Nash bargaining with downward rigid wages

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Abstract

We study the effect of downward wage rigidity in a dynamic model when wages are negotiated according to Nash bargaining. Downward rigidity causes a decrease in the worker's expected utility. For the firms the effect is ambiguous. © 1997 Elsevier Science S.A.

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This note studies the effect of downward rigidity of wages, when wages are negotiated bilaterally in a dynamic model. Such type of wage rigidity arises, for example, when workers' compensation is regulated by an industry-wide pay scale and no demotions are allowed. Many countries have, in different degrees, institutional settings that approximate this description. Calmfors and Driffill (1988) provide an index of the degree of centralization of wage bargaining and its impact on macroeconomic performance. Blanchflower (1989), using data from the British Social Attitudes Surveys and Holzer and Montgomery (1993), with data from the Employment Opportunity Pilot Project Survey of Firms, both from the UK, provide positive evidence for the existence of downward wage rigidity. Nevertheless, after surveying the literature more extensively, Layard et al. (1991) (p. 208) and Jimeno and Toharia (1994) (p. 88) conclude that the evidence for downward wage rigidity as a general phenomenon is inconclusive.

The model has two periods. In each period, a random output is divided, according to Nash's bargaining solution, between a worker and the employer. In the first period, the wage can be chosen arbitrarily. In contrast, the second period wage cannot be lower than the one for the first period. At any time, firm or worker can choose to take an outside option, and the match is terminated.

The firm/worker pair produces an output given by an increasing function $R(s)$ where s is a random variable that represents the quality of the match. This quality varies from period to period. The first period quality is denoted by s_1. We assume the worker has an outside option that gives total utility U

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and the firm and outside option with value normalized to zero. Under these conditions matches such that $R(s_1) > U$ enter a contract.

The firm and the worker negotiate a wage for the first period w_1, after observing s_1, which is determined by Nash bargaining (Nash, 1950). The first period bargaining takes into account the expected value of the whole relationship. The productivity of the match in the second period s_2, is drawn from the cumulative distribution function $F(s)$. After observing s_2, second period wages are determined by Nash bargaining subject to the constraint that $w_2 \geq w_1$. If no agreement can be reached that gives both parties a value at least as large as their outside options, the match is terminated.

The choice of the first period wage w_1 has an effect on second period negotiation and termination decisions. Thus we start our analysis considering the second period negotiation, for a given value w_1. Once the information about the second period output of the match is revealed a decision must be taken concerning the continuation of the relationship. Since the firm cannot lower wages, a layoff will occur if and only if

$$R(s) - w_1 < 0.$$

Otherwise, the salary will be determined by Nash bargaining (subject to the constraint $w_2 \geq w_1$). For simplicity assume equal weights in the Nash bargaining solution. It is easy to check that the second period wage will satisfy

$$w_2 = \max\left\{w_1, \frac{R(s) + U}{2}\right\}.$$

Let $S_1(w)$ be the minimum value of s such that a layoff does not occur, i.e.

$$R(S_1(w)) - w = 0$$

and $S_2(w)$ be defined by

$$\frac{R(S_2(w)) + U}{2} = w.$$

Note that, if $w > U$, $S_2(w) > S_1(w)$ and $w_2 = w$ for $s \in [S_1(w), S_2(w)]$. In contrast, if $w \leq U$, the first period wage has no effect on second period decisions. Provided that productivity is sufficiently high in the first period, the Nash bargaining solution will imply that $w > U$, so the constraint on wages will bind in the second period, with positive probability.

We now turn to the first period wage bargaining.

For simplicity we assume that the rate of discount is zero. Total profits of the firm as a function of w_1 will thus be given by

$$\Pi(s_1, w_1) = R(s_1) - w_1 + \int_{s \in [S_1(w_1), S_2(w_1)]} (R(s) - w_1) F(ds) + \int_{s > S_2(w_1)} \frac{R(s) - U}{2} F(ds)$$

and the total value for the worker will be given by

$$V(w_1) = w_1 + F(S_1(w_1)) \cdot U + [F(S_2(w_1)) - F(S_1(w_1))] \cdot w_1 + \int\limits_{s > S_2(w_1)} \left(\frac{R(s) + U}{2} \right) F(ds).$$

The first period solution to the Nash bargaining problem consists of solving

$$\max_{w_1} \Pi(s_1, w_1) V(w_1).$$

Proposition: *In the Nash bargaining solution, if $w_1 > U$, the total surplus of the match will be lower than the value obtained in the absence of wage inflexibility and $\Pi(s_1, w_1) > V(w_1)$.* **Proof**: See Appendix A.

