

1113 CONTRACTS AND GAME THEORY
Final Exam
Universitat Pompeu Fabra – Spring 1997
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1. Consider the following game:

Show that there is a Perfect Bayesian Equilibrium for that game where all types of player 1 choose action X .

2. Consider a duopoly whose inverse demand function is given by

$$P(Q) = \max\{M - Q, 0\}, \quad M > 0.$$

The cost functions of the firms are $C_i(q_i) = 0$, for $i = 1, 2$, and this is common knowledge. M can take one of two values $M = A$ and $M = B$, where $A > B > \frac{1}{4}A$, both with identical probabilities. Firm 1 is informed about the value of M . Assuming that the firms take their decisions simultaneously, compute the Bayes-Nash equilibrium of this game.

3. In the game of the previous problem assume that $A = 6$ and $B = 4$. Consider now the alternative game where firm 1 decides first how much to produce, while firm 2 takes the production decision after observing the production of firm 1 (but without being informed of M). Find an equilibrium in which the two types of firm 1 produce $q_1 = 1$.
4. Consider the following game:

- (a) Find the three pure strategy Nash equilibria of this game.
- (b) Show that two of the three Nash equilibria are sequential but the other is not.
Hint: Characterize the beliefs that would make this equilibrium sequentially rational and show that they are not consistent.

5. Consider the following game:

- (a) Find a subgame perfect Nash equilibrium of this game. Is it unique? Are there any other Nash equilibria?
- (b) Now suppose that player 2 cannot observe player 1's move. Write down the new extensive form. What is the set of Nash equilibria?
- (c) Now suppose that player 2 observes player 1's move correctly with probability $p \in (0, 1)$ and incorrectly with probability $1 - p$ (e.g., if player 1 says T , player 2 observes T^s with probability p and observes B^s with probability $1 - p$) is common knowledge to the two players. What is the extensive form now? Show that there is a unique Perfect Bayesian Equilibrium.

6. Player 1 is involved in an accident with player 2. Player 1 knows whether she is negligent or not, but player 2 does not know; if the case comes to court the judge learns the truth. Player 1 sends a "take-it-or-leave-it" pretrial offer of compensation that must be either 3 or 5, which player 2 either accepts or rejects. If he accepts the offer the parties do not go to court. If he rejects it the parties go to court and player 1 has to pay 5 to player 2 if he is negligent and 0 otherwise; in either case player 1 had to pay the court expenses of 6. Write the extensive form of this signaling game and find its sequential equilibria.