# Long-term debt and hidden borrowing*

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**Abstract**

We consider borrowers with the opportunity to raise funds from a competitive banking sector that shares information, and from an alternative hidden lender. The presence of the hidden lender restricts the contracts that can be obtained from the banking sector and reduces welfare. In equilibrium some borrowers obtain funds from both the banking sector and the inefficient hidden lender simultaneously. Imposing distributional assumptions, we fully characterize the equilibrium and show that as the cost of borrowing from the hidden lender increases, total welfare increases. We generalize the model to allow for a partially hidden lender and obtain qualitatively similar results.

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## 1 Introduction

Households have many potential sources of credit available, including secured mortgages, installment loans, bank overdrafts, store credit, credit cards, payday loans, borrowing from family, and borrowing from other "informal" sources. Similarly, small and large firms face a number of different financing options ranging from private placements, securitized loans, trade credit, and personal loans to the owner. These alternative forms of financing differ in a number of ways. While some of these differences might be endogenous (such as the interest rate, or the term length for repayment), there are also exogenous differences—for instance, with respect to the seniority of the claims and enforcement.

An empirical puzzle is that borrowers appear to borrow from apparently costly lenders while not fully exhausting cheaper sources. Gross and Souleles (2002), for example, report that in a large sample of credit card holders, almost 70% percent of those borrowing on bankcards have positive housing equity. Similarly, small businesses often use uncollateralized trade credit and personal loans to the owner when

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collateral and collateralized loans are available. Finally, there is both theoretical work and empirical evidence of formal and informal sources of credit coexisting in developing countries (Bell et al (1997), Bose (1998), Jain (1999)). In these cases where some agents simultaneously borrow from both sources, it is unclear whether agents are rationed by the formal sector before they access the informal one.[1]

We suggest that an important consideration in understanding this puzzle is that while some lenders share information about borrowers, others do not.[2] This allows borrowers the chance to conceal liquidity shocks that affect their creditworthiness by borrowing from junior lenders whose loans are hard to observe by senior lenders. Thus, even when seniority is well defined, a senior lender cares about the existence of junior lenders because the possibility that the borrower is using them affects the information obtained through interim repayments.

As an example, missing a lease payment can trigger a renegotiation with the financier and lead to a higher future interest rate. This reflects the financier's renewed assessment of the borrower's ability to repay. An effort to renegotiate the loan may well be costly for the borrower, because of the information revealed in the process. This can be interpreted as an endogenous renegotiation cost.

In order to avoid this penalty, an entrepreneur might borrow from elsewhere, for example taking a personal loan, to conceal the bad news that her enterprise has suffered a liquidity shock. In turn, this makes missing a payment even worse news as it reflects a liquidity shock so large that it is prohibitively costly to conceal. The entrepreneur's opportunity to borrow from a source that the bank does not observe increases this informational penalty and leads to higher renegotiation costs. The resulting overall cost of renegotiation may be sufficiently high so that the financier would repossess the asset or foreclose following a missed payment.

We illustrate these ideas more formally in a two-period model, where heterogeneous agents can access two sources of funds: a competitive banking sector that shares information, and an opaque lending sector. Banks are senior claimants and seek to obtain information regarding borrowers through interim payments. While, most of our discussion views banks as providing flexible long-term (two period) financing, one could also interpret the banking sector as providing a sequence of short-term loans.

Our principal results are that in the absence of the opaque sector realized contracts are complex menus, in which higher levels of interim payments lead to lower final payments. This is not only to take into account that less is owed, but also because a higher interim payment reflects that the borrower is less of a credit risk.[3] However, with a viable alternative hidden lender, a borrower might be tempted

[1] While some of this literature posits a tradeoff between an informal sector, with better information about borrowers' abilities to repay, and a formal sector with a lower cost of capital, the information available to the informal lender does not play any role in our model.

[2] Note that other explanations have been posited to explain this apparent puzzle; for example, Laibson et al. (2001) calibrate a model of life-cycle borrowing with time inconsistent preferences, and Haliassos and Reiter (2003) discuss a model of separate mental accounts. The results of this paper need not contradict such explanations but can be seen as complementary to them. While our results (as any models which assume fully rational consumers) fail to explain the coexistence of credit card debt and liquid assets, they can be seen as suggesting some endogenous illiquidity of certain assets.

[3] This result mirrors the observation in Allen (1985) that long-term contracts allow interim payments to provide information.

to borrow from that source in order to disguise her type. This possibility is anticipated by the original lender in the banking sector. In general, this will lead to a more limited menu of repayment schedules in the optimal contract. Further, some agents borrow from the opaque sector to make this payment. Thus, in equilibrium, these agents are simultaneously borrowing from both the banking and the opaque sectors.

We impose a distributional assumption, that types are uniformly distributed, which allows us to fully characterize equilibrium and in that case, we show that the unique equilibrium results in only a single level of interim payment observed in the banking sector.

