Building socio-economic Networks: How many conferences should you attend?

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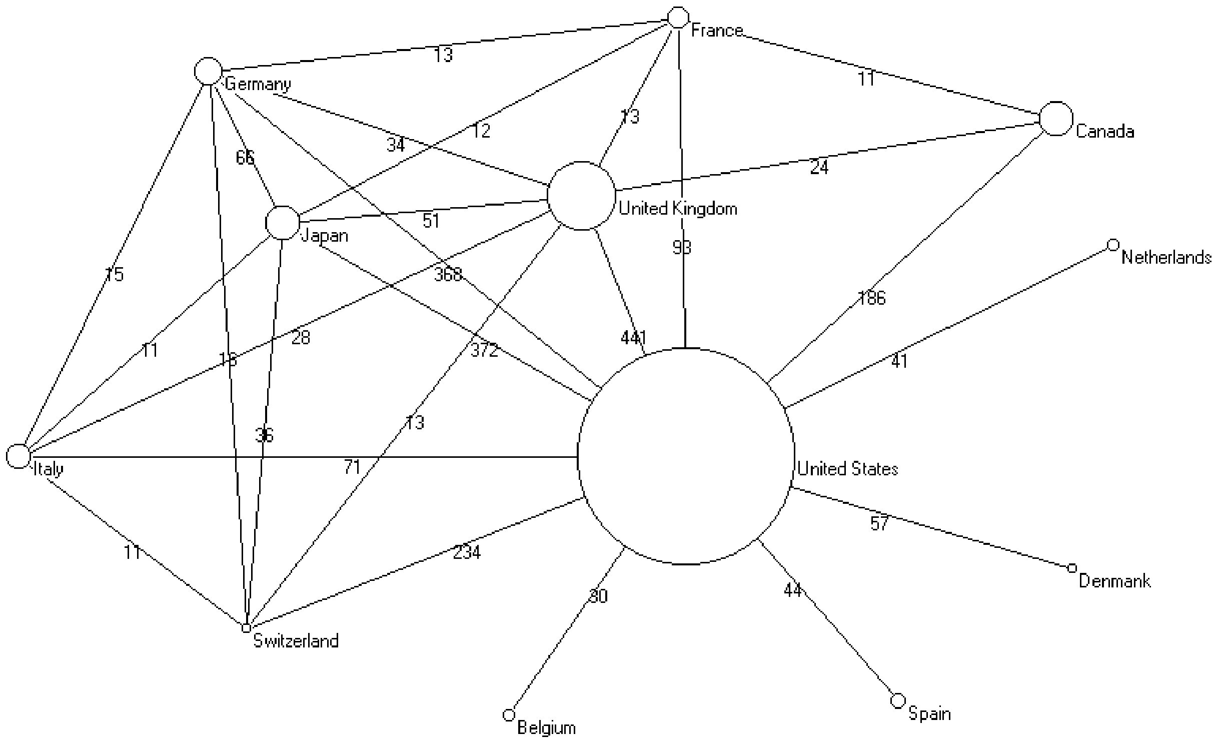
Summary

- Introduction
- The game
- Equilibrium (large economies)
- Response to incentives
- Policies: how should you spend your first dollar?
- A couple of extensions



- Cross-National Network of R&D Projects Involving PROs and Commercial Entities, 1990–1999 (Owen Smith, Riccaboni, Pammolli, Powell 2002).

Introduction *(2/6)*

- Spillovers between different agents generate incentives for "linking."
  - Research and development.
  - Labor Market Information.
  - Friendships and "Social Capital."
- If linking is done "non-cooperatively," inefficiencies arise (overlinking - underwork), so role for policy.

Introduction *(3/6)*

- Prior work:
  - Spillovers (theory): Marshall (1920), D'Aspremont and Jacquemin (1988), Bénabou (1993).
  - Spillovers (empirics): Ciccone and Hall (1996), Cassimand and Veugelers (2002).
  - Spillovers (policy): Motta (1996), Leahy and Neary (1997).
  - Networks (theory): Jackson (2005), Goyal and Moraga (2001).
  - Networks (empirics): Pammolli and Riccaboni (2001), Owen-Smith et al. (2004).

