# Dynamic price competition with capacity constraints and strategic buyers

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## Abstract

We analyze a simple dynamic framework where sellers are capacity constrained over the length of the game. Buyers act strategically in the market, knowing that their purchases may affect future prices. The model is examined when there are single and multiple buyers, with both linear and non-linear pricing. We find that, in general, there are only mixed strategy equilibria and that sellers get a rent above the amount needed to satisfy the market demand that the other seller cannot meet. Buyers would like to commit not to buy in the future or hire agents with instructions to always buy from the lowest priced seller. Furthermore, sellers' market shares tend to be maximally asymmetric with high probability, even though they are *ex ante* identical.

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# 1 Introduction

In many durable goods markets, sellers who have market power and intertemporal capacity constraints face strategic buyers who make purchases over time to match their demands. There may be a single buyer, as in the case of a government that purchases military equipment or awards construction projects, such as for bridges, roads, or airports, and chooses among the offers of a few large available suppliers. Or, there may be a small number of large buyers, such as in the case of airline companies that order aircraft or that of shipping companies that order cruise ships, where the supply could come only from a small number of large, specialized companies.<sup>1</sup> The capacity constraint may be due to the production technology: a construction company that undertakes to build a highway today may not have enough engineers or machinery available to compete for an additional large project tomorrow, given that the projects take a long time to complete; a similar constraint is faced by an aircraft builder that accepts an order for a large number of aircraft. Or, the capacity constraint may simply correspond to the flow of a resource that cannot exceed some level: thus, if a supplier receives a large order today, he will be constrained on what he can offer in the future. This effect may be indirect, if the resource is a necessary ingredient for a final product, with no substitutes (as often in the case of pharmaceuticals). Cases like the ones mentioned above suggest a need to study dynamic oligopolistic price competition under capacity constraints, when buyers are also strategic. Although this topic is both important and interesting, it has not been treated yet in the literature.

To obtain some first insights into the problem, consider the following simple setting. Take two sellers of some homogeneous product, say aircraft, to fix ideas. Each seller cannot supply more than a given number of aircraft over two periods. Suppose that there is only one large buyer in this market, this may be the defense department, with a demand that exceeds the capacity of each seller but not that of both sellers combined. Let the period one prices be lower for one seller than the

<sup>&</sup>lt;sup>1</sup>Anton and Yao (1990) provide a critical survey of the empirical literature on competition in defense procurement - see also Burnett and Kovacic (1989) for an evaluation of relevant policies. In an empirical study of the defense market, Greer and Liao (1986, p.1259) find that "the aerospace industry's capacity utilization rate, which measures propensity to compete, has a significant impact on the variation of defense business profitability and on the cost of acquiring major weapon systems under dual-source competition". Ghemawat and McGahan (1998) show that order backlogs, that is, the inability of manufacturers to supply products at the time the buyers want them, is important in the U.S. large turbine generator industry and affects firms' strategic pricing decisions. Likewise, production may take significant time intervals in several industries: e.g., for large cruise ships, it can take three years to build a single ship and an additional two years or more to produce another one of the same type. Jofre-Bonet and Pesendorfer (2003) estimate a dynamic procurement auction game for highway construction in California - they find that, due to contractors' capacity constraints, previously won uncompleted contracts reduce the probability of winning further contracts.

other. Then, if the buyer's purchases exhaust the capacity of the low priced seller, only the other seller will remain active in the second period and, unconstrained from any competition, he will charge the monopoly price. A number of questions arise. Anticipating such behavior, how should the buyer behave? Should he split his orders in the first period, in order to preserve competition in the future, or should he get the best deal today? Given the buyer's possible incentives to split orders, how will the sellers behave in equilibrium? Should sellers price in a way that would induce the buyer to split or not to split his purchases between the sellers? How do sellers' equilibrium profits compare with the case of only a single pricing stage? Does the buyer have an incentive to commit to not making purchases in the future? Are there incentives for the buyer to vertically integrate with a seller?

An additional set of questions emerges when there is more than one buyer. Would the buyers like to coordinate their purchases? Is buyer coordination possible in equilibrium? Are the seller equilibrium market shares identical, since the sellers are identical?

We consider a set of simple dynamic models with the following key features. There are two sellers, each with fixed capacity over two periods. Sellers set first-period prices and then buyers decide how many units they wish to purchase from each seller. The situation is repeated in the second period, given the remaining capacity of the firms; sellers set prices and buyers decide which firm to purchase from. We examine the cases of a single and of two buyers. In each case, we consider linear, as well as non-linear pricing.

Our main results are as follows. Under monopsony and linear pricing, a pure strategy subgame perfect equilibrium fails to exist. This is due to a combination of two phenomena. First, the buyer has an incentive to split his orders in the first period if the prices are close in order to keep competition alive in the second period. This in turn, gives the sellers incentives to raise their prices. Second, if prices get "high", each seller has a unilateral incentive to lower his price, and sell all his capacity. We characterize the mixed strategy equilibrium and show that the buyer may have a strict incentive to split his orders, in equilibrium. Also, the sellers make a positive economic rent above the profits of serving the buyer's residual demand, if the other seller sold all of his units. There are three main implications that follow from this result. First, the buyer would like to commit to not make purchases in the second period, so as to induce strong price competition in the first period. This is consistent with the practice in the airline industry, where airliners have options to buy airplanes in the future. Second, the buyer has the incentive to instruct its purchasing agents to always buy from the lowest priced firm. This is consistent with many government procurement rules that do not allow discretion to its purchasing officers. In other words, in equilibrium, the buyer is hurt by his ability to behave strategically over the two periods and would like to commit to myopic behavior, if possible. Finally, the firm has a strict incentive to vertically integrate with one of the suppliers. Under non-linear pricing, we show that the ability of each seller to price each of his units separately allows us to derive a pure strategy equilibrium where the sellers make no rents and the buyer does not have an incentive to commit not to buy in the future. These results are due to the fact that the seller's profitability on each unit can be separated with non-linear pricing. We study both linear and non-linear pricing to understand the subtle potential differences that may arise from different seller pricing strategies. We do not attempt to pick what type of pricing will arise in equilibrium.

In the duopsony case, we find that it is now the buyers that must play a mixed strategy, randomizing between which of the two sellers they should buy from in period 1 using either linear or non-linear prices, while it is not required that they split orders in equilibrium. There must be the potential of buyers not coordinating their orders in period one, despite the fact that buyers would like to coordinate and split their orders evenly among sellers to maximize competition between the sellers in period two: competition in period two is most stiff when sellers' period two capacities are close. This inability of buyers to coordinate in equilibrium, makes it highly likely that sellers' markets shares in both the first period and for the entire game can be quite asymmetric, even though sellers are *ex ante* identical. In particular, we show that there is a 50% chance that the final market shares will be extreme, in the sense that one of the sellers will sell all of his capacity, while half of the other seller's capacity will not be used. This is due to the fact that the buyers are strategic and have to play a mixed strategy in equilibrium. We also find, as in the single buyer case, that the sellers make positive rents if the buyers cannot commit not to buy in the future.

Our paper is related to a few distinct literatures. First, to the literature on pricing with capacity constraints. It is already known from the classic work of Edgeworth (1897) that capacity constraints may dramatically alter the nature of price competition in oligopolistic markets, possibly leading to a "nonexistence of equilibrium" or, as sometimes described, "cycles".<sup>2</sup> Mixed strategy pricing equilibria under capacity constraints are derived in Beckmann (1965), Levitan and Shubik (1972) and Osborne and Pitchik (1986).<sup>3</sup> Dudey (1992) derives the price equilibrium when capacity-constrained

 $<sup>^{2}</sup>$ Maskin and Tirole (1988) provide game theoretic foundations for "Edgeworth cycles" in a somewhat different setting, without capacity constraints.

<sup>&</sup>lt;sup>3</sup>Kirman and Sobel (1974) prove equilibrium existence in a dynamic oligopoly model with inventories.

sellers face buyers that arrive to the market sequentially. There is a well-known literature on firms that choose their capacities, in anticipation of an oligopoly competition stage; see e.g. Dixit (1980), Spulber (1981), Kreps and Scheinkman (1984), Davidson and Deneckere (1986a), Deneckere and Kovenock (1996), and Allen, Deneckere, Faith, and Kovenock (2000).<sup>4</sup> Relative to these papers, a crucial difference in our analysis is that we consider strategic behavior also on the buyers' side and that sellers have intertemporal capacity constraints. In particular, we examine how the sellers' capacities evolve over time, as interrelated with their pricing strategies and the buyers' decisions.

A second related set of papers examines when a monopsonist influences the degree of competition among (potential) suppliers, in particular they focus on the buyer's incentives to act strategically when facing competing sellers, as in the context of "split awards" and "dual-sourcing". Rob (1986) studies procurement contracts that would allow selection of an efficient supplier, while also providing incentives for product development. Anton and Yao (1987, 1992) consider models where a buyer can buy either from one seller or split his order and buy from two sellers. They find conditions under which a buyer will split his order and characterize seemingly collusive equilibria. Related studies on dual-sourcing are offered by Riordan and Sappington (1987) and Demski, Sappington and Spiller (1987). Our work differs in two important ways. The intertemporal links are at the heart of our analysis: the key issue is how purchasing decisions today affect the sellers' remaining capacities tomorrow. In contrast, the work mentioned above focuses on static issues and relies on cost asymmetries. Strategic purchases from competing sellers and a single buyer in a dynamic setting are also studied under "learning curve" effects; see e.g. Cabral and Riordan (1994) and Lewis and Yildirim (2002, and 2004 for switching costs). One general difference that should be noted is that, in our case, by buying a larger quantity from one seller you make that seller less competitive in the following period (at the extreme case: inactive) - in the learning curve case, the more you buy from a seller, the more competitive you make that seller, as his unit cost decreases.<sup>5</sup>

Gehrig (1990, ch.2) studies non-linear pricing with capacity constrained sellers. Lang and Rosenthal (1991) characterize mixed strategy price equilibria in a game where contractors face increasing cost for each additional unit they supply.

<sup>&</sup>lt;sup>4</sup>A number of papers have also analyzed the effect of capacity constraints on collusion - see e.g. Brock and Scheinkman (1985), Davidson and Deneckere (1986b), Lambson (1987), Rotemberg and Saloner (1989), and Compte, Jenny and Rey (2002), to mention a few.

<sup>&</sup>lt;sup>5</sup>Bergemann and Välimäki (1996, 2002) examine models where in each period sellers set prices and buyers choose which seller to purchase from. Buyers' decisions affect how competitive each seller could be in subsequent periods, however this is in a different setting where the action comes from experimentation and learning, not from capacities. Strategic competition with capacity constraints is also part of Yanelle's (1997) model of financial intermediation.

Finally, our analysis has implications related to vertical integration strategies and is, thus, related to the literature on the issue (see e.g. Innes and Sexton, 1994, on strategic buyers and exclusionary contracts, Ma, 1997, for an analysis of vertical integration under option contracts, and Rey and Tirole, 2003, on foreclosure).

A third set of papers that our work is related to is on bilateral oligopolies, where both sellers and buyers are large players and act strategically. While in many important markets players have significant power on both sides of the market, such situations have not generally received enough attention.<sup>6</sup> In our analysis, with two sellers, we examine both the case of a single and that of multiple buyers and we emphasize that the sellers' behavior changes qualitatively when we move from the former case to the latter.

The remainder of the paper is organized as follows. The model is set up in Section 2. Section 3 characterizes the equilibrium with one buyer and linear pricing. In Section 4 we characterize the equilibrium with one buyer and non-linear pricing. The duopsony case is presented in Section 5, first under linear and then under non-linear prices. We conclude in Section 6. Some proofs, not required for the continuity of the presentation, and other material, not directly related to the core arguments, are relegated to an Appendix.

## 2 The basic model

The game lasts two periods. There are N + 2 firms in total, two sellers and N buyers. We will consider the case of monopsony (N = 1) and that of duopsony (N = 2). The product is perfectly homogeneous and the sellers are identical. Each seller has a capacity to produce, for the two periods, a total of 2N units at marginal cost of 0.<sup>7</sup> The goods are durable over the lifetime of the model. A buyer values each the first two units V in *each* of the periods and a third unit at  $V_3$  in period 2. We assume that  $V \ge V_3 > 0.^8$  The key features about the relative demands of consumers and potential supplies by firms is that no firm can supply the entire market, but the total available

<sup>&</sup>lt;sup>6</sup>Different aspects of bilateral oligopoly have been studied in a small but growing literature - see e.g. Horn and Wolinsky (1988), Dobson and Waterson (1997), Bloch and Ghosal (1997), Hendricks and McAfee (2000) and Inderst and Wey (2003).