We consider how the welfare of consumers and the transparent sector vary with the cost of borrowing from the opaque source. In particular, a lower cost of borrowing benefits consumers for a fixed level of borrowing, but it also encourages a greater number of inefficient types to continue to borrow rather than terminate the debt contract and would lead the contract in the transparent sector to change. Overall welfare falls.[4]

A key element of the model is that a lender may not perfectly observe all the loans that a borrower may hold. Empirically, this is certainly the case. For example, although information sharing takes place through credit bureaus, there are many lenders who choose neither to pay for access to credit bureaus nor to provide information to them. Trade credit, informal, black market lending, payday lenders and personal loans to entrepreneurs subsequently used in their firms are clear examples. Further examples include consumer credit, store credit and other sources that do not participate in formal information-gathering credit bureaus, both in developing countries and elsewhere currently and historically. Even when a lender has access to a credit bureau, the costs associated with accessing and processing the relevant information may lead lenders to obtain and use this information only in particular circumstances. Such circumstances would include the loan approval stage, missed payments, and renegotiation. Otherwise there is unlikely to be continual monitoring. In this paper, we simply take it for granted that some types of borrowing are not commonly observed by all lenders.[5]

The banking sector cannot write contracts that make payments depend on the agent's borrowing from the hidden lender. This is a natural consequence of the assumption that the banking sector cannot observe borrowing from the hidden lender. This paper is therefore related to a growing literature on non-exclusive contracts and on hidden savings, which includes Allen (1985), Cole and Kocherlakota (2001), Bisin, and Guaitoli (2004), and Doepke and Townsend (2004). Our model differs from those in a number of respects and in its motivation. In particular, we consider different lending sectors that vary in the information that they have, we allow for adverse selection rather than presenting a pure moral hazard model with a single lending sector (Bizer and DeMarzo (1992) and Arnott and Stiglitz (1991) for example), and we model agents to be risk-neutral with limited liability, rather than risk-averse and seeking to smooth consumption or buy insurance—an important focus of this literature.

[4] This result is to some an extent an application of the general theory of the second best, discussed by Lipsey and Lancaster (1956). This states that in the presenece of some market imperfections that cannot be eliminated, there is no guarantee that a move towards eliminating other market imperfections will make the market more efficient and indeed it may make the market less efficient.

[5] For further information on consumer credit reporting in the US and further references both on the theory and development of credit bureaus and reporting institution, the interested reader is referred to Hunt (2002).

A key aspect of our analysis is varying the cost of borrowing from the hidden source.[6] Allen (1985) and others focus on the case where this cost is equal to the social planner's rate. Innes (1990), in order to generate monotonicity in repayment schedules, considers the case where money can be repaid immediately so that essentially the cost of borrowing is zero.

Section 2 of this paper introduces the model and elaborates the key assumptions. In Section 3, we solve for the equilibrium and characterize the principal results; in particular, we highlight and distinguish results which are distribution free and discuss comparative statics with respect to the cost of borrowing from the opaque sector. Then, we assume a uniform distribution for agent´s type in order to fully characterize equilibria and show their implications for welfare. We briefly discuss an extension to the case where borrowing from the opaque sector can be observed with some probability in Section 4. The final section concludes.

## 2 The Model

Although the underlying economic mechanisms have wider applicability, we focus the model on the particular example of a start-up firm that is raising funds for an investment project. We first present the basic set-up, timing, and structure of the model. We then introduce additional assumptions that rule out uninteresting cases and simplify the analysis of the model.

### 2.1 Set-up

We introduce a two-period model to consider the interaction between alternative sources of borrowing, a transparent banking sector, and an opaque hidden lending sector. In the transparent sector, credit is provided by a continuum of agents that we call banks. Banks are risk-neutral deep pockets, and there is competition among them. Banks share information, and so the borrowing position of any agent with a bank is perfectly observable and verifiable among all banks. We normalize the gross riskless market interest rate of this formal sector to one. The principal assumptions on the banking sector can be summarized as follows:

*Assumption 1:* The total amount of loanable funds in the banking sector exceeds demand.

*Assumption 2:* A borrower can repay her outstanding balance and switch to another bank at any point in time.

*Assumption 3:* Banks perfectly share the information about the borrower's outstanding loans.

These assumptions guarantee both that banks do not make a profit on average and that conditional on the information known at any point in time, every contract offered must break even. In short, there can be no observable cross-subsidies between borrowers. If a set of borrowers knew and were able to prove to a third party that they are subsidizing other borrowers, they would switch to another bank, leaving their previous bank with only subsidized borrowers and so losses.[7] Note that we assume that

[6] The general model of Doepke and Townsend (2004), as illustrated in their example in Section 7.1, allows for this more general interest rate; however, as in Cole and Kocherlakota (2001) and Ljungqvist and Sargent (2003), they consider hidden saving and insurance rather than hidden borrowing and focus on numerical rather than analytical solutions.

[7] Note that the assumption of perfect competition within the banking sector is not crucial for our results. If there were small costs in switching banks, for example, in the absence of an opaque sector, banks will offer contracts that are contingent on type (although not fully contingent). The introduction of a hidden sector would still lead to less contingent

banks are committed to sharing information. In particular this implies that they cannot simply replicate the hidden lender as they have no means to hide such contracts from other banks.

In addition to the transparent banking sector we introduce an alternative opaque lending sector that lends at a flat repayment rate $r$ ($r > 1$), this rate is composed of the endogenous break even rate plus an additional markup $\rho$ per unit borrowed.[8] The introduction of this exogenous additional markup can be justified as arising from alternative uses of the hidden source or a relative inefficiency of the hidden lender in obtaining funds or processing loans. It allows us to show some interesting comparative statics on the relative inefficiency of the hidden lender. A situation in which the hidden lender is fully efficient ($\rho = 0$) is therefore a particular case of the characterized equilibria.