Introduction *(4/6)*

- They do not look very much at endogenous and costly network formation.

- When they do, they simplify away the game after forming the network.

- Reason: Analytical intractability.

Introduction *(5/6)*

- We analyze a network formation game in two stages:
  - First - Socialization effort.
  - Second - Productive effort.

- The key simplification is: undirected socialization.
  - Each link created with probability equal to product of socialization efforts.
  - Thus random network.

- Strategy space much simpler (one dimensional for each player - rather than $n-1$-dimensional), so equilibrium is a smaller-sized fixed point.

Introduction *(6/6)*

- Equilibrium: for "large" groups - unique and symmetric.

- An increase in the returns to "success" makes socialization effort relatively stronger.

  - An explanation for the explotion of R&D collaboration.

  - Perhaps also for the decrease in social capital.

- Public policy: where should you put your first euro?

The game

Let $N = \{1, \ldots, n\}$ be a set of players.

We consider a two-stage game:

Stage one: Players select $k_i > 0$. i and j interact with probability:

$$g_{ij}(k) = g_{ji}(k) = \frac{k_i k_j}{\sum_{l \in N} k_l} = \frac{k_i k_j}{n \langle k \rangle}.$$

Interim stage: Learn k and i.i.d. shocks on $[\underline{\varepsilon}, \overline{\varepsilon}]$, expected value ε, and variance σ_{ε}^2.

Stage two: Players select $s_i > 0$. Let $p_{ij} = g_{ij}$ if $i \neq j$, and $p_{ii} = g_{ii}/2$.

Player i's utility:

$$u_i(s, k) = [b + \varepsilon_i + \alpha \sum_j p_{ij} s_j] s_i - \frac{1}{2} s_i^2 - \frac{1}{2} k_i^2$$

where $b > 0$ and $\alpha \geq 0$.

Productive effort

Let $\mathbf{G}(k) = [g_{ij}(k)]_{i,j \in N}$ be matrix of random links. Define:

$$\lambda(k) = \frac{\alpha \langle k \rangle}{\langle k \rangle - \alpha \left\langle k^2 \right\rangle}.$$

Lemma 1 *When $p_{ii} < 1/2\alpha$, the unique interior Nash equilibrium in pure strategies of the second-stage game is:*

$$\mathbf{s}^*(k) = b\beta(k) + \mathbf{M}(k) \cdot \varepsilon \tag{1}$$

-

$$\mathbf{M}(k) = [\mathbf{I} - \alpha \mathbf{G}(k)]^{-1} = \sum_{p=0}^{+\infty} \alpha^p \mathbf{G}^p(k).$$

- $m_{ij}(\mathbf{k})$ counts the total number of direct and indirect paths in the expected network $\mathbf{G}(\mathbf{k})$, where paths of length p are weighted by the decaying factor α^p.

- $$m_{ij}(k) = \begin{cases} \lambda(k) g_{ij}(k), \text{ if } i \neq j \\ 1 + \lambda(k) g_{ii}(k), \text{ if } i = j \end{cases}$$

- Define $\beta_i(\mathbf{k}) = m_{i1}(\mathbf{k}) + ... + m_{in}(\mathbf{k})$. This is the sum of all paths stemming from i in the expected network where links are independently and randomly drawn with probability $(g_{ij}(\mathbf{k}))$.

- $\beta_i(k){=}1 + \lambda(k) k_i$ is a measure of centrality in the random graph $\mathbf{G}(\mathbf{k})$, reminiscent of the Bonacich centrality measure for fixed networks.