<sup>&</sup>lt;sup>7</sup>This is independent of when these units are supplied during the two periods.

<sup>&</sup>lt;sup>8</sup>This specification is consistent with growing demand. In general, the first and second units could have different values (say  $V_1 \ge V_2$ ). Also, we could allow the demand of the third unit to be random. It is straightforward to introduce either of these cases in the model, with no real change in the results, only at the cost of some additional notation.

capacity by the two sellers is larger than the potential market demand of the buyers. The basic economic results would survive in more general environments with these features.

In each period, each of the sellers sets a price for each of his available units of capacity.<sup>9</sup> We consider competition both with linear and with non-linear pricing. For simplicity, we assume non-discriminatory pricing by sellers.<sup>10</sup> Each buyer chooses how many units he wants to purchase from each seller at the price specified, as long as the seller has enough capacity. If the demand by buyers is greater than a seller's capacity, then they are rationed. The rationing rule that we use is that each buyer is equally likely to get his order filled. The rationed buyers can buy from the other seller as many units as they want.<sup>11</sup> We assume that sellers commit to their prices and that all information is common knowledge and symmetric. All firms have a common discount factor  $\delta$ . In each case examined, we are looking for a symmetric subgame perfect equilibrium.<sup>12</sup>

The interpretation of the timing of the game is straightforward in case the sellers' supply comes from an existing stock (either units that have been already produced, or some natural resource that the firm controls). In case there is production taking place in every period, there are more than one possible interpretations of the intertemporal capacity constraint, depending on the details of the technology. One simple way to understand the timing, in such a case, is illustrated in Figure 1. The idea here is that actual production takes time. Thus, orders placed in period one are not

<sup>&</sup>lt;sup>9</sup>Regarding the assumption that sellers set prices, note that, even in a monopsony situation, we often see the sellers making offers, like when the department of defense (DOD) is purchasing weapon systems. The DOD may do this to solve possible agency problems between the agent running the auction and the DOD. If an agent can propose offers, it is much easier for sellers to bribe the agent to make high offers than if sellers make offers, which can be observed by the regulator. This is because the sellers can bribe the agent to make high offers to each of them, but competition between the sellers would give each seller an incentive to submit a bid to grab all the sells and it would be quite difficult for the agent to accept one offer that was much higher than another. Further, when there are more than one buyers, this assumption seems the most natural.

<sup>&</sup>lt;sup>10</sup>Clearly, this assumption does not matter when there is a single seller. The flavor of our results would be the same if discriminatory pricing was allowed with multiple buyers.

<sup>&</sup>lt;sup>11</sup>Our results would not change qualitatively if the sellers could choose which buyer to ration, as long as each buyer has a positive probability of being rationed.

<sup>&</sup>lt;sup>12</sup>To clarify, given the discounting and demand structure, the maximum value that a buyer could obtain over both periods and evaluated at the beginning of the first period is equal to  $2V(1 + \delta) + \delta V_3$  (if the units were sold at zero price). It is also convenient to observe, before proceeding to the analysis, that negative prices cannot be part of an equilibrium. Suppose in equilibrium some seller charged a negative price in some period. Then, either a buyer would have a strict incentive to buy all the available units of that seller or would choose to wait and purchase those units at a later time if the relevant price was expected to be even lower. Either way, this seller could do better by increasing his price to zero, thus increasing his profit from a negative level to zero. This observation allows us to simplify the presentation of the arguments, by focusing on non-negative prices.



Figure 1: Timing

completed before period two orders arrive. Since each seller has the capacity to only work on a limited number of units at a time, units ordered in period one restrict how many units could be ordered in period two. In such a case, since our interpretation involves delivery after the current period, the buyers' values specified in the game should be understood as the present values for these future deliveries (and the interpretation of discounting should be also accordingly adjusted).

# **3** Monopsony with linear pricing

We first examine the single buyer case (N = 1), that is, monopsony and consider competition when the two sellers are restricted to linear pricing. We then allow for non-linear pricing under monopsony in the next section.

We are constructing a subgame perfect equilibrium, and thus we work backwards by starting from period 2.

## **3.1** Second period

There are several cases to consider, depending on how many units the buyer has bought from each seller in period one. We will use, throughout the paper, the convention of calling a seller with i units of remaining capacity seller i.

Buyer bought two units in period 1. If the buyer bought a unit from each of the sellers in period 1, then the price in period 2 would be 0, due to Bertrand competition. If the buyer bought both units from the same firm, then the other firm would be a monopolist in period 2 and charge  $V_3$ . Thus, period 2 equilibrium profit of a seller that has one remaining unit of capacity is 0 and that of a seller with two remaining units of capacity is  $V_3$ .

Buyer bought one unit in period 1. In this case, the buyer has demand for two units, one of the sellers has capacity of 1 unit, seller 1, while the other has a capacity of 2 units, seller 2. We demonstrate that there is no pure strategy equilibrium in period 2 by the following Lemma.

**Lemma 1** If the buyer bought one unit in period 1 in the linear pricing monopsony model, then there is no pure strategy equilibrium in period 2.

**P roof.** First, notice that the equilibrium cannot involve seller 2 charging a zero price: that seller could increase his profit by raising his price (as seller 1 does not have enough capacity to cover the buyer's entire demand). Thus, seller 1 would also never charge a price of zero. Suppose now that both sellers charged the same positive price. One, if not both, sellers have a positive probability of being rationed. A rationed seller could defect with a slightly lower price and raise his payoff. Suppose that the prices are not equal:  $p_i < p_j \leq V_3$ . Clearly, seller *i* could increase his payoff by increasing his price if  $p_i < V_3 \leq p_j$ . Finally, if  $V_3 \leq p_i < p_j$ , seller *j* makes 0 profit and can raise his payoff by undercutting firm i's price.

There is a unique mixed strategy equilibrium which we provide in the following Lemma (see also Figure 2 for an illustration).

**Lemma 2** If the buyer bought one unit in period 1 in the linear pricing monopoly model, then there is a unique mixed strategy equilibrium. Both sellers mix on the interval  $[V_3/2,V_3]$ . Seller 1's price distribution is  $F_1(p) = 2 - \frac{V_3}{p}$ , with an expected profit of  $V_3/2$ . Seller 2 has price distribution  $F_2(p) = 1 - \frac{V_3}{2p}$  for  $p < V_3$ , with a mass of 1/2 at price V<sub>3</sub>, and expected profit equal to V<sub>3</sub>. Seller 2's price distribution first order stochastically dominates seller 1's distribution.

#### **P roof.** See Appendix A1. ■

By Lemma 2, we obtain two key insights that run throughout the paper. The first concerns the calculation of the equilibrium sellers' profits and the second regards the ranking of the sellers' price distributions. The seller with two units of capacity can always guarantee himself a payoff of at least  $V_3$ , since he knows that, no matter what the other seller does, he can always charge  $V_3$  and sell at least one unit. This is the high-capacity seller's *security profit level*. The high-capacity seller's security profit puts a lower bound on the price offered in period 2. In the situation examined at Lemma 2, the lowest price is  $V_3/2$ : the seller will never charge a lower price because he can at



Figure 2: Mixed strategy equilibrium

most sell two units and would do better by selling one unit at  $V_3$ . This puts a lower bound of  $V_3/2$  on the period 2 profit of seller 1, the low-capacity seller; this, in turn, is equal to the profit that the low-capacity seller can guarantee to himself, given that the high-capacity seller will not choose a strictly dominated price.<sup>13</sup> Competition between the two sellers fixes their profits at their respective security levels.<sup>14</sup>

The second insight deals with the incentives for aggressive pricing. We find that the seller with larger capacity will price *less* aggressively than the seller with smaller capacity in period 2. The larger capacity seller knows that he will make sales even if he is the highest price seller, while the smaller capacity seller makes no sales if he is the high price seller. So the low capacity seller always has incentives to price more aggressively. More precisely, the high-capacity seller prices distribution first-order stochastically dominates the price distribution of the low capacity seller. This general property has important implications for market shares of the entire game.

We now examine the remaining period-two case (subgame).

Buyer bought no units in period 1. Each seller enters period 2 with 2 units of capacity, while the buyer demands 3 units. Using arguments similar to the ones in Lemma 1, we can show that there is no pure strategy equilibrium. There is a unique symmetric mixed strategy equilibrium.

<sup>&</sup>lt;sup>13</sup>Note that, while  $V_3/2$  is not the "security" profit of the low-capacity seller, it becomes that after one round of elimination of strictly dominated strategies.

<sup>&</sup>lt;sup>14</sup>We can generalize the analysis presented just above for any case where there is a low-capacity seller that cannot cover the demand and a high-capacity seller that can cover the demand. Suppose in period 2 there is demand for B units with value  $V_3$  and the capacity of the low-capacity seller is C, with C < B. Then there is no pure strategy equilibrium. In the unique mixed strategy equilibrium, the high-capacity seller's profit is  $V_3(B-C)$  and the low capacity seller's profit is  $C \frac{V_3(B-C)}{B}$ . The support of the prices is from  $V_3(B-C)/B$  to  $V_3$ . We provide further details and discussion in Appendix A5.

Each player's expected second-period equilibrium payoff is  $V_3$ ; this is the security profit of each seller. Using arguments similar to those in Lemma 2, we find that the players mix on the interval  $\left[\frac{V_3}{2}, V_3\right]$ , with no mass points or gaps. The buyer's demand is always met and he buys two units from the lowest priced seller and one from the highest priced seller. The sellers' distribution of prices satisfies<sup>15</sup>

$$p[F(p) + 2(1 - F(p))] = V_3, \tag{1}$$

or

$$F(p) = 2 - \frac{V_3}{p}.$$
 (2)

The equilibrium behavior in the second period is now summarized:

**Lemma 3** Second period competition for a monopsonist facing linear pricing falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price,  $V_3$ , and extracts the buyer's entire surplus. (ii) If each seller has enough capacity to cover by himself the buyer's demand then there is (Bertrand) pricing at zero. (iii) If the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity, then there is no pure strategy equilibrium. In the mixed strategy equilibrium, a seller with two units of capacity has expected profit equal to  $V_3$  and a seller with one unit of capacity has expected profit equal to  $V_3/2$ .

Note that case (i) in the above Lemma occurs when the buyer bought two units from the same seller in period 1; then the seller that has remaining capacity sets his price equal to  $V_3$ . Case (ii) occurs when the buyer bought one unit from each seller in period 1. Case (iii) occurs when the buyer bought one unit in period 1 or when the buyer bought no units in period 1.

#### **3.2** First period

Now, we go back to period 1. First, we demonstrate that the buyer will always buy two units in equilibrium and that there is no pure strategy equilibrium. We then characterize equilibrium payoffs and discuss the properties of equilibria.

#### **Proposition 1** The buyer buys two units in period 1.

 $<sup>^{15}</sup>$ The analysis underlying this expression is along similar lines to that of the subgame above (see Appendix A1 for details) and is, thus, omitted.

We sketch the proof here; the formal proof is in Appendix A2. First, prices must be positive, since the seller knows that even if he does not sell a unit in period 1, he will make positive profits in period 2. We then show that a buyer will never buy three units in period 1. For the buyer to buy three units, he must buy two units from the low priced seller at a positive price and one from the other seller. If he only buys one unit from each seller in period 1, the price for the third unit bought in period 2 is zero due to Bertrand competition, thus the buyer will never buy three units. We next argue that there is a maximal price by the highest price seller such that the buyer prefers buying one unit from each seller as opposed to only one unit from the low priced seller and that the sellers always set prices less than this price. This is because this price is greater than  $\delta V_3$ , a seller's expected profit in period 2 if he makes no sales in period 1. Thus, two units will always be purchased in any equilibrium.

A feature of the equilibrium is the incentive of the buyer to split his order. This is captured by the following result.

**Lemma 4** The buyer prefers to buy one unit from each seller as opposed to buying two units from the lowest priced seller if the difference in prices is less than  $\delta V_3$ .

