Advanced Microeconomics I
Final Exam
Universidad Carlos III de Madrid – Fall quarter 2006
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- The total score is 100 points.
- Each question is labeled with the number of points it is worth.
- GOOD LUCK!

1. Consider a strategic form game $G = \{N, S, \Pi\}$ where iterative elimination of strictly dominated strategies results in strategy subsets $\widetilde{S}_1$ and $\widetilde{S}_2$, each consisting of *two* strategies. Define *better-response* dynamics so that for all $i \in N$:

$$s_i(t) \in BR_i(s(t-1)) = \{s_i \in S_i | \pi_i(s_i, s_{-i}(t-1)) \geq \pi_i(s_i(t-1), s_{-i}(t-1))\}$$

and *best-response* dynamics so that for all $i \in N$:

$$s_i(t) \in B_i^*(s(t-1)) = \{s_i' \in S_i | \pi_i(s_i', s_{-i}(t-1)) \geq \pi_i(s_i, s_{-i}(t-1)), \text{ for all } s_i \in S_i\}$$

(a) (15) Show, by example, that.the *better response* dynamics need not have all of its limit strategy profiles in $\widetilde{S}_1 \times \widetilde{S}_2$.

Solution Let the game

1,2	a	b
F	-1.5,1	0.5,-1
A	-1,1	1,-1
B	1,-1	-1,1

The following sequence is a solution to the *better response* dynamics:

$$\{(F, a), (B, a), (B, b), (F, b), (F, a), ...\}$$

(b) (10) Show that, in your example, the *best response* dynamics must have all of its limit strategy profiles in $\widetilde{S}_1 \times \widetilde{S}_2$.

Solution Starting at any initial condition, since F is not a best-response to any strategy in S_2, we must have that $s_1(t) \neq F$ for all $t > 1$. This immediately implies that $(s_1(t), s_2(t)) \in \widetilde{S}_1 \times \widetilde{S}_2$ which only excludes F.

2. Consider a strategic form game $G = \{N, S, \Pi\}$ where $S = \{1, 2, ..., \overline{s}\}$, and

$$\pi_i(s_1, ..., s_n) = \left\{\min_{j \in N} s_j\right\} - \gamma s_n$$

(a) (7) What are the (pure-strategy) equilibria of this game?

Solution For $\gamma \geq 1$, only $(s_1, ..., s_n) = (1, ..., 1)$ is an equilibrium. For $\gamma < 1$, every homogeneus strategy profile $(s_1, ..., s_n) = (a, a, ..., a)$ with $a \in S$ is an equilibrium.

(b) (18) What is the stochastically stable set of this game?

Solution For $\gamma \geq 1$, trivially the equilibrium. For $\gamma < 1$, just one mutation (any player changing into $s_i = 1$) is necessary to move play to $(s_1, ..., s_n) = (1, ..., 1)$, whereas moving from $(s_1, ..., s_n) = (1, ..., 1)$ to any other equilibrium $(s_1, ..., s_n) = (a, a, ..., a)$ requires all players except one moving to $s_i = a$. Thus $(s_1, ..., s_n) = (1, ..., 1)$ is the stochastically stable state.

3. Suppose there are $n \geq 3$ players. For any $k \geq 3$, let a k-player circle be a network of the form $\{12, 23, ..., ii+1, ..., k1\}$, or any permutation of such a network. Consider a network where the total value to a component is k^2 if it is a k-person circle with $k \geq 3$, and 0 otherwise. Suppose that the value of each component is allocated in an egalitarian way within the component.

(a) (7) What is the set of *pairwise stable* networks for this game?

Solution All the graphs composed by some k_l-person circles where the sum of k_l adds up to n..

(b) (8) What is the efficient network in this game?

Solution A single n-person circle.

A network g is *strongly stable* if for any $S \subset N$, g' that is obtainable from g via deviations by S, and $i \in S$ such that $u_i(g') > u_i(g)$, there exists $j \in S$ such that $u_j(g') < u_j(g)$.

(c) (10) What is the *strongly stable* network of this game?

Solution A single n-person circle.

4. Muench and Walker's (1984) paper, "Are Groves-Ledyard Equilibria Attainable?" shows that the Groves-Ledyard mechanism may be unstable for certain values of the key parameter γ, specially for large populations. When the parameter γ is small enough (of the order of $1/n$) the dynamics are stable.

(a) (6) What is the (new) problem with a small γ?

Solution That incentives for optimization are so small that agents may get stuck outside an equilibrium.

(b) (7) What is the reason behind the contrasting result in Sandholm's (2005), "Negative Externalities and Evolutionary Implementation"?

Solution That the payoffs are constructed to be a potential game, and thus a Lyapunov function (and global stability) are immediate.

5. (12) What are the main differences and similarities between Kranton and Minehart's (2001) "A Theory of Buyer-Seller Networks," and Corominas' (2004) "Bargaining in a Network of Buyers and Sellers"?

   Solution Both deal with bi-partite graphs with buyers and sellers and detect equilibria of games that are somehow "Walrasian". The main difference is that Kranton and Minehart resorts to auctions and is thus superficially more centralized. On the other hand, the centralization allows to think about investment in link formation and in capital, which makes it more general and allows to think of other issues beyond the equilibrium outcome.