Socialization effort

The expected payoffs are:

$$
\begin{aligned}
Eu_i(k) &= (b+\varepsilon)E(s_i) + \alpha E(\sum_j p_{ij} s_i s_j) - \frac{1}{2}E(s_i^2) - \frac{1}{2}k_i^2 \\
&= (b+\varepsilon)^2 \beta_i + \alpha \sum_j p_{ij}\omega_{ij} - \frac{1}{2}\omega_{ii} - \frac{1}{2}k_i^2
\end{aligned}
$$

Obtaining the equilibrium profile k^* is messy. The first-order conditions are:

$$
k_i = (b+\varepsilon)^2 \frac{\partial \beta_i}{\partial k_i} + \alpha \sum_j \left[p_{ij}\frac{\partial \omega_{ij}}{\partial k_i} + \frac{\partial p_{ij}}{\partial k_i}\omega_{ij} \right] - \frac{1}{2}\frac{\partial \omega_{ii}}{\partial k_i} \tag{2}
$$

where:

$$\frac{\partial \lambda}{\partial k_i} = \frac{\alpha^2}{n} \frac{2k_i - \langle k^2 \rangle}{\left(\langle k \rangle - \alpha \langle k^2 \rangle\right)^2}$$

$$\frac{\partial \beta_i}{\partial k_i} = \lambda(k) + k_i \frac{\partial \lambda}{\partial k_i}$$

$$\frac{\partial p_{ij}}{\partial k_i} = \frac{\partial g_{ij}}{\partial k_i} = \frac{1}{n}\left[\frac{k_j}{\langle k \rangle} - \frac{k_i k_j}{n \langle k \rangle^2}\right], \text{ if } i \neq j$$

$$\frac{\partial p_{ii}}{\partial k_i} = \frac{1}{2}\frac{\partial g_{ii}}{\partial k_i} = \frac{1}{2n}\left[\frac{k_i}{\langle k \rangle} - \frac{k_i^2}{n \langle k \rangle^2}\right]$$

$$\frac{1}{\sigma_\varepsilon^2}\frac{\partial \omega_{ij}}{\partial k_i} = \left(2 + \lambda \frac{\langle k^2 \rangle}{\langle k \rangle}\right)\left[\frac{\partial \lambda}{\partial k_i} g_{ij} + \frac{\partial g_{ij}}{\partial k_i}\lambda\right] + \lambda g_{ij}\left[\frac{\partial \lambda}{\partial k_i}\frac{\langle k^2 \rangle}{\langle k \rangle} + \frac{1}{n}\left[2\lambda \frac{k_i}{\langle k \rangle} - \lambda \frac{\langle k^2 \rangle}{\langle k \rangle^2}\right]\right]$$

$$\frac{1}{\sigma_\varepsilon^2}\frac{\partial \omega_{ii}}{\partial k_i} = \left(2+\lambda\frac{\left\langle k^2\right\rangle}{\langle k\rangle}\right)\left[\frac{\partial\lambda}{\partial k_i}g_{ii}+\frac{\partial g_{ii}}{\partial k_i}\lambda\right] + \lambda g_{ii}\left[\frac{\partial \lambda}{\partial k_i}\frac{\left\langle k^2\right\rangle}{\langle k\rangle}+\frac{1}{n}\left[2\lambda\frac{k_i}{\langle k\rangle}-\lambda\frac{\left\langle k^2\right\rangle}{\langle k\rangle^2}\right]\right]$$

In a symmetric equilibrium:

$$k = (b+\varepsilon)^2\lambda+\frac{(b+\varepsilon)^2}{n}(2-k)\lambda^2 + \frac{\lambda}{n}\sigma_\varepsilon^2\left[2\frac{\lambda^2}{n^2}(2-k)\left(1+\lambda k\right)+\frac{\lambda^2 k}{n}+\frac{2\lambda}{n}\left(1-\frac{1}{n}\right)\right]\left[\alpha\frac{k}{n}\left(n-\frac{1}{2}\right)-\frac{1}{2}\right] + \alpha\sigma_\varepsilon^2\frac{1}{n}\left(1-\frac{1}{n}\right)\left[\frac{1}{2}+\left(n-\frac{1}{2}\right)\left[\lambda\frac{k}{n}\left(2+\lambda k\right)\right]\right] \quad (3)$$