A key feature of this alternative borrowing source is that it does not share information with the rest of the financial system. That is, the borrowing position of any agent in the opaque sector is not observable by banks. Further, we model the opaque sector as a junior lender. This is certainly consistent with an interpretation as trade credit, credit cards or a concealed loan to a firm by the firm owner.[9] In our model lenders exogenously belong to either the banking sector or the opaque sector. Pagano and Japelli (1993) discuss determinants of belonging to either group as an endogenous decision.

Demand for funds comes from borrowers who require these funds for an investment project, and who are heterogeneous in the quality of their projects. They are risk neutral and maximize total consumption across periods.[10] We assume that they have all the bargaining power and propose contracts before knowing their types. These assumptions may not be crucial for the qualitative insights; however, they are convenient in characterizing a unique equilibrium outcome.

The timing of the model is as follows:

At $t = 0$ each borrower does not know her type. In order to raise $D$ units of funding necessary to invest in the project, the borrower proposes a menu of first and associated second period repayment schedules $\{(p, q)\}$ to a bank.

At $t = \frac{1}{2}$, each borrower learns the type of her project, which is parametrized by $\alpha$, where $\alpha$ is distributed on $[0, 1]$. At this point, the borrower can either liquidate the project and fully repay the loan or continue with the project.[11]

At $t = 1$ agents realize a cash flow $\alpha$ that corresponds to their type. They can choose to borrow $d$ from the informal lending source. The informal lender is junior to the bank loan, and banks do not observe $d$. Agents can use these funds to either consume or choose one of the repayment schedules from the menu and repay $p$ to the bank. Agents consume anything left over, so residual income cannot be used to repay future debts.

Note that $\alpha$ is neither observable nor verifiable in Period 1 and it need not be in Period 2 in case of a

contracts and reduce welfare.

[8] The endogenous break even rate is always equal to $\frac{1}{\nu}$ as discussed in the paragraph preceding Section 3.1 on page 8.

[9] Other types of hidden lending, including black market lending may be more ambiguous with respect to seniority.

[10] In particular, while most of the literature on hidden savings (Allen (1985), Cole and Kocherlakota (2001) and Doepke and Townsend (2004)) has been concerned with risk-sharing, here borrowers require access to funds and have limited liability.

[11] We model this option to stop the project as a costless liquidation in a very early stage; but supposing that the agent was able to recover a sufficiently large salvage value at an early stage would generate similar qualitative results.

successful project outcome. We do assume however, that in case the project fails, triggering liquidation and investigation, it becomes verifiable. Introducing a small verification cost in Period 2, in the spirit of the costly state verification literature (Gale and Hellwig (1985)) would not affect the qualitative results.

At $t = 2$ the project is successful and delivers $B + \alpha$ with probability $\nu$. Otherwise the project fails and delivers only $\alpha$. In both cases, seniority of debt is such that the borrower repays $q(p)$ to the bank first and then repays opaque lenders up to $rd$. The borrower consumes all the remaining funds.

The parameter $\alpha$ represents the creditworthiness of the borrower, since the expected final cash flow of the project is positively correlated with its interim cash flow. Note that overall, a project of type $\alpha$ generates $-D + \alpha + \nu(B + \alpha) + (1 - \nu)\alpha = -D + \nu B + 2\alpha$. In particular, the worst potential project, a project of type $\alpha = 0$ generates $\nu B - D$ in expectation. It is convenient to define $z = 1 + (\nu B - D)/2$ as a measure related to the average profitability of a project. Low values of $z$ suggest that the worst potential project is inefficient and more generally (and depending on the distribution) a high proportion of inefficient projects. In particular, $z \leq 0$ implies that no projects should be funded, while $z \geq 1$ implies that all projects are efficient and should be funded. With intermediate values of $z$, only projects with $\alpha \geq 1 - z$ are efficient. For notational purposes it is also useful to define $y = (\nu B - D)$ which can be seen as the part of the returns of the project that is common to all types or, alternatively, as the average return of the worst possible project.

Figure 1 summarizes both the borrower's actions and the payoffs required and generated by the investment project.

Figure 1: Time Line

This concludes the set-up of the model that contains the basic elements required to consider the implications of hidden borrowing for the structure of formal lending contracts. In particular, agents must be heterogeneous, there must be two periods in order that interim payoffs play a role, and there must be some possibility of default for the agent's type to have any meaningful consequences.

## 2.2 Further Assumptions

In this section, we add three auxiliary assumptions that help to simplify the analysis. The first assumption ensures that contracts are renegotiation-proof. The second assumption is a transversality condition that precludes unlimited borrowing. The final assumption imposes parametric restrictions that rule out uninteresting cases.

Specifically we begin by making the following assumption, which ensures that banks in the transparent sector break even both at the *ex-ante* and interim stages, as stated in Lemma 1.[12]

*Assumption 4:* Some infinitesimal proportion of borrowers already know their own types at $t = 0$.