This is an important result. It says that a buyer prefers to split his order if the discounted price differential is lower than the discounted price of a third unit when facing a monopolist. The price of a third unit when splitting an order is zero, while if the buyer does not split an order it is  $V_3$ . This value is the expected discounted payoff to a seller of not selling a unit in period 1, which makes sense since the third unit will always be bought by the buyer so there is no efficiency loss.

The next proposition demonstrates that there is no pure strategy equilibrium (symmetric or asymmetric) in the entire game.

**Proposition 2** There is no pure strategy equilibrium in the monopsony, linear pricing model.

## **P roof.** See Appendix A3. $\blacksquare$

The result of no pure strategy equilibrium is due to two phenomena. First, as depicted in Lemma 4, the buyer's incentive to split his orders if the prices are close, within  $\delta V_3$ . This gives the sellers incentives to raise price. On the other hand, if prices get "high", then sellers have incentives to drop their prices, and sell two units immediately. This cycling feature is common in games with capacity constraints.



Figure 3: Period 1 acceptances as a function of prices

Thus far, we have proved there is no pure-strategy equilibrium and the buyer always buys two units in period 1. Now we further characterize the (mixed-strategy) symmetric equilibria of the game.<sup>16</sup> First, we prove in Appendix A4 that the seller's price distributions must be sufficiently wide, so that the buyer will accept either 0, 1, or 2 units from a particular buyer. Figure 3 illustrates what can happen with the equilibrium prices distributed on the interval  $[\underline{p}, \overline{p}]$ . If a seller sets a price between  $\underline{p}$  and  $\underline{p} + \delta V_3$  he will sell either 1 or 2 units; the other buyer will never undercut his price by more than  $\delta V_3$  so the seller will always sell at least one unit and if the other seller's price is greater than his price by more than  $\delta V_3$  he will sell two units. If the seller sets a price between  $\underline{p} + \delta V_3$  and  $\overline{p} - \delta V_3$  he will sell either 0,1 or 2 units. If the prices are within  $\delta V_3$  of each other, the buyer will want to split his order between the sellers. Otherwise, the buyer will buy two units from the low priced seller. Finally, for prices between  $\overline{p} - \delta V_3$  and  $\overline{p}$ , the seller can never sell two units since the other seller's price will never be more than  $\delta V_3$  above his price. He will sell 1 unit if the prices are within  $\delta V_3$ , and 0 units otherwise.

In Appendix A4, we also prove the following Proposition.

**Proposition 3** In the monopsony model with linear prices, the lowest price offered in equilibrium, p, is greater than  $\delta V_3$ . Thus, the expected profit of a seller is greater than  $\delta V_3$ .

Thus, in equilibrium, the sellers receive rents above satisfying the residual demand after the buyer bought the other seller's capacity (or the static Bertrand competition),  $\delta V_3$ . Why is this the case? A seller knows that, if he makes no sells in period 1, his expected profit is  $\delta V_3$ . This gives a seller the incentive to raise his price above  $\delta V_3$  to take a chance of no sells in period 1, since by Lemma 4 a seller knows that even if he has the highest price he will make a sell as long as the price difference is less than  $\delta V_3$ . Since there is no cost of increasing his price and a potential benefit, the

<sup>&</sup>lt;sup>16</sup>Given that we have well defined payoffs in each of the period two subgames, we can guarantee existence of (a mixed strategy) Nash equilibrium in period one prices and, consequently, of a subgame-perfect equilibrium in the entire game, by use of arguments along the lines of Dasgupta and Maskin (1986a, 1986b).

seller can improve his payoff. Thus, there needs to be strategic uncertainty about what the buyer will do for a given set of prices for the sellers to make an offer with probability 1 in equilibrium. By not having any uncertainty about what a buyer will do, sellers can increase their expected payoff over the static Bertrand competition.

That the equilibrium expected profit is greater than  $\delta V_3$  is an important property and we further discuss some of its implications in the following subsection.

## 3.3 Equilibrium properties and analysis

As we saw above (Proposition 3), in the equilibrium of the linear pricing monopsony model each seller's profit exceeds  $\delta V_3$ . Thus, the buyer's total payment (to the sellers) exceeds  $2\delta V_3$ . We illustrate three strategies that the buyer can use to reduce his expected payments and still preserve efficiency. First, we show that the sellers obtain higher profit than the profit they would obtain if the buyer were behaving myopically; the buyer has an incentive to commit to (myopic) period-byperiod minimization of his purchase costs. Second, the buyer benefits if he can commit to make all his purchases at once, effectively making the game collapse into an one-shot interaction. Third, we show that the buyer may benefit by merging with one of the sellers. These three observations help to demonstrate the fundamental force that drives the equilibrium that we derived: due to strategic considerations, the buyer does not always purchase from the lowest priced seller when he plans to make further purchases, giving sellers the incentive to raise their prices above the static equilibrium level.

Our first observation is as follows :

**Corollary 1** The buyer would like to commit to myopic behavior and to make his purchases on the basis of static optimization in each period.

Suppose that the buyer could commit to behaving myopically (that is, to not behaving strategically across periods). In other words, while valuations are the same as assumed in the model, now the buyer does not recognize the link between the periods and views his purchases in each period as a separate problem. Thus, the buyer within each period purchases a unit from the seller that charges the lowest price (as long as this price is below his reservation price). There are two possible ways to generate a pure strategy equilibrium for this model (but only slightly different). We could have an equilibrium where each seller charges  $\delta V_3/2$  in the first period and the buyer randomizes between purchasing two units from one or from the other seller. Then, the seller that has not sold his two units in the first period, charges a price of  $V_3$  in the second period and the buyer purchases one unit from that seller. Thus, total profit for each seller (in present values terms) over both periods is  $\delta V_3$  and total payment for the buyer is  $2\delta V_3$ . To establish that this is an equilibrium, first note that the buyer indeed behaves optimally, on a period by period basis. Second, neither seller has a profitable deviation. Clearly, the seller cannot increase his profit above  $V_3$  in period 2. Now, in period 1, if a seller lowers his price below  $\delta V_3/2$ , he then sells both units (since he becomes the low priced seller) and obtains a profit below (at best, just a bit below)  $V_3$ . If he raises his price, he sells no units in the first period but obtains a profit equal to  $V_3$  in the second. Thus, there are no profitable deviations and this is an equilibrium.

The possibility that the buyer may split his order (he is indifferent, given the myopia assumption, between splitting his order and not splitting) may be viewed as a weakness of the equilibrium described just above. But this can be easily addressed: there is an alternative way to generate a pure strategy equilibrium in this case, if we introduce a smallest unit of account. We can assume that there is a smallest unit of account,  $\Delta$ , so that now the buyer has a strict preference for not splitting his order. The equilibrium has one seller charging  $\delta V_3/2 - \Delta$  and the other seller charging  $\delta V_3/2$  in the first period and the buyer buying two units from the low priced seller. The seller that made no sales in the first period, charges  $V_3$  in the second period and the buyer purchases one unit from that seller. Thus, total payment in present value terms for the buyer is  $2\delta V_3 - 2\Delta$ .<sup>17</sup> Clearly, the equilibrium payoffs are essentially the same under both approaches.

By combining this result with Proposition 3, we conclude that, when the buyer behaves strategically and recognizes that his current purchasing decisions affect the intensity of competition in the subsequent period, in equilibrium *his surplus is lower* than when he behaves myopically.

What drives this result is that now a seller knows that if he sets a higher price than his rival he cannot sell a unit in period one (and can only obtain a second period profit of  $V_3$ ). The above comparison may provide a rationale for purchasing policies that large buyers have in place that

<sup>&</sup>lt;sup>17</sup>To establish that this is an equilibrium, first note that the buyer indeed behaves optimally, chooses the lowest price in each period. Second, neither seller has a profitable deviation. Clearly, a seller cannot increase his profit above  $V_3$  in period 2. In period 1, if the low price seller lowers his price below  $\delta V_3/2 - \Delta$ , he reduces his profit. If the high priced seller reduces his price he either sells 0, 1, or 2 units if he charges  $\delta V_3/2 - \Delta$  or 2 units at a lower price. In any case, his profits fall. Clearly, neither seller can gain by raising their price above  $\delta V_3/2$ . If the low priced seller raises his price to  $\delta V_3/2$ , the equilibrium can have the buyer splitting his order (as the prices would be equal) and lowering this seller's profit. Thus, there are no profitable deviations.

We note that the equilibrium with a strategic buyer is *not* affected if there is a smallest unit of account, since the buyer will want to split his order as long as the gap between the two prices is less than  $\delta V_3$ . The mixed strategy equilibrium would generate the same payoffs without this assumption.

require purchasing at each situation strictly from the lowest priced seller. In particular a government may often assume the role of such a large buyer. It is often observed that, even when faced with scenarios like the one examined here, governments require that purchasing agents absolutely buy from the low-priced supplier, with no attention paid to the future implications of these purchasing decisions. While there may be other reasons for such a commitment policy (such as preventing corruption and bribes for government agents), our analysis suggests that by "tying its hands" and committing to purchase from the seller that sets the lowest current price, the government manages to obtain a lower purchasing cost across the entire purchasing horizon. We find, in other words, that delegation to such a purchasing agent that maximizes in a myopic way is beneficial, since it ends up intensifying competition among sellers.<sup>18</sup>

A related observation is the following.

**Corollary 2** In the monopsony model with linear prices, the buyer would like to commit to not buying any units in period 2.

In other words, instead of breaking the link between the periods (as discussed in the previous corollary), the buyer faces a lower purchasing price if he could make all his purchases at once (in a single period) and rule out the possibility of purchasing a unit in the second period. The idea is that the equilibrium profit level described in Proposition 3 is larger than in the static equilibrium (when the buyer commits to buying all goods in period 1). Let us now show that the seller's profit level in that, one-shot, case would be equal to  $\delta V_3$ . We found in Lemma 3 that the second-period equilibrium, if no units are sold in period one, has each seller making an expected profit of  $V_3$ . If now all competition took place in one period, the sellers' expected payoff would be again  $\delta V_3$ , since the strategic situation would be exactly the same as the last period with all sellers having full capacity (and the buyer's valuation for the third unit, as of period 1, equal to  $\delta V_3$ ). Thus, each seller's profit in the one-shot situation would be exactly  $\delta V_3$ . Since the allocation is always efficient, lower seller profit implies higher buyer profit. In the equilibrium of the game with purchases (potentially) over two periods, the expected buyer's payments is strictly above  $2\delta V_3$  and, thus, strictly exceeds his expected payment in case of competition in only the first period.

The behavior described in the Corollary above would require, of course, some vehicle of com-

<sup>&</sup>lt;sup>18</sup>Strategic delegation has been also shown to be (unilaterally) beneficial by providing commitment to some modified market behavior in other settings (see e.g. Fershtman and Judd, 1987, and Vickers, 1985). In our case, the key is the separation from the subsequent period and the commitment to myopia.

mitment that would make future purchases not possible. This is an interesting result and can be viewed as consistent with the practice of airliners placing a large order that often involves the option to purchase some planes in the future at the same price for firm orders placed now. Such behavior is sometimes attributed to economies of scale - our analysis shows that such behavior may emerge for reasons purely having to do with how sellers compete with one another.<sup>19</sup>

A further implication of Proposition 3 is:

**Corollary 3** In the monopsony model with linear prices, the buyer has a strict incentive to buy one of the sellers, that is, to become vertically integrated.

This result is based on the following calculations. By vertically integrating, and paying the equilibrium profit of a seller,  $\pi$ , the total price that the buyer will pay is  $\pi + \delta V_3$  since the other buyer would change the monopoly price  $V_3$  for a third unit (sold in period 2). This total payment is strictly less than the total expected payment  $(2\pi)$  that he would otherwise make in equilibrium. Thus, even though the other seller will be a monopolist, the buyer's payments are lower, since the seller that has not participated in the vertical integration now has lower profits.

# 4 Monopsony with non-linear pricing

In this section, we assume that a firm can offer a menu of prices in each period; a price if it sells one good and a price if it sells two goods.

## 4.1 Second period

First, we consider equilibrium in the possible period 2 subgames.