Lemma 2 *For n large, k^* is $O(n^0)$.*

Proof. When k^* is $O(n^p)$ for $p > 0$, $\lim_{n\to+\infty} \lambda k = -1$, and $\lim_{n\to+\infty} \lambda = 0$. Thus:

$$\lim_{n\to+\infty} ((b+\varepsilon)^2\lambda + \frac{(b+\varepsilon)^2}{n}(2-k)\lambda^2) = 0$$

$$\lim_{n\to+\infty} \frac{\lambda}{n}\sigma_\varepsilon^2 \left[2\frac{\lambda^2}{n^2}(2-k)(1+\lambda k) + \frac{\lambda^2 k}{n} + \frac{2\lambda}{n}\left(1-\frac{1}{n}\right)\right]\left[\alpha\frac{k}{n}\left(n-\frac{1}{2}\right)-\frac{1}{2}\right] = 0$$

$$\lim_{n\to+\infty} \alpha\sigma_\varepsilon^2\frac{1}{n}\left(1-\frac{1}{n}\right)\left[\frac{1}{2}+\left(n-\frac{1}{2}\right)\left[\lambda\frac{k}{n}(2+\lambda k)\right]\right] = 0$$

Right hand side tends to zero, but left hand side goes to infinity. ∎

Proposition 3 *For n large, there is a unique symmetric subgame perfect equilibrium. The actions $\mathrm{s}^*(k)$ at the second stage are given in Lemma 1 while the equilibrium value of k in the first stage tends to*

$$\lim_{n\to+\infty} k \equiv k^* = \frac{1}{2\alpha}\left(1 - \sqrt{\left(1 - 4(b+\varepsilon)^2\alpha^2\right)}\right)$$

Proof. By the previous lemma, k^* is $O(n^0)$. This implies that in the limit we have to satisfy:

$$k = (b+\varepsilon)^2\lambda$$

So the equilibrium candidate must solve:

$$k - \alpha k^2 - (b+\varepsilon)^2\alpha = 0$$

∎

Lemma 4 *For n large, there are no asymmetric equilibria.*

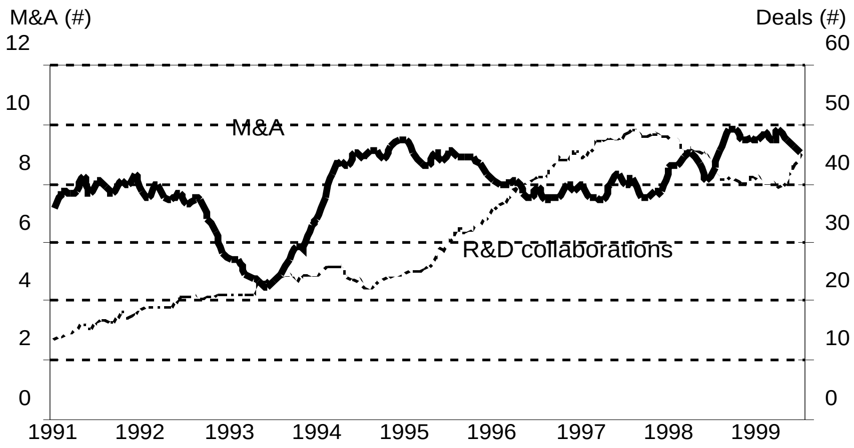
Equilibrium (large economies) *(8/8)*

The equilibrium approximation for $(\alpha, b+\varepsilon, \sigma_\varepsilon) = (1, 0.1^{0.5}, 1)$

n	k^*
10	.294
20	.178
50	.136
100	.124
200	.116
1000	.114
∞	.113

Relative response of s and k to a change in $(\alpha, b+\varepsilon, \sigma_\varepsilon)$

Proposition 5 *Let $(\alpha, b+\varepsilon, \sigma_\varepsilon)$ be scaled by factor $1/\sqrt{2\alpha(b+\varepsilon)} > \delta \geq 1$. Then, equilibrium k^* increases more than $s^*(k^*)$ in percentage terms.*



- Number of Mergers and Acquisitions and R&D Collaborations per Month in the pharmaceutical industry. One-Year Moving Averages (Pammolli and Riccaboni, 2001).