**Lemma 1** *: The menu of repayment schedules* $\{p, q(p)\}$ *is renegotiation proof and breaks even at all future possible stages.*

**Proof.** Competition between banks ensures that banks break even at the *ex-ante* stage.

Assumption 4 guarantees that the initial menu is already contingent on all the future public information about the borrower's type so borrowers will effectively not switch (or renegotiate on the threat of switching) to another bank. This follows since if, contingent on some possible future information, a repayment schedule contained cross subsidies then among the small subset of agents that already know their type, subsidizing agents would propose alternative schedules to other banks while subsidized agents would stay.[13] Thus, those banks offering schedules which allowed for cross subsidies would suffer losses. ■

As a transversality condition we make the following assumption:

*Assumption 5:* An agent cannot borrow from the opaque lender if the nominal value of the loan is higher than the highest possible residual income.

This assumption can be understood as a "no fraud" condition. For example, it might be appropriate if agents can be punished beyond limited liability if it were found (perhaps with some probability) that they did not intend to repay in any possible state of the world. This is a sensible borrowing limit, since most legal systems allow for punishment above limited liability (i.e., prison or personal liability) whenever a borrower takes a loan that she does not intend to repay even in the best possible situation. Alternatively if one thinks of the alternative lender as an informal lender in development economies, as friends and family, or as a black market, one could think that while limited liability may still hold, it would not apply to agents that use the system without any intention of repayment. Assumption 5 ensures that borrowing from the hidden source in order to consume will not occur. Borrowing from

[12] The renegotiation-proof condition is effectively equivalent to an exclusivity-proof contract—that is, a contract that guarantees that at any point in time the borrower does not want to switch to another bank (see Rampini and Bisin 2004). In the absence of Assumption 4, more general contracts could arise in period 0, but renegotiation would lead to the outcomes characterized by the model.

[13] While one might consider that such offers are off-equilibrium offers and so might be rendered meaningless by a modeler with judicious use of off-equilibrium beliefs. We focus on equilibria robust to the intuitive criterion.

the hidden sector to consume and repay in the good state is inefficient given that $r > \frac{1}{\nu}$. Borrowing to consume would only be worthwhile if the agent intends to default for sure and Assumption 5 precludes this possibility.

Finally, we make parametric restrictions that preclude some trivial and uninteresting cases.

*Assumption 6:* $D > 2$ and $0 < z < 1$

The first restriction ensures that no borrower can repay for sure; the second restrictions ensure that all types of borrowers will default to a different extent if the project is unsuccessful (so from the point of view of lenders they really are different types) and in particular, some projects are efficient and some are not.

## 3 Equilibrium

The feasible strategies for the borrower are any offer of a menu of repayment schedules $\{p, q(p)\}$ at $t = 0$. Furthermore the borrower has to decide whether to pursue the project at $t = \frac{1}{2}$ or liquidate. Finally the borrower has to decide which schedule and requisite first payment from the menu to choose, funding any shortfall for the first payment through borrowing from the hidden source. The bank has to choose whether to agree to the proposed menu.

In order to characterize the equilibrium, we can draw on the revelation principle at $t = 1$ and think of the borrower's choice from the menu $\{p, q(p)\}$ as a function of her type—that is, we could think of offering a menu $\{p(\alpha), q(\alpha)\}$. As discussed above, in Lemma 1, any meaningful contract on the menu—that is any contract that is ever taken up in equilibrium—will break even at all the stages of the contract and so will not contain any observable cross subsidies. Since there is competition among banks and given that borrowers make offers, the equilibrium menu will maximize the *ex-ante* welfare of consumers. Finally, associated with each of the payments that are ever made in equilibrium, incentive compatibility must be satisfied (that is, once a borrower has learned her type $\alpha$, she prefers to pay $p(\alpha)$ rather than any other $p(\alpha'))$.

The equilibrium configuration crucially depends on whether the interest rate at which the informal sector lends $r$ is above or below the threshold $\frac{2-\nu}{\nu}$. We separate these two cases in the discussion that follows.

Throughout, the exogenous interest rate $r$ can be thought of as a measure of the degree of inefficiency of the opaque sector. The break-even rate for $r$ is $\frac{1}{\nu}$, and this would be the endogenous rate for the opaque sector if there were no other frictions or inefficiencies. Regardless of the amount borrowed, the opaque lender will always be repaid if the good state is realized and will always face default in the bad state.[14] Therefore, $r = \frac{1}{\nu}$, is indeed the endogenous interest rate when the hidden lender is efficient. However, whether we think of the opaque lender as trade credit, a credit card, personal loans to an entrepreneur, or an informal lender, it is reasonable to believe that the interest rate charged could be above this break-even rate. Causes of an apparently high interest rate include that the opaque lender is not as specialized as the bank in lending money and that informal credit is also used for purposes other

[14] This follows from the seniority of bank debt, the size of the project, and Assumption 5. Note that this is independent of the type of the project, and this is precisely the reason why the information held by the hidden lender is irrelevant to our analysis.

than concealing liquidity shocks. We therefore allow for the possibility that $r > \frac{1}{\nu}$. For this reason we allow for the possibility that the hidden lender may charge a markup $\rho$ over the break even rate, therefore $r = \frac{1}{\nu} + \rho$.

### 3.1 Very inefficient informal sector

In this section we explore the implications of a very inefficient opaque sector. In particular, we explore the resulting equilibrium when $\rho > \frac{1-\nu}{\nu}$; that is, when the resulting endogenous interest rate $r$ is bigger than $\frac{2-\nu}{\nu}$. We begin by characterizing an equilibrium where there is full separation among those types that borrow—that is, each different type repays the formal sector a different interim payment and there is no borrowing from the opaque sector. We then go on to briefly discuss other equilibria.