Notice, for comparison, that in absence of wage inflexibility the Nash bargaining solution gives $\Pi(s_1, w_1) = V(w_1)$. This proposition thus implies that wage inflexibility reduces total surplus of workers and that it is more detrimental to workers than firms. The intuition behind the result is that higher wages in the first period imply a larger potential loss of total surplus in the second period. This effect is analogous to introducing a tax on wages, thereby reducing the share of workers.

An interesting additional question is to see whether the utility of firms is necessarily reduced by the wage rigidity. We first provide an intuitive argument and then give an example in which profits actually increase with the introduction of downward wage rigidity.

Nash bargaining maximizes a social welfare function subject to a utility possibility frontier. Wage rigidity produces a change in the marginal relation of transformation of utilities which makes the worker's utility more 'expensive' (if w_1 were equal to U there would be no distortion and loss of utility and the larger the salary the larger the loss). Under these conditions it is not surprising that the worker loses, since the substitution effect and the income effect both go towards making the 'consumption' of worker utility smaller (the utility function is homothetic so the income effect is negative for a 'price increase').

For the case of the firm the conclusion is more ambiguous, since the substitution effect leads to an increased utility for the firm but the income effect is negative. We present now a numerical example which shows that in fact it is possible both that the firm wins and that the firm loses by having downward inflexible wages. The answer depends on the initial productivity. With low initial productivity the income effect is small because the constraint is not binding very often, so the distortion is small and the firm wins. With high initial productivity the distortion is large often enough that the firm loses.

Let the distribution of second period shocks be uniform on $[0,A]$, and $R(s) = s$ so that $S_1(w) = w$ and $S_2(w) = 2w - U$. Then the value for the firm will be

$$\Pi(s_1, w_1) = s_1 - w_1 + \int\limits_{w_1}^{2w_1 - U} (s - w_1) \frac{1}{A} \, ds + \int\limits_{2w_1 - U}^{A} \frac{s - U}{2} \frac{1}{A} \, ds$$

$$= s_1 - w_1 + \frac{1}{A} \left[\frac{A^2}{4} - \frac{UA}{2} - \frac{w_1^2}{2} + w_1 U - \frac{U^2}{4} \right]$$

and the total value for the worker will be given by

$$V(w_1) = w_1 + \frac{w_1}{A} U + \left[\frac{2w_1 - U}{A} - \frac{w_1}{A} \right] w_1 \int_{2w_1-U}^{A} \left(\frac{s+U}{2} \right) \frac{1}{A} \, ds = w_1 + \frac{1}{A} \left[\frac{A^2}{4} + \frac{UA}{2} + \frac{U^2}{4} \right]$$

By Nash bargaining we have that

$$s_1 - w_1 + \frac{1}{A} \left[\frac{A^2}{4} - \frac{UA}{2} - \frac{w_1^2}{2} + w_1 U - \frac{U^2}{4} \right] = \left(1 + \frac{1}{A} (w_1 - U) \right) \left(w_1 + \frac{1}{A} \left[\frac{A^2}{4} + \frac{UA}{2} \right. \right.$$
$$\left. \left. + \frac{U^2}{4} \right] \right) \tag{1}$$