Buyer bought two units in period 1. In this case, non-linear pricing is the same as linear pricing (since at most one unit can be bought in period 2). If the buyer bought a unit from each seller in

<sup>&</sup>lt;sup>19</sup>It is also easy to see that the buyer would be better off if he could commit to *reduce* his demand to only two units. By committing to not purchasing a third unit (in any period), the value he obtains gets reduced by  $\delta V_3$ , while his payment gets reduced by an amount strictly higher than that (each seller's equilibrium profit drops from  $\pi > \delta V_3$  to zero). Still, it should be noted that commitment to such behavior may be difficult: once the initial purchases have been made, the buyer would then have a strict incentive to "remember" his demand for a third unit. A related point is that the sellers would have an incentive not to reveal some of their available capacity, as such a strategic move (if credible) would lead to higher profit for them. Remarks similar to the ones made just above about the (non) credibility of such strategies hold.

period 1, the price that each seller charges in period 2 for a single unit is 0. If the buyer bought two units from one of the sellers, then the price for a unit of the other seller is  $V_3$ .

Buyer bought one unit in period 1. The equilibrium price triples satisfy  $\{p_1, p_2^1, p_2^2\} = \{0, k, V_3\}$ , where  $k \ge V_3$ . The firm that has one unit of capacity (seller 1) charges  $p_1 = 0$  for his unit and the firm with two units of capacity (seller 2) charges  $V_3$  if the buyer buys two units  $(p_2^2)$  from the seller and a price that is not lower than that if the buyer buys one unit  $(p_2^1)$ . In equilibrium, the buyer buys both units from seller 2.

First, we show that the above strategy profiles are an equilibrium, next we argue that these are the unique pure strategy equilibrium payoffs. The buyer is indifferent between buying both units from seller 2 or one unit from seller 1: either way, his net surplus is equal to  $V_3$ . If seller 1 raises his price, he will still sell no units and his profit remains at zero; lowering his price would result in a loss. If seller 2 raises  $p_2^2$  he will sell nothing and his profit drops to zero (compared to the profit of  $V_3$  at the candidate equilibrium). If seller 2 lowers either prices below  $V_3$ , then the buyer will accept one of them and the seller will have a payoff lower than  $V_3$ . Thus, this is an equilibrium.

Now, we argue that the equilibrium payoffs are unique. First, in any equilibrium the buyer will buy two units: if we had a candidate equilibrium where the buyer bought only one unit (from either seller 1 or seller 2), then seller 2 could charge an incremental price of less than  $V_3$  and sell his second unit, thus increasing his profit. Second, there cannot be an equilibrium where the buyer gets one unit from each seller. For the buyer to buy one unit from each seller, it must be that  $p_1 + p_2^1 \le p_2^2$  and each price must not exceed  $V_3$ . Seller 2, has a profitable deviation to setting  $p_2^2$ equal to the original  $p_2^1$  plus  $\frac{p_1}{2}$  and setting the price for one unit so that the buyer will never buy only one unit from him. Thus, seller 1 cannot sell a unit. This can only occur if  $p_1 = 0$ . Finally, we must have  $p_2^2 = p_1 + V_3$ . Thus, we have unique equilibrium payoffs.

Buyer bought no units in period 1. The prices would be each seller charging  $V_3$  whether the buyer buys one or two units and the buyer buys two units from one seller and one from the other. The buyer's net surplus is  $2V + V_3 - 2V_3$  and each seller's profit is  $V_3$ . Let us establish that this is an equilibrium. If a seller increased his price he would sell no units. If a seller decreased either of the prices he charges, this price would get accepted and the seller's payoff would drop from  $V_3$  to the new price level.

The equilibrium is unique. To see this, first note that we cannot have an equilibrium where the buyer buys fewer than 3 units. This is because, if that were the case, one seller would have a strict incentive to lower one of his prices. Furthermore, a seller can always guarantee himself a profit of  $V_3$  since the buyer is always willing to pay this amount for the third unit. Finally, using simple arguments we can show that Bertrand competition will induce both sellers to price using two part-tariffs with the fixed portion of  $V_3$  and marginal costs of 0.

Equilibrium behavior in period 2, under non-linear pricing, can be summarized as follows.

**Lemma 5** Second period competition in the case of a monopsonist under non-linear pricing falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price,  $V_3$ , and extracts the entire seller's surplus. (ii) If both sellers have enough capacity to cover the buyer's demand, there is (Bertrand) pricing at zero. (iii) If the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity then there is a pure strategy equilibrium: a seller with two units charges  $V_3$  for two units (and a price at least as high for one unit) and a seller with one unit charges 0.

It follows from the above analysis that a seller gives up second-period profit  $V_3$  when, by selling one (or two) units in period 1, his remaining capacity drops from 2 units to 1 (or 0, respectively). Clearly, he would demand at least the discounted present value of that amount to sell one unit in period one.

Discussion. Comparing competition under linear and non-linear pricing in period 2, we observe that there is a critical difference in the case when the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity. Under linear pricing, a pure strategy equilibrium fails to exist because the seller with two units of capacity cannot prevent the price of the first unit affecting his sales of a second unit, while he can achieve this under non-linear pricing. To see why, suppose that under linear pricing, the seller with two units of capacity, seller 2, charges  $V_3$ . The seller with one unit of capacity, seller 1, would respond by charging  $V_3 - \epsilon$  to guarantee a sell. But, seller 2's best reply would be to slightly undercut seller 1 and sell both units. This undercutting process would take place until both prices reached  $V_3/2$  because, at that point, seller 2 would prefer to sell only one unit at a price of  $V_3$ . On the other hand, with non-linear prices, seller 2 can implicitly keep the undercutting process going by bundling his two units. This, essentially, puts the price of the first unit that each seller has to a zero price, while maintaining the price for seller 2's second unit at his monopoly price of  $V_3$ . To guarantee that the buyer buys seller 2's bundle, seller 2 raises the price of buying only a single unit to at least  $V_3$ .

#### 4.2 First period

Now we go back to period 1. There is a unique pure strategy equilibrium price paid for the goods. The equilibrium prices are for each seller to charge  $\delta V_3$  for both a single unit and two units. The period 1 actions are that the buyer either buys one good from each seller, two units from one of the sellers and none from the other, or two units from one seller and one unit from the other. The total amount paid by the buyer and the revenue that each seller receives is the same, no matter which of the three actions the buyer takes in period 1.

First, we argue that these strategy profiles constitute an equilibrium. If a seller raised his price for a single unit, then the buyer would buy both units from the other seller in period 1. The seller then becomes the only one with available capacity in period 2 and hence his overall payoff over the two periods is  $\delta V_3$ . Thus, there is no improvement in the seller's payoff. Increasing the price for two units to some level  $\tilde{p}$ , has no effect on the buyer's choice, since the buyer prefers buying one unit from each seller and obtaining payoff to paying  $\tilde{p}$  whenever  $\tilde{p} > \delta V_3$ . Thus, such a price increase cannot improve the seller's payoff. Clearly, lowering the price cannot improve a seller's payoff. Thus, this is an equilibrium.

Why are the equilibrium payoffs unique? Let us take a candidate equilibrium where each seller demanded a different price for a single unit. It is easy to show that both prices are at least  $\delta V_3$ . Then either the lower price seller could increase his price, if the buyer split his order, or the higher priced seller who got no orders could reduce his price and make a sell. So, each seller has to offer the same price for a single unit.

Suppose now that each seller demanded a price  $p_1 > \delta V_3$  for a single unit and  $p_2 \ge p_1$  for two units. If the buyer buys one unit from each seller, his payoff over both periods is

$$2V(1+\delta) - 2p_1 + \delta V_3,$$

since second period competition implies he will then get the third unit at zero price. If the buyer buys two units from the same seller, his payoff is

$$2V(1+\delta) - p_2,$$

since the seller with remaining capacity will be a monopolist in period 2 and charge  $V_3$ . The buyer's payoff is higher if he buys one unit from each seller if

$$p_2 > 2p_1 - \delta V_3$$

and he will buy two units from one of the sellers otherwise.

If  $p_2 < 2p_1 - \delta V_3$ , the buyer would accept both units from one of the sellers. The other seller would then have a payoff of  $\delta V_3$ . He could improve his payoff by lowering his price for two units and having the buyer accepting both his units, since  $p_2 \ge p_1 > \delta V_3$ . If  $p_2 > 2p_1 + \delta V_3$ , the buyer would split his order. A seller could improve his payoff by lowering the price for two units to some price less than  $2p_1 - \delta V_3$ , the buyer will accept both his units and the seller will improve his payoff.

We summarize as follows:

**Proposition 4** With a monopsonist under non-linear pricing, there are unique pure strategy equilibrium payoffs. In period 1, both sellers charge  $\delta V_3$  for both a single unit and two units and the buyer buys either two or three units. Period 2 equilibrium is as stated in Lemma 5.

The equilibrium involves a two-part tariff with the fixed fee equal to a seller's discounted monopoly profit in the next period and all units are priced at marginal cost, which we have normalized to 0. This equilibrium was not possible with linear prices, because each seller would have an incentive to raise his price to induce the buyer to split his order.

Remark. Unlike the case when linear prices are used, under non-linear prices the sellers make no positive rents: their equilibrium payoffs,  $\delta V_3$ , are equal to the profit from satisfying the residual demand, after the buyer bought the other seller's capacity. Further, the buyer has no incentive to commit to not making purchases in period 2, and to hire an agent to commit to buy only a single unit, and has no incentive to vertically integrate by buying one of the sellers. The reason for these results in the monopsony case is that with non-linear pricing the sellers can price "as if" all purchases are done in a single period. This makes competition more stiff and results in a better outcome from the buyer's point of view.

## 5 Duopsony

Now, we study strategic issues raised when there are two buyers. Each buyer has the same demand as in the monopsony case, and each seller has now doubled his capacity to four units. A buyer coordination issue which was not present in the monopsony case is now present. First, we examine linear pricing. Then we show that a non-linear pricing equilibrium induces the same payoff as the linear pricing equilibrium that we focus on.

## 5.1 Linear pricing - second period

We first consider equilibrium behavior in the possible period-two subgames. Mixed strategy equilibria arise in most cases (unless either both sellers can cover the market or one seller is a monopolist). The construction of equilibria is similar to that for the monopsony case and their sketch is in Appendix A4. The key results from the period 2 analysis, required for our subsequent analysis of period 1, are summarized in the following Lemma.

**Lemma 6** In the second period of a duopsony under linear pricing (i) the highest expected payoff for a seller with full capacity is  $2V_3$ ; (ii) If one of the buyers bought 2 units in period 1 and the other buyer bought none, then the expected price is more than  $V_3/2$  per unit in period 2; (iii) If each buyer bought 2 units in period 1, with one of the sellers selling 3 units and the other 1 unit, then the price in period 2 exceeds  $V_3/2$ .

## 5.2 Linear pricing - first period

Now, we go to period 1. As in the monopsony case, there will be no pure strategy equilibrium, but the equilibrium behavior will have a very different flavor. We focus on the following equilibrium. Each seller asks  $\delta V_3$  for each unit and the buyers mix with equal probability between buying two units from either seller. First, we argue why this is an equilibrium and then discuss its properties.

Suppose that one seller (say seller L) is charging  $p_L$  per unit and the other seller (seller H) is charging  $p_H$ . Suppose that the buyers mix between buying two units from seller L with probability  $\alpha$  and buying two units from seller H with probability  $(1 - \alpha)$ . The payoff of a buyer buying from seller L is

$$2V(1+\delta) - 2p_L + (1-\alpha)\delta V_3.$$

This expression is derived as follows. With probability  $\alpha$ , the other buyer also buys from seller L, in which case only seller H will have available capacity in period 2 and, acting as a monopolist will leave the consumers with zero surplus. With probability  $1 - \alpha$ , the other buyer buys from seller H, in which case there is Bertrand competition between the two sellers in period 2, leaving surplus  $V_3$ to each of the buyers. Similarly, the payoff of a buyer buying from seller H can be calculated to be

$$2V(1+\delta) - 2p_H + \alpha \delta V_3.$$

Thus, for the buyer to be indifferent between buying from seller L and H we must have

$$\alpha = \frac{p_H - p_L}{\delta V_3} + \frac{1}{2}.$$

Note, that at equal prices buyers mix with probability 1/2. We will argue later why the buyers cannot coordinate perfectly and choose different sellers even if the prices are the same.