**Proposition 1** *When the opaque sector lends at a sufficiently high interest rate ( $r > \frac{2-\nu}{\nu}$), then there exists a fully separating equilibrium where consumers offer the menu $\{p(\alpha), q(\alpha)\}$ with the interim payment equal to the first period cashflow $p(\alpha) = \alpha$ and the corresponding final payment $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$. All types $\alpha < 1 - z$ liquidate at $t = \frac{1}{2}$.*

**Proof.** The banks' equilibrium beliefs are consistent with the borrower behaviour—that is, banks believe that a type that pays $p = \alpha$ is an $\alpha$-type (note that some types will simply prefer to liquidate at the $t = \frac{1}{2}$ stage).

This fully contingent contract has to fulfill the break-even and incentive compatibility conditions.

The break-even condition, given that the first payment $p = \alpha$ reveals the type of the agent as $\alpha$, is that $D = \alpha + \nu q + (1 - \nu)\alpha$ so that in expectation the bank recovers its investment. This determines that the break-even second payment is $q = \frac{D-p-(1-\nu)p}{\nu}$.

We analyze the incentive compatibility condition by considering two deviations: imitating a lower type and imitating a higher type.

Incentive compatibility condition 1: The contract needs to guarantee that no agent wants to imitate a lower quality agent. Suppose that an agent of quality $\alpha$ claims to be a lower quality agent $\alpha' < \alpha$ by paying a first payment $p = \alpha'$; in that case, her total utility would be $(\alpha - \alpha') + \nu(B - \frac{D-\alpha'-(1-\nu)\alpha'}{\nu} + \alpha)$. Note that $(\alpha - \alpha')$ is the additional consumption at $t = 1$ from reporting a lower type, while $(B - \frac{D-\alpha'-(1-\nu)\alpha'}{\nu} + \alpha)$ is the net consumption in the good state (which occurs with probability $\nu$) after repaying $q(\alpha')$. Instead, by revealing her own type she would get $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between these two terms is $-(1 - \nu)(\alpha - \alpha') < 0$, and so it cannot be optimal to claim to be an agent of a lower type.

Incentive compatibility condition 2: The contract also needs to guarantee that no agent wants to imitate a higher quality agent by borrowing from the hidden source and paying a first payment $p > \alpha$. Suppose for contradiction that an agent claims to be a higher quality agent by paying a first payment $p = \alpha'' > \alpha$ and borrowing $\alpha'' - \alpha$ from the hidden source to fund this payment. The total utility of the agent would be $\nu(B - \frac{D-\alpha''-(1-\nu)\alpha''}{\nu} - r(\alpha'' - \alpha) + \alpha)$ instead of $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between the two is:

$$(2 - \upsilon - \upsilon r)(\alpha'' - \alpha) \tag{1}$$

Which is negative if and only if $r > \frac{2-\nu}{\nu}$, so this is the necessary and sufficient condition for this incentive compatibility condition to hold.

Notice that off-equilibrium beliefs only apply to $p > 1$, and even assigning the most optimistic beliefs to such offers (that is $\alpha = 1$) agents prefer their equilibrium contracts. ■

**Lemma 2** *The above equilibrium achieves first best.*

**Proof.** In the first best a borrower should be funded if and only if she generates sufficient expected revenues—that is, if and only if $\alpha + v(B + \alpha) + (1 - \nu)\alpha \geq 0$. This is precisely the marginal borrower in the equilibrium described above. ■

**Corollary 1** *In the absence of an opaque sector, the first best can be achieved.*

**Proof.** The absence of an opaque sector is equivalent to $r \longrightarrow \infty$, and so the results above apply. ■

Formally, beyond the equilibrium described in Proposition 1, there are many other equilibria. First, there are some that are essentially observationally equivalent in the sense that many other redundant $(p, q(p))$ schedules could be included in the offered menu that are never taken up and that have no effect on outcomes (for example schedules with very high $p$'s and $q$'s). Henceforth we ignore such equilibria. A more substantive source of multiplicity of equilibria arises from the private information on the part of a (small) proportion of borrowers. As is common in these sorts of games, this opens the possibility of equilibria where there is no borrowing; for example, supported by the beliefs that the only offers are from those borrowers who know that their own types are $\alpha = 0$ (and such beliefs are never challenged because offers are off-equilibrium). We note that such equilibria exist but could be refined away assuming trembles or other equilibrium refinements. We focus instead on more efficient equilibria—indeed we highlight above an equilibrium that achieves first best.

To summarise this section, when the hidden lender is sufficiently inefficient, borrowers do not attempt to conceal low interim cashflows and do not borrow from the hidden lender. The resulting bank contract is flexible and allows for different first payments with corresponding final payments. An alternative interpretation for the schedule of possible payments is to suppose instead that only one contract from the schedule is initially agreed upon, but that the contract is renegotiated following the cashflow realization in the interim period. The flexibility, under this interpretation, would therefore reflect low (endogenous) costs of renegotiation.

### 3.2 Relatively Efficient Informal Sector

In the previous section, we supposed that the opaque sector was so inefficient, or equivalently that the cost of borrowing from the opaque sector was so high, that it had no effect on outcomes and on the contracts taken up in the transparent sector. In this section, we explore the equilibrium outcome when the opaque sector is more efficient, that is when $\rho < \frac{1-\nu}{\nu}$ or, equivalently whenever $r < \frac{2-\nu}{\nu}$. Note in particular, that this regime includes the case where there are no frictions in the opaque sector and $\rho = 0$.