When $U=0$ this reduces to,

$$\frac{3w_1^2}{2A} + \frac{9w_1}{4} - s_1 = 0 \tag{2}$$

When wages are not constrained (and using Nash bargaining) we have that the value to the firm is,

$$\Pi^u = \frac{s_1 - U}{2} + \int_U^A \frac{s - U}{2} \frac{1}{A} \, ds = \frac{s_1 - U}{2} + \frac{1}{A} \left[\frac{A^2}{4} - \frac{UA}{2} + \frac{U^2}{4} \right] \tag{3}$$

and the value to the worker is

$$V^u = \frac{s_1 + U}{2} + \int_U^A \frac{s + U}{2} \frac{1}{A} \, ds = \frac{s_1 + U}{2} + \frac{1}{A} \left[\frac{A^2}{4} + \frac{UA}{2} - \frac{3U^2}{4} \right] \tag{4}$$

Now let $A=10$ and $U=0$. This implies an expected value of the second period's productivity equal to five. We compute the Nash bargaining solution for an initial productivity shock below and above this value.

For $s_1 = \frac{12}{5}$, it follows that $w_1 = 1$, $\Pi(s_1, w_1) = \frac{77}{20}$, and $(\Pi^u) = \frac{74}{20}$ and thus the firm benefits from the rigidity of wages.

On the other hand for $s_1 = \frac{81}{10}$ it follows that $w_1 = 3$, $\Pi(s_1, w_1) = \frac{62}{20}$, and $(\Pi^u) = \frac{131}{20}$, so the firm loses from the rigidity of wages.

1.1. The effect of a severance payment

In this section we show that a severance payment, appropriately chosen, can correct the inefficiency generated by the wage inflexibility. Assume now that there is a severance payment T, that the firm would have to incur if the match is terminated in the second period (and only at that time).

In this case a layoff will occur if and only if

$$R(s) - w_1 \leq -T.$$

Otherwise, the salary will be determined by Nash bargaining (again subject to the constraint $w_2 \geq w_1$). We assume that if there is no agreement in the second period, the firm must pay the worker

the severance payment T and thus the disagreement point in the corresponding Nash bargaining problem is $\{U + T, \ -T\}$. It follows immediately that

$$w_2 = \max\left\{w_1, \frac{R(s) + U}{2} + T\right\}.$$

As before, let $S_1(w)$ be the minimum value of s such that a layoff does not occur, i.e.

$$R(S_1(w)) - w = -T$$

and $S_2(w)$ be defined by

$$\frac{R(S(w)) + U}{2} + T = w.$$

Notice that by setting $T = w - U$, it follows that $S_1(w) = s^*$, where

$$R(s^*) = U$$

and thus the decision of terminating the match is the efficient one. For values of $s \in (s^*, \ S_2(w))$, the constraint on second period wages will be binding, so $w_2 = w$. Otherwise w_2 will be determined by the standard equal weights Nash bargaining solution, for the given disagreement point.

Given that under this particular choice of severance payment termination is efficient, the downward rigidity of wages has no effect on total surplus. As a result, the choice of first period wage does not affect total surplus, and thus the equal weights Nash bargaining solution implies that $V(w_1) = \Pi(s_1, w_1)$. Consequently, the combination of a policy of wage rigidity with this severance payment is neutral with respect to total surplus and its distribution. (However, it will typically affect the wage profile by decreasing the first period wage and increasing the expected wage in the second period.) This example thus illustrates that multiple distortions in the labor market are not necessarily reinforcing, but could indeed have some countervailing effects.

Acknowledgements

Appendix A

Proof of proposition

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The first-order conditions for the Nash bargaining problem are

$$-\Pi_2(s_1, w_1)V(w_1) = V'(w_1)\Pi(s_1, w_1).$$

We now proceed to characterize this solution. First note that

$$- \Pi_2(s_1, w_1) = 1 + F(S_2(w_1)) - F(S_1(w_1))$$

and

$$V'(w_1) = 1 + F(S_2(w_1)) - F(S_1(w_1)) + F'(S_1(w_1))S_1'(w_1)(U - w_1)$$

where

$$S_1'(w_1) = 1/R'(S_1(w_1))$$

This implies that

$$V(w_1) = \Pi(s_1, w_1)\left[\frac{1 + F(S_2(w_1)) - F(S_1(w_1)) + F'(S_1(w_1))(U - w_1)/R'(S_1(w_1))}{1 + F(S_2(w_1)) - F(S_1(w_1))} \right].$$

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