Let  $f(p_L)$  be the density of prices offered by seller L which range in some interval  $\underline{p}_L$  to  $\overline{p}_L$ . Seller H's expected payoff over the two periods is

$$\pi_H(p_H) = 2\delta V_3 \int_{\underline{p}_L}^{\overline{p}_L} \alpha^2 f(p_L) dp_L + 2p_H \int_{\underline{p}_L}^{\overline{p}_L} 2\alpha (1-\alpha) f(p_L) dp_L + 4p_H \int_{\underline{p}_L}^{\overline{p}_L} (1-\alpha)^2 f(p_L) dp_L$$

The first term is the profit if both buyers buy from seller L in period 1, the second if the buyers buy from different sellers, and the third if both buyers buy from seller H. Differentiating with respect to  $p_H$ , we find  $p_H = \delta V_3$  for any density f(p). This is a local maximum, since  $\pi_H$  is strictly concave in  $p_H$ . Furthermore, it is easy to show that no deviation by a seller that makes the probability of acceptance either 0 or 1 will improve his payoff, which can be calculated in equilibrium to be equal to  $\frac{5\delta V_3}{2}$ . Hence, this is a global maximum for seller H. Since the same argument can be used for seller L, it is a unique best response for the sellers to each ask  $\delta V_3$ , given the buyers' purchasing strategies. In particular no seller can improve his payoff by deviating given the buyers' symmetric mixing probability as characterized by  $\alpha$ .

We also need to make sure that each buyer is acting optimally, given the strategies of the other players. Suppose that buyer 1 is following the putative equilibrium strategy. The payoff for buyer 2 from following the putative equilibrium strategy (that is, to "not split" his order - hence indexed by ns) is

$$S_{ns} = 2V(1+\delta) - 2\delta V_3 + 1/2\delta V_3, \tag{3}$$

calculated as follows: the buyer obtains 2 units in period one, of total value  $2V(1 + \delta)$ , paying  $\delta V_3$  for each of these units; with probability 1/2 both buyers buy their first period 2 units from the same seller and then their period two surplus is zero because the other seller has become a monopolist; with probability 1/2 the two buyers buy 2 units in period one from different sellers and then Bertrand competition in period two implies each buyer enjoys surplus  $V_3$ . Let us now compare this payoff to that of buyer 2 if he "split" his order (hence indexed by s), by buying one unit from each seller, which is

$$S_s = 2V(1+\delta) - 2\delta V_3 + \delta (V_3 - Ep_2).$$

This payoff is calculated as follows. Again, the buyer obtains 2 units in period one for a total net surplus of  $2V(1+\delta) - 2\delta V_3$ ; now, entering period two, one seller has sold three units and the other one unit: buyer 2 then obtains one unit at a net surplus of  $V_3 - Ep_2$ , where  $Ep_2$  is the expected

price that the buyer will have to pay in period 2, given 1 unit of remaining capacity for one seller and three for the other. Payoff  $S_{ns}$  is larger than  $S_s$ , since  $Ep_2 > \frac{\delta V_3}{2}$  by Lemma 6. It is also easy to show that the buyer prefers to buy two units in period 1 to any other quantity. Thus, we have an equilibrium.

We summarize as follows:

**Proposition 5** In the linear pricing duopsony game, there is a symmetric equilibrium. In period 1, each seller demands  $\delta V_3$  per unit and the buyers mix equally between buying two units from one or the other seller. In period 2, the prices are 0 per unit if the buyer bought from different sellers and  $V_3$  if they bought from the same seller.

Properties of the equilibrium. Each seller obtains higher profit than what he would get by not selling any units in period 1, and then becoming a monopolist in period 2. Such a strategy would generate a profit of  $2\delta V_3$ , which is less than the equilibrium profit of  $\frac{5\delta V_3}{2}$ . This is because the buyers are mixing their purchasing decisions between the sellers. By mixing, there is the possibility that the sellers sell 0 or 4 units with probability 1/4 and selling 2 units with probability 1/2. This is desirable from the sellers' points of view, since if they do not make a sell in period 1, then they become a monopolist in period 2 and receive a payoff of  $2\delta V_3$ . Thus, the sellers like that the buyers do not coordinate their behavior.

Thus, we need strategic uncertainty from the sellers' points of view to get them to charge prices equal to  $\delta V_3$ . In contrast, it was not possible to have strategic uncertainty when there was only a single buyer. This is because of two reasons. First, the single buyer did not have anyone to "miscoordinate" with on purchases. Second, if the buyer mixed, then a seller could improve his payoff by changing his price. Thus, sellers must mix to get an equilibrium with a single buyer.

From the above analysis (in particular, from the calculated sellers' expected equilibrium profit), we obtain two Corollaries, similar to these stated in the monopsony case.

#### **Corollary 4** The buyers have a strict incentive to commit not to buy in the future.

Using similar arguments as in the monopsony model each seller's equilibrium profit and, thus, each buyer's equilibrium payment, if buyers only buy in period 1 is  $2\delta V_3$ . This is lower than the equilibrium payment,  $\frac{5\delta V_3}{2}$ , when purchases can be made (potentially) over both periods.

Also,

#### **Corollary 5** A buyer has a strict incentive to vertically integrate with a seller.

Suppose that a buyer unilaterally buys a seller. This increases his expected profit because a buyer can buy a seller, by paying the equilibrium profit of  $\frac{5\delta V_3}{2}$ . This is the buyer's expected cost. He can then satisfy his demand for 3 units and have one extra unit left that he can supply to the other buyer and obtain an additional positive profit.

It is important to note that it would *not* be an equilibrium if the buyers coordinated their behavior by either buying from different sellers with probability 1 or by splitting their orders, buying one unit each from each seller. Such a behavior would be in the buyers' best interests, given the sellers' prices, since then they would pay  $2\delta V_3$  in period 1 and nothing for the third unit in period 2. Essentially, the buyers would avoid the cost of miscoordinating associated with choosing the same seller in period 1, thus creating a monopoly in period two. However, this coordination by buyers cannot be part of an equilibrium. Suppose that buyers did coordinate their behavior when the sellers each asked  $\delta V_3$  per unit. A seller could raise his price by  $\epsilon$  and improve his payoff. There are two possible responses by buyers to this deviation. They could make the same purchasing decisions as before, either splitting or each buying from one seller with probability one. Then, in either case, this improves the deviator's payoff. Or, the buyers could mix their purchasing decisions.<sup>20</sup> In this case, the seller would sell either 0, 1, 2, 3, or 4 units. If he sells a positive number of units in period 1, he improves his payoff, since the payoff is at least  $2\delta V_3 + \epsilon$ . This is because, if a seller sells only one unit in period 1, his payoff is  $\delta V_3 + \epsilon$  in period 1 and  $\delta V_3$  in period 2. If he sells no units, then his payoff is  $2\delta V_3$ . Since the buyers must be mixing, the expected number of units sold by the seller is positive, thus he improves his payoff by such a deviation.<sup>21</sup> Further, it is easy to show that if the sellers offered the same price, but one that was different than  $\delta V_3$ , than at least one of the sellers could improve their payoff by defecting.

It is interesting to examine the equilibrium market shares of the firms. In period 1, there is a 50% chance that both sellers sell two units and a 50% chance that one seller sells all his units. If both sellers sell two units, then the equilibrium market shares for the entire game are "essentially symmetric," since both will set a price of zero in period 2. If in the first period one seller sold 4 units, then the other will sell two units in period 2. Thus, the final market shares can be quite asymmetric for the entire game with 50% probability.

<sup>&</sup>lt;sup>20</sup>Note that there is never an equilibrium where both buyers would buy from the same seller with probability one, since a buyer would do better by accepting the other seller's offer.

 $<sup>^{21}</sup>$ Note that it has to be the case that, in equilibrium, each buyer buys two units in period 1.

One may also wonder if there is an asymmetric pure strategy equilibrium where each buyer buys two units from a different seller. We now argue why this is not the case. For a buyer to be willing to buy two units from the high priced seller charging  $p_H$  instead of two units from the low priced seller charging  $p_L$ , it must be the case that

$$2p_H \le 2p_L + \delta V_3$$

since the price for the third unit in period 2 is 0 if the buyer buys from seller H and  $\delta V_3$  if he buys from seller L. Thus,  $p_L$  must be at least

$$p_H - \delta V_3/2 \le p_L$$

The low priced seller must not be willing to lower his price that induces both seller to buy two units from him in period 1. The price of the low priced seller must satisfy

$$2(2p_H - \delta V_3) \le 2p_L,\tag{4}$$

otherwise, the low priced seller could deviate and get both buyers to buy from him. Clearly, to satisfy condition (4) one would want to minimize the difference between  $p_H$  and  $p_L$ . But, by substituting  $p_L = p_H$  into (4) we find that  $p_H$  must be lower than  $\delta V_3$ . Since there is no equilibrium where prices are below  $\delta V_3$ , otherwise a seller could profitably defect, there can be no asymmetric pure strategy equilibrium. A similar style argument can be used to show that there is no pure strategy asymmetric equilibrium where buyers split their orders.

There is another equilibrium where both sellers set the same price and the buyers mix between buying two units from each seller or splitting their orders and getting one unit from each. We present this equilibrium in Appendix A5. It gives the sellers a lower profit than the one that we have just derived but, importantly, it still gives them a profit greater than  $2\delta V_3$ . Furthermore, in this modified equilibrium, the key implication concerning the asymmetry of market shares continues to hold.

This result, that market share asymmetries should be expected with significant probability when there are two buyers, is complementary to other studies of asymmetries in the literature - see e.g. Saloner (1987), Gabszewicz and Poddar (1997) and Besanko and Doraszelski (2004) to mention just a few. In these studies, capacity asymmetries are typically due to the selling firms' incentives to invest strategically in anticipation of a market competition stage. In contrast, in our analysis, the asymmetries are due to the fact that the buyers are large players and choose strategically which firms they should purchase from - the fact that they cannot fully coordinate their behavior in equilibrium, but need to mix, gives rises to asymmetric sellers' market shares. We now state an equilibrium when the sellers use non-linear pricing; it has a very similar structure and the same payoffs as the equilibrium with linear pricing. The equilibrium is derived in Appendix A7. We then explain why the monopsony and duopsony differ so much when comparing linear and non-linear pricing.

**Proposition 6** In the duopsony model with non-linear pricing, there is an equilibrium where each seller demands  $\delta V_3$ ,  $2\delta V_3$ , and  $\frac{8\delta V_3}{3}$  for one, two and three units, respectively, in period 1. Each buyer mixes with probability 1/2 between buying two units from either seller.

The basic idea behind the result is similar to the case with linear pricing. The complications arise due to the need to check for possible seller deviations when he can manipulate the prices for selling one, two, and three units. Thus, we have demonstrated that the equilibrium payoffs and market shares do not have to depend on whether pricing is linear or non-linear in a duopsony. This result differs dramatically from the monopsony model. The key to the duopsony result is that buyers must mix in equilibrium. This strategic uncertainty is good from the sellers' points of view, since this gives them an expected premium above satisfying the residual demand left after the other seller has sold his units. When there is only one buyer, there can be no strategic uncertainty.

## 6 Conclusion

Capacity constraints play an important role in oligopolistic competition. In this paper, we have examined markets where both sellers and buyers act strategically. Sellers have intertemporal capacity constraints, as well as the power to set prices. Buyers decide which sellers to buy from, taking into consideration that their current purchasing decisions affect the intensity of sellers' competition in the future. In the monopsony case with linear prices, capacity constraints imply that a pure strategy equilibrium fails to exist. Instead, sellers play a mixed strategy with respect to their pricing, and the buyer may split his orders. Importantly, we find that the sellers enjoy higher profits than what they would have in an one-shot interaction (or the competitive profit from satisfying residual demand). The buyer is hurt, in equilibrium, by his ability to behave strategically over the two periods, since this behavior allows the sellers to increase their prices above their rival's and still sell their products. Thus, the buyer has a strict incentive to commit not to buy in the future, or to commit to myopic, period-by-period, maximization (perhaps by delegating purchasing decisions to agents), as well as to vertically integrate with one of the sellers. When non-linear pricing is feasible, however, a pure strategy equilibrium exists under monopsony and the sellers do not obtain excess profits. Turning to the case of two buyers, we find that it is the buyers that must randomize their actions. As a result, equilibrium behavior does not need to prescribe that buyers split their orders. Furthermore, the equilibrium with non-linear pricing has the same transacted prices as with linear pricing, which was due to the strategic uncertainty that occurs when buyers mix in duopsony. We find that, since there has to be some buyers' miscoordination, the equilibrium under duopsony implies asymmetric market shares for the sellers. Also, as in the monopsony model, there are strict incentives for buyers to commit not to buy in the future or to vertically integrate.