Reaching the giant component: phase transition.

Proposition 6 *Let* $(\alpha, b+\varepsilon, \sigma_\varepsilon)$ *be scaled by* $1/\sqrt{2\alpha(b+\varepsilon)} > \delta \geq 1$. *There exists a threshold* $\overline{\alpha}$ *such that, for* $\alpha < \overline{\alpha}$, *when* δ *reaches a threshold value of* δ^*, *the equilibrium network jumps from a fragmented graph to a highly connected graph (single giant component).*

Symmetric equilibrium is Erdös-Rényi random graph (each link is binomial parameter k^*/n. The transition happens when $k = 1$, i.e. when the following holds:

1. $2\alpha(b+\varepsilon) < 1$. **2.** $\alpha < \frac{1}{1+b+\varepsilon}$. **3.** $1 + 4\alpha^2 + 2\alpha > 8\alpha(b+\varepsilon)$.

For example, when $b+\varepsilon < 1$, this happens when $\alpha < \left(3 - \sqrt{5}\right)/4$.

Policies: how should you spend your first dollar? *(1/2)*

Relative impact of k and s subsidy

The technology for producing k and s is: $L_k = \frac{1}{2}\sqrt{k}$ and $L_s = \frac{1}{2}\sqrt{s}$.

Subsidies are a fraction of the cost of the labor input $((1-\theta),(1-\tau))$:

$$u_i = \left(b + \varepsilon_i + \alpha \sum_j p_{ij} s_j\right) s_i - \frac{1}{2}\theta s_i^2 - \frac{1}{2}\tau k_i^2$$

In second stage we have: $s_i^*(\frac{\alpha}{\theta}, \frac{\beta}{\theta}, \frac{\sigma^2}{\theta^2})$.

In first stage: $\tau k = \frac{(b+\varepsilon)^2}{\theta^2}\lambda(\theta)$, which implies that $k^* = \frac{\theta}{2\alpha}\left[1 - \sqrt{1 - 4\frac{(b+\varepsilon)^2\alpha^2}{\theta^4\tau}}\right]$,
so that in particular

$$Eu_i(k) = \frac{(b+\varepsilon)^2}{\theta^2} - \frac{1}{2}\frac{\sigma^2}{\theta^2} + \frac{1}{2}\tau k^2$$

Now we will show the effect of the first unit of subsidy, on k and on s.

Policies: how should you spend your first dollar? *(2/2)*

That is, we compute $\frac{\partial E u_i(k)}{\partial T}$, where $T = (1-\theta)s^2 + (1-\tau)k^2$ for $d\tau > 0, d\theta = 0$ and for $d\tau = 0, d\theta > 0$ and compare.

Theorem 7 *When $\alpha^2\sigma^2 > 3/4$, the first unit of subsidy is always optimally allocated to socialization effort, k_i. When $\alpha^2\sigma^2 < 3/4$ the first unit of subsidy is optimally allocated to socialization effort k_i if and only if the expected marginal return to own investment, $b + \varepsilon$, is low enough.*

A couple of extensions

1. Decisions taken simultaneously - No qualitative changes.

2. Heterogeneity - $\mathbf{b} = (b_1, ..., b_n)$

   (a) A mean-preserving spread of $\mathbf{b}$ leads to a mean-preserving spread of both $\mathbf{s}$ and $\mathbf{k}$, and a shift upwards in the mean.

   (b) k_i is i's expected connectivity. So, we ca map distribution of fundamentals into distribution of connectivity (beyond Erdös-Renyi).

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