In the proof of Proposition 1, we argued that in the case where types were fully separating in their payments and paid exactly their period 1 incomes, then no type (at this interim stage) would want to imitate a higher type if and only if $r \geq \frac{2-\nu}{\nu}$. In particular, this implies that the outcomes described in Proposition 1 can no longer be an equilibrium. Even though a full characterization of the equilibrium when $r < \frac{2-\nu}{\nu}$ requires assuming a given distribution of types, we can still describe some general features of any existing equilibriums. In particular we can determine that there will be some pooling among different types of agents with regard to their interim payments. As banks are not able to distinguish the different types within a pool, it follows that there will be some cross subsidization between agents and therefore liquidation decisions might be inefficient. Inefficient liquidation implies that first best is not attained.

Before we present a more formal characterization of the outcome when $r < \frac{2-\nu}{\nu}$, we state a couple of preliminary results: a "continuity of pools" lemma and a result on the weak monotonicity of payments with type. Then we are able to show in Proposition 2 that individual separation cannot be achieved.

**Lemma 3** *(Continuity of p) For every three borrowers with types $\alpha$, $\beta$, and $\gamma$ such that $\alpha > \beta > \gamma$ where $p(\alpha) = p(\gamma)$, it must be the case that $p(\alpha) = p(\beta) = p(\gamma)$.*

**Proof.** See Appendix. ■

**Lemma 4** *(Monotonicity of p) For every type $\alpha > \beta$ that does not liquidate, $p(\alpha) \geq p(\beta)$.*

**Proof.** See Appendix. ■

**Proposition 2** *When $r < \frac{2-\nu}{\nu}$ there cannot be an equilibrium where a continuum of borrowers are able to fully separate.*

**Proof.** See Appendix. ■

Underlying the proofs of these distribution-free results are the incentive constraints of borrowers to choose the appropriate schedule from the menu. By characterizing each of the incentive constraints (imitating an agent of a higher type or imitating an agent of a lower type) in different cases we complete the proofs.

Additionally, we can determine that the marginal agent that does not liquidate cannot be consuming after paying the first payment. Let $l$ denote the type that is "just indifferent" between liquidating and continuing the project with the $(p(l), q(l))$ repayment schedule that corresponds to the lowest pool of agents.

**Proposition 3** *Consider the lowest type not to liquidate $l$, it must be the case that $l \leq p(l)$*

**Proof.** By contradiction. Conditional on $l > p$, the utility of the indifferent agent $l$ can be expressed as $\nu(B - q(l) + l + (l - p(l)))$. Given that liquidating provides utility equal to zero and that the agent is indifferent, this implies that

$$\nu(B - q_0 + l + (l - p(l))) = 0. \tag{2}$$

As $l > p$ then $(l - p(l)) > 0$. This implies jointly with (2) that $B - q(l) + \alpha < 0$ which violates limited liability. ■

Propositions 2 and 3 imply that when $r < \frac{2-\nu}{\nu}$ if there exist an equilibrium it must be one in which all agents belong to some pool. That is, no agent is able to fully separate.[15] This means necessarily that there will be some cross subsidies from the best agents in each pool to the worst agents in each pool. Given that these subsidies will exist in the bottom pool of agents that decide to invest, liquidation decisions will not be efficient and first best will not be achieved.

To progress and give a full characterization of equilibrium, we introduce a specific distributional assumption.

*Assumption 7:* $\alpha \sim U[0, 1]$

We maintain this assumption throughout the remainder of the paper.

Under this assumption the outcome will be full pooling in the sense that all types that borrow from the transparent sector will choose the same contract from the schedule. In equilibrium only one level of repayment to the transparent sector will be observed. Rather than the menu of contracts actually taken up in the previous section, borrowing from the transparent sector will entail the same payment $p$ at $t = 1$ for all types who have not liquidated and the same remaining debt $q$ due at $t = 2$ (which will be fully repaid in the good state and only partially repaid—depending on type—in the bad state).

**Proposition 4** *When the lending rate from the hidden sector is sufficiently low ($r \leq \frac{2-\nu}{\nu}$), then all borrowers who do not liquidate pay the same interim payment $p(\alpha) = p$ and owe the same amount, $q$, to the bank in period 2.*

The proof, which appears in the Appendix, has a simple structure. We conjecture that there must be at least two types that make different interim payments and find a contradiction. We focus on the highest two payments (and by Lemma 4 these will correspond to the highest differing types). We find that borrowers, at the ex-ante stage where the menus are determined, would prefer that the top two pools be combined as a single pool, in order to maximize their anticipated surplus. It is at that stage that we use the distributional assumption on types, since it allows to quantify the ex-ante (at period 0) surplus. The uniform distribution helps in keeping the analysis simple.

Since borrowers propose these menus, the equilibrium outcome will indeed maximize their surplus and so combine these top two pools. An induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract.[16] There are a number of different cases that must be considered (depending on the level of $p$ and the size of the pools), but working through each of them is relatively straightforward.

Proposition 4 states that the equilibrium contract has only one possible first payment $p$ and a second payment $q$ that makes the bank break even on average, given the pool of agents that do not liquidate. In particular, the proposition shows that there are no ways by which higher types can

[15] Note however that we cannot rule out the existence of multiple pools.