While we have tried to keep the model as simple as possible, our qualitative results appear robust to modified formulations. Perhaps the most important ones refer to how the capacity constraints function. In the model, if a seller sells one unit today, his available capacity decreases tomorrow by exactly one unit. In some of the cases for which our analysis is relevant, like the ones mentioned in the Introduction, it may be that the capacity decreases by less than one unit, in particular, if we adopt the view that each unit takes time to build and, thus, occupies the firm's production capacity for a certain time interval. Similarly, instead of the unit cost jumping to infinity once capacity is reached, in some cases it may be that the unit cost increases in a smoother way: cost curves that are convex enough function in a way similar to capacity constraints. We believe the spirit of our main results is valid under such modifications, as long as the crucial property that by purchasing a unit from a seller you decrease this seller's ability to supply in the subsequent periods holds.

This is, to our knowledge, the first paper that considers capacity constraints and buyers' strategic behavior in a dynamic setting. A number of extensions are open for future work. While non trivial, these present theoretical interest and, at the same time, may make the analysis more directly relevant for certain markets. First, one may wish to examine the case where the products offered by the two sellers are differentiated. Is there a distortion because buyers strategically purchase products different from their most preferred ones, simply with the purpose of intensifying competition in the future? A second interesting extension is when the sellers have asymmetric initial capacities. Which seller sells faster? Do buyers have an incentive to favor a seller with a larger or with a smaller remaining capacity? Is competition more intense when capacities are more or less symmetric? A third extension, based on the one described just above, is that of endogenizing the sellers' capacities. Previous work has examined this issue as a two-stage game, where capacities are chosen first and then firms compete for the final demand. However, regardless of whether final-stage competition is in quantities (e.g. Kreps and Scheinkman, 1983) or in prices (e.g. Allen, Deneckere, Faith and Kovenock, 2000), the issue of strategic buyers has not been treated. Fourth, it may be interesting to extend the intuition from our model with two buyers to the case of a larger number of buyers. Our analysis already makes clear how the presence of buyer competition (moving from monopsony to duopsony) significantly affects the equilibrium behavior - the multiple buyers (possible also with multiple sellers) case may offer some additional insights. However, calculating the probabilities of coordinating at different outcomes, given independent purchasing randomizations, would become increasingly involved. Finally, in our model, price determination takes the form that sellers set prices (linear or non-linear) in each period. Alternative pricing formulations are also possible. For instance, sellers may be able to make their prices dependent on the buyers' purchasing behavior e.g. by offering a lower price to a buyer that has not purchased in the past (or does not currently purchase) a unit from the rival seller. Our setting may allow us to examine such "loyalty discounts." The buyers may also participate more actively in the determination of the prices, e.g. in principle these could be determined by bargaining between the two sides of the markets.

# Appendix

# Appendix A1: Proof of Lemma 2.

First, we argue that the players choose prices in the interval  $\begin{bmatrix} V_3 \\ 2 \end{bmatrix}$ . Suppose that seller 2, asked a price p less than  $V_3/2$ . If seller 1 charges a price less than p, then seller 2 will sell 1 unit, while if seller 1 charges a price higher than p, seller 2 sells 2 units. Seller 2 could improve his payoff no matter what prices seller 1 asks by asking for  $V_3 - \epsilon$  for  $\epsilon$  very small and selling at least one unit for sure, since  $V_3 - \epsilon > 2p$ . Since seller 2 will charge a price of at least  $V_3/2$ , then so will seller 1; otherwise, seller 1 could increase his price and still guarantee a sell of 1 unit. Thus, both sellers charge at least  $V_3/2$ . Now, we argue that price will be no more than  $V_3$ . Take the highest price p offered in equilibrium greater than  $V_3$ . First, assume that there is not a mass point by both sellers at this price. This offer will never be accepted by the buyer, since he will always buy the second unit from the lower priced seller and his valuation for a third unit is  $V_3 < p$ . The seller could always improve his payoff by charging a positive price less than  $V_3/2$ . Second, if there is a mass point by both sellers, then at least one of them is rationed with positive probability and a seller can slightly undercut his price and improve his payoff. Thus, all prices will be between  $V_3/2$  and  $V_3$ .

Now, we argue that the expected equilibrium period 2 payoffs are  $V_3/2$  for seller 1 and  $V_3$  for seller 2. Given that the equilibrium prices are between  $V_3/2$  and  $V_3$ , we know that the profits for seller 1 is at least  $V_3/2$  and for seller 2 at least  $V_3$ . First, we argue that it can never be the case that *both* sellers will have an atom at the highest price  $p_H$ ; later we further show that seller 2 will have a mass point at  $p_H$ . If both did, then there is a positive probability of a seller being rationed, and a seller could improve his payoff by slightly lowering his price. Thus, a seller asking  $p_H$  knows that he will be the highest priced seller. If he is seller 1 he will not make a sell, while if he is seller 2 he will make a sell of one unit. If seller 2 charges  $p_H$  he knows that his payoff will be  $p_H$ , thus  $p_H$ must equal  $V_3$ . If the lowest price offered in equilibrium,  $p_L$ , were greater than  $V_3/2$ , then seller 2 could improve his payoff by offering  $p_L - \epsilon > V_3/2$ , with the buyer buying two units from the seller and thus improve his payoff above  $V_3$ . Thus, the lowest price is  $V_3/2$ . Since both sellers must offer this price, seller 1's expected payoff must be  $V_3/2$ .

We now find the equilibrium price distributions. Let  $F_i$  be the distribution of seller i's price offers. Seller 1's price distribution is then determined by indifference for seller 2:

$$p[F_1(p) + 2(1 - F_1(p))] = V_3, \tag{A1.1}$$

since seller 2's expected payoff is  $V_3$  by the earlier argument. Seller 2's payoff is calculated as

follows. When seller 2 charges price p, then with probability  $F_1(p)$  seller 1's price is lower and seller 2 sells one unit, while with probability  $1 - F_1(p)$  seller 1's price is higher and seller 2 sells both his units. Solving equation (A1.1), we obtain:

$$F_1(p) = 2 - \frac{V_3}{p}.$$

Seller 2's price distribution is a little more complicated. For  $p < V_3$ , it is determined by

$$p[1 - F_2(p)] = \frac{V_3}{2}.$$
 (A1.2)

Seller 1 sells one unit if his price is lower than the rival's and this happens with probability  $1-F_2(p)$ ; otherwise, he sells no units. This equals seller 1's expected profit  $V_3/2$  by Lemma 2. Condition (A1.2) implies

$$F_2(p) = 1 - \frac{V_3}{2p}.$$

There is a mass of 1/2 at price  $V_3$ . Simple arguments can be used to establish that the equilibrium pricing distributions must be continuous and that the only mass point may be located at  $V_3$  for seller 2.

## Appendix A2: Proof of Proposition 1.

First, we define some notation and buyer payoffs. Then we proceed to prove the proposition in a series of Lemmas. Suppose that the prices in period 1 are  $p_H$  and  $p_L$ , with  $p_H \ge p_L$ . Note that pricing in period one could, in principle, be determined via either pure or mixed strategies. In the former case,  $p_H$  and  $p_L$  are the prices set by the two sellers, whereas in the latter these are realizations of the mixed strategies. We use Lemma 3 in computing the payoffs.

The buyer's payoff if he buys one unit from each of the firms in period 1 is

$$W_1 \equiv 2V(1+\delta) - p_H - p_L + \delta V_3.$$

In this case, the buyer gets one unit for free in the following period (since competition drives the price to zero).

The buyer's payoff if he buys both units from firm L is

$$W_2 \equiv (1+\delta)2V - 2p_L.$$

In this case, the buyer faces a monopolist and pays  $V_3$  in period 2.

The buyer's expected payoff if he buys only one unit from firm L is

$$W_3 \equiv (1+\delta)V - p_L + \delta \left[ V + V_3 - E \min[p_1, p_2] - E p_2 \right].$$

In the following period, the buyer will buy two additional units. He will pay the lowest price offered in the following period for the second unit and will buy the third unit from the seller who has two units of capacity in period 2, since either he is the low priced seller or the other seller has no more capacity.

The buyer's expected payoff if he buys two units from the lowest priced seller and one from the highest priced seller is

$$W_4 \equiv 2V(1+\delta) - 2p_L - p_H + \delta V_3.$$

Now, the series of Lemmas. These are numbered (A1-A5) independently from the Lemmas in the main body of the paper.

Lemma A1 The sellers set strictly positive prices in period 1.

**P roof.** If a seller set a price of 0, then either the buyer would buy two units from that seller or one from each of the sellers. In either case, the seller makes 0 profit. If the buyer buys two units from that seller, then the seller can sell no more in period 2. If the seller sells one unit, then the buyer must have bought one unit from the other seller, since  $W_1 > W_3$  at a 0 price in period 1. The seller could raise his price so that he gets no sells in period 1, and improve his payoff by Lemma 2.

It follows directly that:

Lemma A2 The buyer never buys three units in period 1.

**P roof.** By Lemma A1, both prices are positive. Since  $W_1 > W_4$  if  $p_L > 0$ , then buying two units always dominates buying three units.

We now argue in the following three lemmas that no price will be above  $V + \delta [E \min[p_1, p_2] + Ep_2] \equiv p^C$  from which the Proposition will be proven.

**Lemma A3** The buyer prefers to buy one unit from each of the sellers instead of only one unit from the low price seller if  $p_H \leq p^C$ . Thus the buyer will not buy any units from a seller charging  $p_H > p^C$ , when  $p_H > p_L$ .

**P roof.** Compare  $W_1$  and  $W_3$ .

Lemma A4 In any equilibrium, each price offered by each seller in period 1 is an offer which results in his selling at least one unit with positive probability.

**P roof.** Let  $\overline{p}$  be the highest price offered in any (possibly mixed strategy) equilibrium by a

seller. Suppose that in equilibrium  $\overline{p}$  is never accepted. By Lemma 3 the seller's expected payoff of making this offer is  $\delta V_3$ . Let  $\underline{p}$  be the lowest price offered in equilibrium by the other seller. By Lemmas 3 and A1 if the difference between the two prices set is less than  $\delta V_3$ , then the highest price will always be accepted, as long as it is less than  $p^C$ . By Lemma 1,  $\underline{p} > 0$ . A player offering  $\overline{p}$ can defect and offer a price p that is the minimum of  $[\underline{p}+\delta V_3, p^C]$  and know that it will be accepted, and increase his payoff. Thus, no offer is made that is always rejected.

**Lemma A5** In any equilibrium, no seller will offer a price above  $p^{C}$ .

**P roof.** Let  $\overline{p_i}$  be the highest price offered in any equilibrium by seller *i*. Suppose that  $\overline{p_i} \ge \overline{p_j}$ ,  $i \ne j$ , and  $\overline{p_i} > p^C$ . If  $\overline{p_i} > \overline{p_j}$ , then seller *i*'s offer will never be accepted by Lemma A3. Then seller *i*'s payoff is  $\delta V_3$ . Seller *i* can clearly improve his payoff by making an offer of  $\delta V_3 + \epsilon$  (note that  $\delta V_3 + \epsilon < p^C$ ). Suppose now that  $\overline{p_i} = \overline{p_j} \equiv \overline{p}$ . There could not be a mass point at  $\overline{p}$  by each seller, since only one unit will be bought and that seller could increase his payoff by a slight undercut in price. If there is no mass at  $\overline{p}$ , then there is no possibility that the offer will be accepted. But, this contradicts Lemma A4.

Thus, we have proved Proposition 1.

