MICROECONOMICS II
Problem set 3
Universitat Pompeu Fabra – Winter 2006
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1. Assume we have the following game:

	α_2	β_2
α_1	X, X	$X, 0$
β_1	$0, X$	$8, 8$

where X is a random variable which takes values in the set $V = \{9, 8, 7, 6, 5\}$ with equal probability for all values in V. At the beginning of the game each of the two players receives independently a signal $\sigma_i \in \Sigma = \{T, U, W, Y, Z\}$ about the value of X. The following table explains how signals are related to the value of X.

Value of X	9	8	7	6	5
Signal	Tor U	Tor Uor W	Uor Wor Y	Wor Yor Z	Yor Z

Given a value of X, each one of the possible hints for that value is equally probable.

(a) What is the expected value of playing strategy s_i given a signal σ_i?

(b) Does an agent who receives a signal $\sigma_i = T$ have a strictly dominant strategy? What about an agent with signal $\sigma_i = U$?

(c) Is some strategy dominated for agents with other signals, (perhaps after the agents with $\sigma_i = T$ and $\sigma_i = U$ eliminate some strategy)?

(d) What are the Nash equilbria of this game?

2. Consider the following *war of attrition* between two animals (or firms) which are trying to win a certain resource (food or mate, in the case of animals; market power or a patent in the case of firms).

The value v_i that each player $i = 1, 2$, puts on the resource is private information and it is taken from a certain probability distribution $F(.)$ on R^+. At each instant $t \in [0, \infty]$ if the player i is the first to abandon the fight the resource goes to the other player $j \neq i$. Player i experiments a cost c per unit of time he stays in the race, so that the payoff for i will be $\pi_i = -ct$. Once player i has given up, the other player, j, obtains the resource and its value v_j. Given that the players discount the future at a rate δ the payoff for player j in the game is $\pi_j = v_j e^{-\delta t} - ct$.

(a) Consider a symmetric equilibrium given by the differentiable function $\gamma : R^+ \to R^+$ which for each valuation v for each individual determines the instant $\gamma(v)$ in which the individual gives up the chase. Show that $\psi(.) = \gamma^{-1}(.)$ is an strictly increasing function which satisifes for each t the following differential equation:

$$e^{-\delta t}\psi(t)\psi'(t)f(\psi(t)) = c(1 - F(\psi(t))).$$

(b) Assume that $F(v) = 1 - e^{-v}$ and compute the equilibrium.

3. Reinterpret the buyer and the seller in the double auction studied in class as a firm who knows a worker's marginal product m and a worker who knows her outside opportunity v. In this context trade means that the worker is employed and the price at which parties trade is the worker's wage w. If there is trade the firm's payoff is $m - w$ and the worker's is w; If there is no trade then the firm's payoff is zero and the worker's is v.

   Suppose that m and v are independent draws from a uniform distribution on [0, 1], as in class.

   (a) Compute the players' expected payoffs in the linear equilibrium of the double auction.

   (b) Before the parties learn their private information, they sign a contract specifying that if the worker is employed by the firm, then the worker's wage will be w, but also that either side can escape the employment relationship at no cost. After the parties learn the values of their respective pieces of private information, they simultaneous announce either that they Accept the wage w or that they Reject that wage. If both announce Accept then trade occurs; otherwise it does not. Given an arbitrary value of w from [0, 1], what is the Nash equilibrium of this game? Find the value of w that maximizes the sum of the players' expected payoffs and compute the maximized sum.

4. Each of two players receives a ticket on which there is a number from a finite subset s of the interval [0, 1]. The number on a player's ticket is the size of a prize that she may receive. The two prizes are identically and independently distributed, with distribution function $F(.)$. Each player is asked independently and simultaneously whether she wants to exchange her prize for the other player's prize. If both players agree then the prizes are exchanged; otherwise each player receives her own prize. Each player's objective is to maximize her expected payoff. Model this situation as a Bayesian game and show that in any Nash equilibrium the highest prize that either player is willing to exchange is the smallest possible prize.