MICROECONOMICS II
Final Exam (Game Theory)
Universitat Pompeu Fabra – Winter quarter 2004
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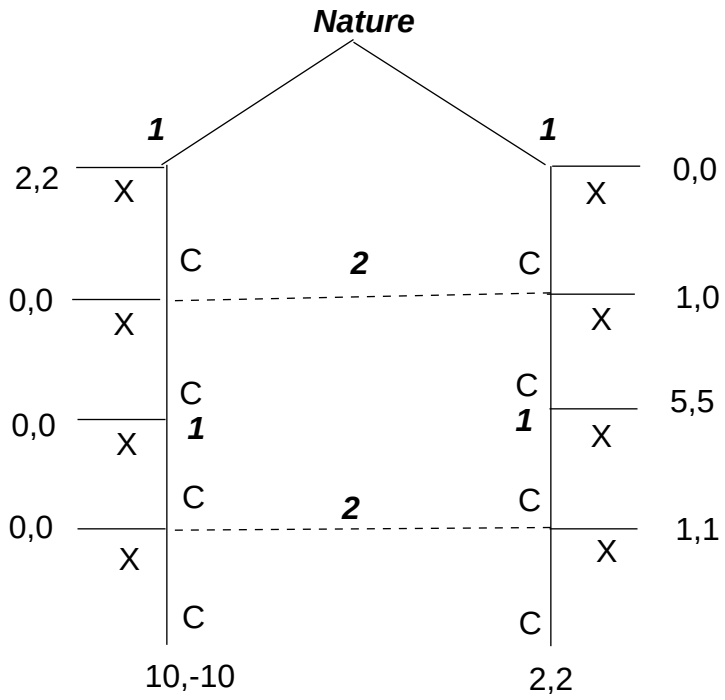
1. Consider the strategic game $\{\{1,2\},\{A_1,A_2\},\{u_1,u_2\}\}$ in which $A_i = [0,1]$ and $u_i(a_1,a_2) = a_i(1-a_1-a_2)$ for $i = 1,2$. Show that each player's only strategy which survives the iterated deletion of strictly dominated strategies is his unique Nash equilibrium strategy, and indicate which strategy is that one.

2. Let the game:



   (a) Describe a sequential equilibrium for this game and show it is indeed a sequential equilibrium.

   (b) Describe as many actions as you can for each player which cannot be part of a sequential equilibrium and explain why.

3. Consider a homogeneous good industry where n firms produce at zero cost and play a Bertrand game (that is, the simultaneously choose prices p_i) for an infinite number of periods. When all firms choose the same price, they earn a per-period profit $\Pi(p) = p\alpha D(p)/n$. When a firm i charges a price lower than the price of all the other firms, it earns a profit $\Pi(p_i) = p_i\alpha D(p_i)$ and all other firms obtain zero profits. The parameter α represents the state of demand. Imagine that in the

current period demand is characterized by $\alpha = 1$, but starting from the following period demand is characterized by $\alpha = \theta$ in each of the following periods. All the players know exactly the evolution of the demand state at the beginning of the game. Firms have the same common discount factor, δ.

(a) What is the Nash equilibrium of the stage game?

(b) Assume $\theta > 1$ and consider the following strategies. Each firms plays the monopoly price p_m in the first period of the game and continues to charge such a price until a profit equal to zero is observed. When this occurs, each firm charges a price equal to zero forever. Under which conditions is this profile of strategies a subgame-perfect equilibrium? In particular, show how θ and n affect such a condition, and give an economic intuition for this result.

(c) Can other prices be sustained at equilibrium under strategies similar to the ones above? Under which condition?

(d) Assume now $\theta < 1$, and find the conditions under which the profile of strategies delineated above represent an equilibrium.