## Appendix A3: Proof of Proposition 2.

Suppose we have a pure strategy equilibrium with prices  $p_H$  and  $p_L$ , where  $p_H \ge p_L$ . A pure strategy equilibrium could exist only if both sellers offered  $p^C$  and the buyer was purchasing a unit from each seller. At any other price, at least one of the sellers could defect and improve their payoff. To see this, we need to look at various cases. First, suppose that the lower offer in equilibrium,  $p_L$ , is greater than  $\delta V_3$ . If  $p_L > p_H - \delta V_3$ , then the buyer will split his order by Lemma 4. Seller L could improve his profit by increasing his offer. If  $p_L < p_H - \delta V_3$ , then the buyer will buy both units from seller L. Seller H will have a payoff of  $\delta V_3$ . Seller H can improve his payoff by making an offer that is accepted. If  $p_L = p_H - \delta V_3$ , then either seller L is not selling 2 units or seller His not selling any units. One of the sellers has an incentive to defect. To see this, suppose that  $\alpha \in [0, 1]$  is the probability that the buyer splits his order between the sellers. Then the payoff to seller L is  $\pi_L = \alpha p_L + (1 - \alpha) 2p_L$ . The payoff to seller H is  $\pi_H = \alpha p_H + (1 - \alpha) \delta V_3$ , which equals  $\pi_H = \alpha p_L + \delta V_3 - \epsilon$  for all positive  $\epsilon$ , since he could always guarantee an acceptance by dropping his price  $\epsilon$ . Thus,  $\alpha$  would have to equal 1. But, if  $\alpha = 1$ , then seller L could improve his payoff by raising his price. Now, suppose that  $p_L \leq \delta V_3$ . If  $p_L > p_H - \delta V_3$ , then the buyer will split his order by Lemma 4. Seller *L* could improve his profit by increasing his offer. If  $p_L < p_H - \delta V_3$ , then the buyer will buy both units from seller *L*. If  $\frac{\delta V_3}{2} < p_L$ , then seller *H* can improve his payoff by making an offer that is accepted. If  $p_L < \frac{\delta V_3}{2}$ , then seller *L* could raise his offer to  $p_H$  the buyer will split his order and the low seller's profit increases. As before, if  $p_L = p_H - \delta V_3$ , then one of the sellers could do better by defecting.

An equilibrium with both sellers offering  $p^C$  could arise only if  $p^C \ge 2(p^C - \delta V_3)$  or if  $2\delta V_3 \ge p^C$ . This is because a defection by a seller that gets the buyer to buy two units from the seller will reduce his profits. Otherwise a seller would defect. This is equivalent to  $2\delta V_3 \ge V_2 + \delta [E \min[p_1, p_2] + Ep_2]$ . But, this condition never can hold, since both  $p_1$  and  $p_2$  are greater than  $V_3/2$ .

## Appendix A4: Proof of Proposition 3.

We know that  $\overline{p} - \underline{p} \geq \delta V_3$ ; otherwise a seller could increase his payoff by moving mass from lower parts of the price distribution to higher parts and still get accepted. Suppose that  $\overline{p} - \underline{p} < 2\delta V_3$  and for now assume that the equilibrium price distribution is continuous. Define three regions as follows: region 1 where  $p \in [\underline{p}, \overline{p} - \delta V_3]$ , region 2 where  $p \in [\overline{p} - \delta V_3, \underline{p} + \delta V_3]$  and region 3 where  $p \in [\underline{p} + \delta V_3, \overline{p}]$ . A price offered in region 1 will be accepted for either 1 or for 2 units. A price in region 2 will always be accepted for 1 unit. A price in region 3 will be accepted either for a single unit or no units. But, if there is an offer in region 2, then a seller can always improve his payoff by moving all the probability mass in region 2 to a price of  $\underline{p} + \delta V_3$ . Thus, there would be a gap in the price offer distribution.

Suppose that there was a gap in the price offer distribution in region 2. Then prices offered in region 1 would all be moved to the top of region 1 at a price of  $\overline{p} - \delta V_3$ , since whether the offer is accepted either once or twice is independent of the price in region 1. But, if sellers move up all their mass to  $\underline{p} + \delta V_3$ , then the price distribution would only be  $\delta V_3$ , but then any price in the interior distribution is inferior to either a price at the bottom or the top of the distribution. Thus, we would have a two-point distribution. But this cannot be an equilibrium. Suppose one player made an offer of  $\underline{p}$  and the other at  $\overline{p}$ . Then either the buyer accepts 2 units at the low price or splits his order. In the former case, the high bidder could increase his payoff by reducing his offer slightly, while in the latter case the low bidder could increase his payoff by a bid reduction.

Let  $\pi$  be the equilibrium payoff. The equilibrium pricing equations are as follows. If p ,

$$p[2 - F(p + \delta V_3)] = \pi.$$
 (A4.1)

If  $p + \delta V_3 ,$ 

$$\delta V_3 F(p - \delta V_3) + P \left[2 - F(p + \delta V_3) - F(p - \delta V_3)\right] = \pi.$$
(A4.2)

If  $p > \overline{p} - \delta V_3$ ,

$$\delta V_3 F(p - \delta V_3) + P(1 - F(p - \delta V_3)) = \pi.$$
(A4.3)

Some further important facts about the equilibrium follow.

Substituting  $p = \underline{p}$  and  $p = \underline{p} + \delta V_3$  into (A4.1), setting the two resulting values equal to each and manipulating the equation, we obtain

$$\delta V_3 \left[ 2 - F(\underline{p} + 2\delta V_3) \right] = \underline{p} \left[ F(p + 2\delta V_3) - F(p + \delta V_3) \right].$$

Since  $2 - F(\underline{p} + 2\delta V_3) \ge 1$  and  $F(\underline{p} + 2\delta V_3) - F(\underline{p} + \delta V_3) < 1$ , it must be the case that  $\underline{p} > \delta V_3$ . This completes the proof.

Appendix A5: Equilibrium behavior in the second-period subgames of the duopsony model under linear pricing.

We need to examine the equilibrium behavior in the various second-period cases (subgames). Before proceeding to each of these cases, it is useful to provide some unified treatment of such cases. This generalizes the analysis presented for Lemma 2 to *any* case where there is a low-capacity seller that cannot cover the demand and a high-capacity seller that can cover the demand, including the case where there are multiple (two, here) buyers. The steps in the analysis are the same as the ones presented for Lemma 2. We can then state:

**Result (mixed strategy equilibria).** Suppose that in period 2 the buyers have value for B units and the capacity of the low-capacity seller is C, with C < B. Then there is no pure strategy equilibrium. In the unique mixed strategy equilibrium, the high-capacity seller's profit is  $V_3(B-C)$  and the low capacity seller's profit is  $C\frac{V_3(B-C)}{B}$ . The support of the prices is from  $V_3(B-C)/B$  to  $V_3$ .

Note that given the structure of demand and capacity, the situation described here will always be the case whenever we have asymmetric capacities in period 2: the low capacity seller's capacity will be strictly lower than the demand while the high capacity seller's capacity will be at least as high as the demand.

To understand the above result note that the high-capacity seller's security profit, is  $V_3(B-C)$ . This is so because the low-capacity seller can supply only up to C of the B units that the buyer demands and the buyer is willing to pay up to  $V_3$ . This high-capacity seller's security profit puts a lower bound on the price offered in period 2. Given the high-capacity seller can sell at most B units (that is the total demand), he will never charge a price below  $V_3(B - C)/B$ , since a lower price would lead to profit lower that his security profit (that seller could sell at most B units and would do better to sell (B - C) units at a price of  $V_3$ ). Since the high-capacity seller would never change a price below  $V_3(B - C)/B$ , this level also puts a lower bound on the price the low-capacity seller would charge and, as that seller has C units he could possibly sell, his profit becomes  $C\frac{V_3(B-C)}{B}$ . The details of the formal proof are identical to Lemma 2.

Now we turn to each of the second-period subgames. We can employ the Result stated just above to obtain the characterization of the equilibria.

Each buyer bought two units in period 1. If each of the two sellers sold two units in period 1, then the equilibrium has both sellers charging 0 in period 2. If one seller sold 4 units in period 1, then the equilibrium is for the other seller to charge  $V_3$  and for each buyer to buy a unit. If one of the sellers sold 3 units in period 1, and the other sold 1 in period 1, then there is a unique mixed strategy equilibrium. Let seller 1 be the seller who has 1 unit of capacity in period 2 (that is, sold 3 units in period 1) and seller 3 be the one who has three units of capacity in period 2 (that is, sold 1 unit in period 1). The "security" profits (see also Lemma 3), which are the unique equilibrium profits for seller 1 and 3, are  $V_3/2$  and  $V_3$ . Denoting by  $F_1$  and  $F_3$  the distribution functions employed at the mixed strategy equilibrium by the two firms, these satisfy the conditions

$$p_1 \left[ 1 - F_3(p_1) \right] = \frac{V_3}{2}$$

and

$$p_3 \left[ 2 \left( 1 - F_1(p_1) \right) + F_1(p_3) \right] = V_3,$$

where the prices are from  $\left[\frac{V_3}{2}, V_3\right]$ . It follows that seller 1's distribution is  $F_1(p_1) = 2 - \frac{V_3}{p_1}$ . As in the monopsony case of Lemma 2, seller 3's distribution will have a mass point at  $V_3$ : it satisfies  $F_3(p_3) = 1 - \frac{V_3}{2p_3}$ , with a mass point of 1/2 at  $V_3$ .

One seller sold 2 units in period 1 and the other sold 1 unit in period 1, with each buyer buying at least one unit. Let seller 2 be the seller that has 2 units of capacity remaining and seller 3 have 3 units of capacity. The equilibrium profits are  $2V_3/3$  for seller 2 and  $V_3$  for seller 3. Now, the equilibrium pricing equations satisfy

$$2p_2\left[1 - F_3(p_2)\right] = \frac{2V_3}{3}$$

and

$$p_3 \left[ 3 \left( 1 - F_2(p_3) \right) + F_2(p_3) \right] = V_3,$$

where the prices are from  $\left[\frac{V_3}{3}, V_3\right]$  and are distributed according to  $F_2(p_2) = \frac{3}{2} - \frac{V_3}{2p_2}$  and  $F_3(p_3) = 1 - \frac{V_3}{3p_3}$ , with a mass point of 1/3 at  $V_3$ .

Each seller sold 1 unit in period 1 and each buyer bought one unit. Then, the equilibrium prices would be distributed on  $\left[\frac{V_3}{3}, V_3\right]$  and satisfy

$$p[3(1-F(p))+F(p)] = V_3$$

or,

$$F(p) = \frac{3}{2} - \frac{V_3}{2p},$$

with each seller having an expected payoff of  $V_3$ .

One seller sold two units and the other none, and either each of the two buyers bought one unit or one buyer bought two units. Then the equilibrium payoff of the seller who sold no units is  $2V_3$ , while the equilibrium payoff of the other seller is  $V_3$ .

One seller sold 1 unit and the other none. Suppose that Seller 3 sold 1 unit in period 1, he has 3 units of capacity in period 2, and seller 4 sold none in period 1, he has 4 units of capacity in period 2. Then, the equilibrium prices are distributed on  $\left[\frac{V_3}{2}, V_3\right]$  and the corresponding distributions satisfy

$$p_3[3(1 - F_4(p_3)) + F_4(p_3)] = 3V_3/2$$

and

$$p_4 \left[ 4(1 - F_3(p_4)) + 2F_3(p_4) \right] = 2V_3.$$

Seller 3's price distribution is  $F_3(p_3) = 2 - \frac{V_3}{P_3}$  and seller 4's distribution is  $F_4(p_4) = \frac{3}{2} - \frac{3V_3}{4p_4}$  with a mass point of 1/4 at  $V_3$  for  $F_4$ . The equilibrium payoffs are  $3V_3/2$  for seller 3 and  $2V_3$  for seller 4.

Neither seller sold a unit in period 1. Then the equilibrium price distribution for each seller satisfies

$$p_4 \left[ 4(1 - F(p_4)) + 2F(p_4) \right] = 2V_3$$

or,

$$F(p_4) = 2 - \frac{V_3}{p_4}$$

on  $\left[\frac{V_3}{2}, V_3\right]$ . The equilibrium expected payoffs are  $2V_3$ .

Appendix A6: A modified equilibrium in the duopsony model under linear pricing.

Here, we present another equilibrium in the duopsony model under linear pricing. This has a very similar flavor as the one we have focused on in the main text of the paper: each buyer buys with some probability two units from (either) one of the sellers and with some probability splits his order.

