analysis base on the results from OUTPUT 7.

(a) Only (i) and (ii) are true.

(b) The three statements are true.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

36. **(Problem 1)** According the results presented in the specifications (1) and (2), we are able to conclude:
(i) Income (Y) is not relevant to explain the real money demand (M/P).
(ii) At a 3% significance level, we reject the hypothesis that the income elasticity (Y) of the real money demand (M/P) is one.
(iii) Income has significant impact on the velocity of money : for each percentage point that income increases, keeping constant the interest rate, the velocity of money increases in approximately 0.04%.

(a) Only (i) and (ii) are true.

(b) The three statements are false.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

37. **(Problem 1)** According the results presented in the specifications (1) and (2), we are able to conclude:
(i) The real money demand (M/P) depends on Income (Y) and on the short–run interest rate (RS).
(ii) At a 3% significance level, we reject the hypothesis that the income (Y) elasticity of the real money demand (M/P) is one.
(iii) for each percentage point that income (Y) increases, keeping constant the short–run interest rate, the real money demand (M/P) increases on average in 1%.

(a) Only (i) and (ii) are true.

(b) The three statements are false.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

38. **(Problem 1)** According the results presented in the specifications (1) and (2), we are able to conclude:
(i) Estimations of specification (1) are inconsistent.
(ii) We should select the results from OUTPUT 7.
(iii) Estimations of specification (2) are inconsistent.

(a) Only (i) and (ii) are true.

(b) The three statements are false.

(c) Only (i) and (iii) are true.

(d) Only (ii) and (iii) are true.

39. **(Problem 1)** According the results presented, the preferred estimation of the impact of a percentage increases in the short–run interest rate on the velocity of money is::

(a) -7%.

(b) 7.33%.

(c) -7.33%.

(d) 7%.

40. **(Problem 1)** According the results presented, the preferred estimation of the income (Y) elasticity of the real money demand (M/P) is:

(a) -1.

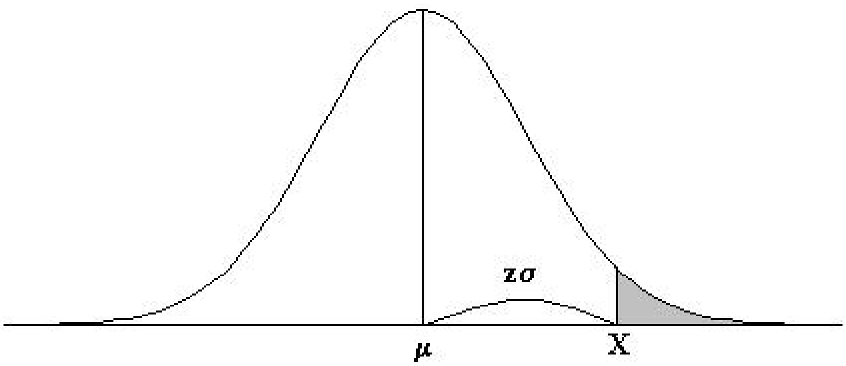
(b) 1.04.

(c) -0.96.

(d) 1.

TABLE 1: STANDARDIZED NORMAL DISTRIBUTION

Areas under the normal curve



Example:

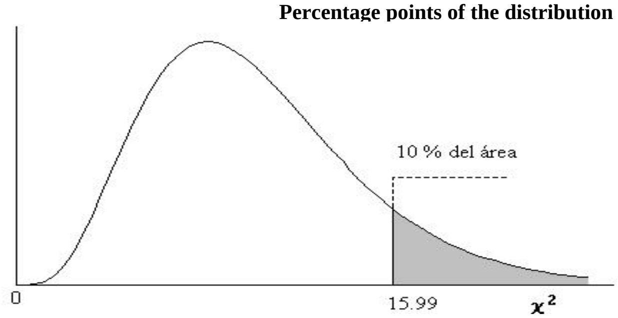
$$Z = \frac{X - \mu}{\sigma}$$

P [Z > 1] = 0.1587
P [Z > 1.96] = 0.0250

Dev. normal Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

TABLE 2: DISTRIBUTION χ^2

Percentage points of the distribution χ^2



Example:
For ϕ = 10 degrees of freedom

$P [\chi^2 > 15.99] = 0.10$

π / ϕ	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005	π / ϕ
1	3.93E-05	1.57E-04	9.82E-04	3.93E-03	1.58E-02	0.102	0.455	1.323	2.71	3.84	5.02	6.63	7.88	1
2	1.00E-02	2.01E-02	5.06E-02	0.103	0.211	0.575	1.386	2.77	4.61	5.99	7.38	9.21	10.60	2
3	7.17E-02	0.115	0.216	0.352	0.584	1.213	2.37	4.11	6.25	7.81	9.35	11.34	12.84	3
4	0.207	0.297	0.484	0.711	1.064	1.923	3.36	5.39	7.78	9.49	11.14	13.28	14.86	4
5	0.412	0.554	0.831	1.145	1.610	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75	5
6	0.676	0.872	1.237	1.635	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55	6
7	0.989	1.239	1.690	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.3	7
8	1.344	1.647	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.1	22.0	8
9	1.735	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.7	23.6	9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.5	23.2	25.2	10
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.9	24.7	26.8	11
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.0	23.3	26.2	28.3	12
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.4	24.7	27.7	29.8	13
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.1	23.7	26.1	29.1	31.3	14
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.3	25.0	27.5	30.6	32.8	15
16	5.14	5.81	6.91	7.96	9.31	11.91	15.34	19.37	23.5	26.3	28.8	32.0	34.3	16
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.5	24.8	27.6	30.2	33.4	35.7	17
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.6	26.0	28.9	31.5	34.8	37.2	18
19	6.84	7.63	8.91	10.12	11.65	14.56	18.34	22.7	27.2	30.1	32.9	36.2	38.6	19
20	7.43	8.26	9.59	10.85	12.44	15.45	19.34	23.8	28.4	31.4	34.2	37.6	40.0	20
21	8.03	8.90	10.28	11.59	13.24	16.34	20.3	24.9	29.6	32.7	35.5	38.9	41.4	21
22	8.64	9.54	10.98	12.34	14.04	17.24	21.3	26.0	30.8	33.9	36.8	40.3	42.8	22
23	9.26	10.20	11.69	13.09	14.85	18.14	22.3	27.1	32.0	35.2	38.1	41.6	44.2	23
24	9.89	10.86	12.40	13.85	15.66	19.04	23.3	28.2	33.2	36.4	39.4	43.0	45.6	24
25	10.52	11.52	13.12	14.61	16.47	19.94	24.3	29.3	34.4	37.7	40.6	44.3	46.9	25
26	11.16	12.20	13.84	15.38	17.29	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	26
27	11.81	12.88	14.57	16.15	18.11	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	27
28	12.46	13.56	15.31	16.93	18.94	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	28
29	13.12	14.26	16.05	17.71	19.77	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	29
30	13.79	14.95	16.79	18.49	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	30
40	20.7	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7	66.8	40
50	28.0	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2	79.5	50
60	35.5	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4	92.0	60
70	43.3	45.4	48.8	51.7	55.3	61.7	69.3	77.6	85.5	90.5	95.0	100.4	104.2	70
80	51.2	53.5	57.2	60.4	64.3	71.1	79.3	88.1	96.6	101.9	106.6	112.3	116.3	80
90	59.2	61.8	65.6	69.1	73.3	80.6	89.3	98.6	107.6	113.1	118.1	124.1	128.3	90
100	67.3	70.1	74.2	77.9	82.4	90.1	99.3	109.1	118.5	124.3	129.6	135.8	140.2	100
Z_α	-2.58	-2.33	-1.96	-1.64	-1.28	-0.674	0.000	0.674	1.282	1.645	1.96	2.33	2.58	Z_α

For $\phi > 100$ take $\chi^2 = \frac{1}{2}\left(Z_\alpha + \sqrt{2\phi - 1}\right)^2$. Z_α is the standardized normal deviation that corresponds to the significance level, which is shown in the upper part of the table.