[16] Note that in the case where $r > \frac{2-\nu}{\nu}$ an induction argument would be inappropriate because there could be an infinite number of pools.

efficiently separate from lower types. This implies some cross-subsidies from higher to lower types of agents and therefore involves inefficient liquidation decisions. This fully characterizes the structure of the equilibrium; however, to gain further insight and in particular to analyze welfare, we proceed by precisely calculating the values of $p$, $q$, and the equilibrium liquidation policy.

### 3.2.1 Equilibrium payments and welfare

In this section, we characterize the liquidation policy. We then define welfare. In equilibrium, since borrowers propose menus before knowing their types, total welfare will be maximized. We conclude by characterizing this maximized value.

We begin by restating our notation to discuss the liquidation policy. Recall that $l$ denotes the type that is "just indifferent" between liquidating and continuing the project with the $(p, q)$ repayment schedule. Under perfect information, $l = 1 - z$: however, limited liability and the cross subsidies between agents inside the pool (from higher quality to lower quality ones) will imply that $l < 1 - z$. This reflects an important externality in our model. Whenever there is some pooling between agents, there will be cross subsidies from agents of higher quality to the agents of lower quality. This generates an inefficient liquidation policy, as some inefficient projects are not liquidated due to this implicit subsidy.

Proposition 3 allows us to focus on the case $l \leq p$. First note that in the case that $l = 0$, it is trivial that the optimal choice of $p$ is $p = 0$, and overall welfare in this case is $W = 1 + y = 2z - 1$ (this is simply the average surplus generated by a project, given that all types of projects will be pursued).

Alternatively, it may be optimal to choose an interior $l$. In this case we can characterize $l$ by noting that a couple of conditions must be satisfied. First, by definition, an agent of type $l$ must be indifferent between liquidating or continuing with the project; that is,

$$0 = \nu(B + l - q - r(p - l)). \tag{3}$$

In addition, banks need to break even on average, and so

$$D = p + \nu q + (1 - \nu)\frac{1 + l}{2}. \tag{4}$$

Note that the indifference condition (3) implies that $B + \alpha > q$ for every $\alpha > l$, and so it is appropriate to write the break-even condition as above in (4), being sure that the loan will be fully repaid if the contract is successful for every borrowing type. Substituting for $q$ from (4) into (3), we obtain the following expression for $l$:

$$l = \frac{\nu + 2p(r\nu - 1) - 1 - 2y}{\nu + 2r\nu + 1}. \tag{5}$$

We characterize the equilibrium $p$, under the assumption that both the optimal $p$ and $l$ are interior. Having done so, it is easy to verify conditions under which this is indeed the case and then go on to consider outcomes when these conditions fail.

Continuing under the assumption that $l$ is interior, we consider the first order condition, and we maximize total welfare in order to find the contract offered in the optimal equilibrium (other equilibria exist but as discussed at the end of Section 3.1, we focus attention on the most efficient equilibrium).

We begin with the expression of total welfare.

$$
\begin{aligned}
W = & \int_{l}^{1} (2x + \upsilon B - D)dx - (\nu r - 1) \int_{l}^{p} (p - x)dx \\
= & \, y - ly - l^2 + 1 - \frac{1}{2}(\nu r - 1)(p - l)^2. \qquad (6)
\end{aligned}
$$

The first integral represents the net (positive or negative) welfare from each project financed, while the second integral is the welfare loss out of inefficient borrowing. Note that the above expression (in the upper limit of the second integral) supposes that $p < 1$, which it will be easy to verify is true in equilibrium.

The first order condition that characterizes the optimal $p$ is:

$$\frac{dW}{dp} = (-y - 2l)\frac{dl}{dp} - (\nu r - 1)(p - l)(1 - \frac{dl}{dp}) = 0, \qquad (7)$$

where, by taking the derivative of $l$, as defined in equation (5), with respect to $p$:

$$\frac{dl}{dp} = \frac{2(r\nu - 1)}{2r\nu + 1 + \nu}. \qquad (8)$$

Note that this derivative is strictly positive since $r\nu > 1$. One might expect this to be the case, as the interim payment increases, concealing a low type becomes more costly and so more projects might be liquidated at $t = 1/2$.

Substituting expression (8) into (7), the expression that implicitly defines the optimal first payment $p$ is:

$$\frac{dW}{dp} = (-y - 2l)\frac{2(r\nu - 1)}{2r\upsilon + 1 + \nu} - (\nu r - 1)(p - l)(\frac{3 + \nu}{2r\upsilon + 1 + \nu}) = 0. \qquad (9)$$

Simple algebraic manipulation yields the following equilibrium expression for the optimal $p$:

$$p = \frac{\nu l - 2y - l}{3 + \nu}. \qquad (10)$$

We solve simultaneously for $p$ and $l$ from this equation and equation (5) to obtain:

$$l = \frac{2\nu - 2y\nu - 4ry\nu + \nu^2 - 3 - 2y}{6\nu + 8r\nu + \nu^2 + 1}, \qquad (11)$$

and

$$p = \frac{\nu^2 + 1 - 4y\nu - 2\nu - 4ry\nu}{6\nu + 8r\nu + \nu^2 + 1}. \qquad (12)$$

Substituting these expressions into equation (6) we can calculate a value $W_I$ for welfare. The notation

$W_I$ is intended to highlight that this is the welfare under the optimal interior solution when it is feasible. However this need not be the global optimum since choosing $p = 0 = l$ and generating an expected surplus of $1 + y$ is always feasible.