Let the prices be  $p_L$  and  $p_H$  and let  $\alpha_i$  be the probability that a buyer buys two from seller *i*, with  $(1 - \alpha_L - \alpha_H)$  be the probability that the buyer splits his order

Each buyer's payoff is as follows. If a buyer buys 2 units from seller L, his payoff is

$$2V(1+\delta) - 2p_L + (1-\alpha_L)\delta V_3 - (1-\alpha_L - \alpha_H)\delta E_2,$$
(A6.1)

where  $E_2$  is defined as the expected second-period price if, in period one, one seller sells 3 units and the other 1 unit. If a buyer buys 2 units from H, his payoff is

$$2V(1+\delta) - 2p_H + (1-\alpha_H)\delta V_3 - (1-\alpha_L - \alpha_H)\delta E_2.$$
 (A6.2)

If he splits his order, his payoff is

$$2V(1+\delta) - p_L - p_H + \delta V_3 - (\alpha_L + \alpha_H)\delta E_2.$$
(A6.3)

For the buyer to be indifferent among the three alternatives we must have that, by combining expressions (A6.1) and (A6.2),

$$\alpha_L = \frac{2(p_H - p_L)}{\delta V_3} + \alpha_H,\tag{A6.4}$$

and, by combining (A6.2) and (A6.3),

$$\alpha_H = \frac{V_3(p_H - p_L + \delta E_2) - 4(p_H - p_L)E_2}{(4\delta E_2 - \delta V_3)V_3}$$

Note that  $\frac{\partial \alpha_H}{\partial p_H} = -\frac{1}{\delta V_3}$  and that when  $p_L = p_H$ ,

$$\alpha_H = \alpha_L = \frac{E_2}{4E_2 - V_3}.$$

Now let us turn to the sellers. Expected profit for seller H is:

$$\pi_{H} = 2\delta V_{3}\alpha_{L}^{2} + 4p_{H}\alpha_{H}^{2} + 2p_{H}[(1 - \alpha_{L} - \alpha_{H})^{2} + 2\alpha_{L}\alpha_{H}] + (p_{H} + \delta V_{3})2\alpha_{L}(1 - \alpha_{L} - \alpha_{H}) + (3p_{H} + \delta V_{3})2\alpha_{H}(1 - \alpha_{L} - \alpha_{H}) =$$

$$= \delta V_3(2\alpha_L - 4\alpha_L\alpha_H + 2\alpha_H - 2\alpha_H^2) + 2p_H(1 - \alpha_L + \alpha_H).$$

Dividing by 2, this becomes

$$\delta V_3(\alpha_L - 2\alpha_L\alpha_H + \alpha_H - \alpha_H^2) + p_H(1 - \alpha_L + \alpha_H).$$

Further, by substituting (A6.4) into the expression above, we have

$$\pi_H = \delta V_3 \left[ 2\alpha_H + \frac{2(p_H - p_L)}{\delta V_3} - 3\alpha_H^2 - \frac{4\alpha_H(p_H - p_L)}{\delta V_3} \right] + p_H - \frac{2(p_H - p_H p_L)}{\delta V_3}$$

and

$$\begin{aligned} \frac{\partial \pi_H}{\partial p_H} &= \delta V_3 \left[ -\frac{2}{\delta V_3} + \frac{2}{\delta V_3} + \frac{6\alpha_H}{\delta V_3} + \frac{4(p_H - p_L)}{\delta^2 V_3^2} - \frac{4\alpha_H}{\delta V_3} \right] + 1 - \frac{2(2p_H - p_L)}{\delta V_3} \\ &= \delta V_3 \left[ \frac{2\alpha_H}{\delta V_3} + \frac{4(p_H - p_L)}{\delta^2 V_3^2} \right] + 1 - \frac{2(2p_H - p_L)}{\delta V_3}. \end{aligned}$$

Note that if  $p_H = p_L = \delta V_3$  and there is no splitting  $(\alpha_H = 1/2)$  we return to the mixed strategy equilibrium described in Proposition 5.

Now, suppose that the buyers both split their orders with positive probability. Then, at the symmetric equilibrium,  $p_H = p_L \Rightarrow \alpha_L = \alpha_H = \alpha = \frac{E_2}{4E_2 - V_3} < 1/2$  and the price charged by both firms in equilibrium is

$$p = \delta V_3 \left[ \frac{1}{2} + \frac{E_2}{4E_2 - V_3} \right] < \delta V_3.$$

Each seller's equilibrium profit is then

$$\pi = 2p + \delta V_3(4\alpha - 6\alpha^2).$$

Thus, profits are greater than  $2\delta V_3$ , thus the sellers receive positive rents, but are less than  $\frac{5\delta V_3}{2}$ , thus the sellers make lower profits than when buyers do not split their orders.

# Appendix A7: Proof of Proposition 6.

To be succinct, we analyze only the first period, since we can use earlier analysis to deal with the second period. First, we find what prices each seller must charge for each buyer to buy two units and only two units from a seller. Next, we show that there is no profitable deviation by buyers given the prices by seller. Finally, we check deviations by sellers to demonstrate that they have no profitable deviations and hence we have an equilibrium.

Suppose that each of the sellers is charging  $p_1$ ,  $p_2$ , and  $p_3$  for one, two, and three units, respectively. We want to see if there is an equilibrium where both sellers charge a price  $p_2$  and both buyers buy only two units. Suppose that one seller (say seller L) is charging  $p_2^L$  for two units and that the other seller (seller H) is charging  $p_2^H$  for two units. If  $\alpha$  is the probability that both buyers will buy two units from seller L, it can be shown, using arguments similar to those used in the linear case, that  $\alpha = \frac{p_2^H - p_2^L}{2\delta V_3} + \frac{1}{2}$ . Let  $f(p_2^L)$  be the density of prices offered by seller L, which range in some interval  $\underline{p}_2^L$  to  $\overline{p}_2^L$ . Seller H's expected profit function over the two periods is

$$\pi_H(p_H) = 2\delta V_3 \int_{\underline{p}_2^L}^{\overline{p}_2^L} \alpha^2 f(p_2^L) dp_2^L + p_2^H \int_{\underline{p}_2^L}^{\overline{p}_2^L} \alpha(1-\alpha) f(p_2^L) dp_2^L + 2p_2^H \int_{\underline{p}_2^L}^{\overline{p}_2^L} (1-\alpha)^2 f(p_2^L) dp_2^L,$$
(A7.1)

where the first term is seller *H*'s profits if both buyers buy two units from seller *L*, the second term is when each buyer buys two units from different sellers, and the final term is when both buyers buy two units from seller *H*. Taking the first order condition, we obtain  $p_2^H = 2\delta V_3$ . Thus,  $p_2 = 2\delta V_3$ is the only possible equilibrium price where both buyers buy only two units. This will give a seller an expected equilibrium payoff of  $2.5\delta V_3$ . We want to check to see if the price configuration  $(p_1, p_2, p_3) = \left(\delta V_3, 2\delta V_3, \frac{8\delta V_3}{3}\right)$  is an equilibrium.

First, we examine possible buyer deviations. If buyer 1 is mixing equally between buying from either seller 2 units, then we have already shown in the linear pricing case that the payoff for the buyer is higher than if he split his order between the two sellers and bought two units, since the price for one or two units is the linear price of  $\delta V_3$ . Furthermore, no buyer wants to buy three units from a single seller or buy two units from one seller and one from another. Thus, the buyers are behaving optimally, since the expected discounted price of the third unit is  $\delta V_3$ .

Now, we check for deviations by the sellers. It is easy to demonstrate that we do not need to examine the case when a seller induces no or 1 purchases from a buyer in period 1. There are two possible deviations that a seller can engage in by inducing buyers to buy three units in period 1. One is to induce a buyer to buy two from one seller and one from another. The other is to induce a buyer to buy three units from a seller. Let the deviating seller be seller H and charge prices  $p_1^H$ ,  $p_2^H$ , and  $p_3^{H}$ . First, we check to see if there is a profitable deviation by inducing a buyer, say buyer 1, from buying two units from one seller and one from another, assuming that buyer 2 buys either two units from a single seller, one from each seller, or two units from one seller and one from another. Since buyer 1 knows that his order will always be filled and that he will buy no goods in the future, he will always look for the current cost minimizing bundle. He will prefer to buy two units from H and one from L instead of two units from L and one from H if

$$p_1 + p_2^H < p_1^H + p_2^L = p_1^H + 2\delta V_3 \tag{A7.2}$$

Suppose that (A7.2) holds. Buyer 2 will buy two units from seller H if

$$p_2^H + \delta V_3 \le p_2^L = 2\delta V_3,$$
 (A7.3)

since if he also buys two units from H he will need to pay  $V_3$  in period 2 (he could also buy a unit from seller L in period 1 at  $\delta V_3$ ), while if he bought two units from L the price would be 0. If condition (A7.3) does not hold then buyer 2 prefers to buy two units from seller L, but then buyer 1 will never buy a unit from seller L at a positive price, since the price for the third unit will be zero in period 2. That is, if the buyers know that they will buy two units from different sellers, they will never pay a positive price for the third unit. But, for (A7.3) to hold,  $p_2^H \leq \delta V_3$  and seller H makes a profit of less than  $2\delta V_3$ , thus he does not benefit from the deviation.

Suppose that (A7.2) does not hold. Buyer 1 will buy two units from L and one from H. Buyer 2 prefers to buy two units from seller L than two units from H if

$$2\delta V_3 + \delta V_3 \le p_2^H \tag{A7.4}$$

If (A7.4) did not hold, then the buyers are buying two units from two different sellers and as before buyer 1 will not buy a unit from seller H. So, suppose that (A7.4) holds. Seller H's profit is  $p_1^H + \delta V_3$ . But, buyer 1 will only buy from seller H if  $p_1^H \leq \delta V_3$ . Similarly, if buyer 2 plans to buy a third unit from seller H, she will pay no more than  $\delta V_3$ . Thus, the deviation is not profitable.

Suppose that (A7.2) is an equality, so that buyer 1 is indifferent between how she divides her purchases. If she buys two units from L and one unit from H with probability  $\alpha$  and one unit from L and two from H with probability  $(1 - \alpha)$  to make buyer 2 indifferent between buying two units from either seller, then  $\alpha = \frac{1}{2} + \frac{p_2^H - 2\delta V_3}{2\delta V_3}$ . But this is the same  $\alpha$  that was used when examining (A7.1). We found that the optimal  $p_2^H$  is  $2\delta V_3$ .

Now, we examine deviations where seller H tries to induce a buyer to buy three units from him. There are two possibilities: a buyer buys three units from him and the seller makes no other sales or one buyer buys three units from him and the other buyer a single unit from him. Suppose that seller H tries to induce one buyer to buy three units from him and the other buyer to buy one unit from him. For the seller to benefit  $p_1^H + p_3^H$  must be greater than  $\frac{5\delta V_3}{2}$ . If buyer 1 is going to buy three units from seller H and buyer 2 is going to buy one unit, then buyer 2 will buy either one or two units from seller L. If buyer 2 plans to buy 2 units from seller L, then  $p_1^H$  must equal 0, since the buyer can pick up the third unit for free in period 2. If this is the case, then  $p_3^H$  must be greater than  $\frac{5\delta V_3}{2}$ , but buyer 1 would prefer to buy two units from seller L at  $2\delta V_3$  and one unit from seller H at a price of 0. This reduces seller H's payoff. If buyer 2 plans to only buy one unit from seller L, then  $p_1^H + \delta V_3 + \delta V_3 \leq 2\delta V_3$ , since the left hand side is the price paid to seller Hfor a single unit, the price to seller L for a single unit in period 1, and the price paid to seller L in period 2 for its third unit. Again, this implies  $p_1^H \leq 0$ . We can use the same logic as before to show that seller H does not benefit from this deviation.

Suppose that seller H wants to induce one of the buyers to buy three units from him and make no other sales. Let  $\alpha$  be the mixing probability of a buyer buying two units from seller L, instead of three units from seller H. It can be shown that a buyer is indifferent between buying three units from seller H at price  $p_3^H$  and two units from seller L, who is charging  $2\delta V_3$ , if

$$\alpha = \frac{p_3^H - 2\delta V_3}{4\delta V_3 - p_3^H}.$$

Seller H's expected profit in this case is

$$\pi^H = p_3^H \left[ 2\alpha (1-\alpha) + (1-\alpha)^2 \right] + \alpha^2 2\delta V_3$$

Differentiating with respect to  $p_3^H$ , including taking into account its effect on  $\alpha$ , we find that the optimal price is  $p_3^H = \frac{8\delta V_3}{3}$ . Plugging this price into the  $\alpha$  equation, we find  $\alpha = 1/2$ . Substituting, in turn, this value into the profit function, the expected profit from this deviation is  $\frac{5\delta V_3}{2}$ , which is exactly equal to the equilibrium profit. Thus, the deviation does not increase the seller's profit.

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