3402 CONTRACTS AND GAME THEORY
Midterm Exam
Universitat Pompeu Fabra – Winter 1995
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1. Consider the three-player extensive-form game depicted in the following figure.

(1,1,0)

(3,0,0) (0,3,0) (3,0,0) (0,3,0)

   (a) Show that (A, A) is not the outcome of a Nash equilibrium.
   (b) What are the Nash equilibria in this game?
   (c) Suppose that Player 1 thought that the probability of L was 0 and Player 2 thought that the probability of L was 1. What would be the outcome of the game in this case? Why is this outcome not a Nash equilibrium?

2. The accompanying simultaneous-move game is played twice, with the actions of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that (4, 4) is not an equilibrium payoff in the one-shot game. For what values of x is the following strategy (played by both players) a subgame-perfect Nash equilibrium?

   Play Q_i in the first stage. If the first-stage outcome is (y, Q_2) where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z) where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z) where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

	P_2	Q_2	R_2	S_2
P_1	2, 2	x, 0	-1, 0	0, 0
Q_1	0, x	4, 4	-1, 0	0, 0
R_1	0, 0	0, 0	0, 2	0, 0
S_1	0,-1	0,-1	-1, -1	2, 0

3. In Rubinstein's infinite horizon bargaining game suppose that the players are restricted to proposing either that Player 1 gets the whole dollar or that Player 2 gets the whole dollar.

   (a) Describe a subgame perfect equilibrium (and show that it is subgame perfect) in which Player 1 begins by proposing that Player 1 gets the whole dollar and Player 2 agrees.

   (b) Describe a subgame perfect equilibrium (and show that it is subgame perfect) in which Player 1 begins by proposing that Player 2 gets the whole dollar and Player 2 agrees.

   (c) Describe a subgame perfect equilibrium (and show that it is subgame perfect) in which agreement does not take place immediately.

4. The following game is called the game of Chicken.

	Dove	*Hawk*
Dove	2, 2	0, 3
Hawk	3, 0	-1, -1

Assume this game is repeated 100 times. The repeated game payoffs are just the sum of the stage-game payoffs. Consider a strategy s that tells you always to choose *dove* up until the 100th stage and to use *dove* and *hawk* with equal probabilities at the 100th stage-*unless* the two players have failed to use the same actions at the preceding stage. If such a coordination failure has occurred in the past, s tells a player to look for the first stage at which differing actions were used and then always to use whatever action he or she did *not* play at that stage.

   (a) Why is (s, s) a Nash equilibrium?

   (b) Prove that (s, s) is a subgame-perfect equilibrium.

   (c) What is it about Chicken that allows "folk theorems" results to be possible in the finitely repeated case?