The equilibrium expression for $p$, Equation (12), together with the break even-condition, Equation (4), and the expression for $l$, Equation (11), determine the equilibrium value for the second payment $q$.

**Proposition 5** *Both $l$ and $p$ are interior when $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ and $W_I \geq 1 + y$.*

**Proof.** Note that $p$ is linear in $y$. It is sufficient therefore to consider the two extremes $y = -2$ and $y = 0$. For $y = -2$, $p = 1$. For $y = 0$, $p = \frac{(1-\nu)^2}{6\nu+8r\nu+\nu^2+1}$ which is greater than 0 and less than 1. Furthermore for $y = -2$, $l = 1$ and $l > 0$ as long as $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$.

For values of $y$ higher than $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$ the optimal contract is to set $p = 0$, which leads to $l = 0$.

Thus $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ is required for an interior $l$ and $p$ to be feasible. An interior $l$ would generate more surplus and so would be the equilibrium outcome when the welfare generated is higher than the next best alternative–choosing $l = 0$ and $p = 0$, or equivalently:

$W_I \geq 1 + y$. ■

Note that for $y = -2$ both $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ and $W_I \geq 1 + y$ hold as strict inequalities, and so in particular for small enough $z$, both $l$ and $p$ will be interior.

Equations (11) and (12) show that in the parameter range where $l$ is interior then both $l$ and $p$ are linear and decreasing in $y$. That they should be increasing is quite intuitive. As $y$ goes up, the least efficient and all other projects become more and more attractive, so the optimal first payment $p$ goes down to decrease the liquidation threshold $l$. On the other hand, as $y$ goes down, fewer projects should be funded so $p$ goes up. Further, as $y$ decreases the parameters are more likely to be such that the optimal choice of $l$ is interior and in particular this is always the case when $y = -2$. This is proved explicitly as Lemma 7 in the Appendix.

**Corollary 2** *There are parameter values for which in equilibrium there are agents who simultaneously borrow from both the bank and the hidden lender*

**Proof.** Proposition 5 states that there are parameter values for which $l$ and $p$ are interior. In this case $l$ and $p$ take values as given by expressions (12) and (11). Note that $p > l$ since the expression $\frac{\nu^2+1-4y\nu-2\nu-4ry\nu}{6\nu+8r\nu+\nu^2+1} > \frac{2\nu-2y\nu-4ry\nu+\nu^2-3-2y}{6\nu+8r\nu+\nu^2+1}$ can be simplified as $(4 + 2y)(1 - \nu) > 0$ which is always true. Since $p > l$, some agents — those with type $\alpha$ being an element of $[l, p)$ — need to borrow from the hidden source to satisfy the first payment. ■

### 3.3 Equilibrium summary

There are three equilibrium regimes. When the hidden lender is relatively inefficient ($r > \frac{2-\nu}{\nu}$) there is full separation, where each type of borrower who does not liquidate pays an interim payment equal to the interim cashflow, $p = \alpha$, and a corresponding second period payment that accurately assesses the credit-worthiness of the borrower, $q = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$. Firms get financed as long as $\alpha > -\frac{y}{2}$; equivalently, projects are financed if and only if they are efficient. In this region bank contracts have interest rates between period 1 and period 2 that are contingent on interim payments. These allow the bank to perfectly elicit information about the agent's type.

Instead of interpreting the contract as a long term contingent contract with a menu of different schedules, one could interpret it as an uncontingent contract that specifies maximum repayment $p = 1$ (the maximum level that can be paid without hidden borrowing) that then becomes renegotiated after the borrower learns its type. Along these lines, the fully separating equilibrium would be a situation where the endogenous costs of renegotiation are low. That is, small changes in the actual payment $p$ lead to small changes in $q$.

When the informal sector is relatively efficient ($r < \frac{2-\nu}{\nu}$), pooling cannot be avoided, and in equilibrium only one contract is taken up form the banking sector. The existence of cross subsidies from higher types to lower ones induces too little liquidation. There are two cases to consider.

If the average project is relatively profitable ($-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$), the welfare loss from funding inefficient projects is smaller than is the loss from forcing some borrowers to borrow from the opaque sector. Therefore it is optimal to fund all projects, and in this case there is no interim payment ($p = 0$ and $q = \frac{2D-1+\nu}{2\nu}$).

Finally, if the average project is relatively unprofitable ($-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ and $W_I > 1 + y$), the equilibrium will see some types of projects liquidated ($l = \frac{2\nu+\nu^2-3-2y-2y\nu-4ry\nu}{6\nu+8r\nu+\nu^2+1}$). All types that do not liquidate will make an interim payment to the bank and a corresponding second period payment that takes into account the information implied by the equilibrium liquidation policy ( $p = \frac{\nu^2+1-4y\nu-2\nu-4ry\nu}{6\nu+8r\nu+\nu^2+1}$ and $q = \frac{D-p-(1-\upsilon)\frac{l+1}{2}}{\upsilon}$). In this case some agents borrow simultaneously from both sectors. Both sources of inefficiency operate in this regime—namely, some inefficient projects are conducted, and there is some costly borrowing from the inefficient opaque lender. The bank contract is determined by optimally trading off these two sources of inefficiency.

Again if the contract is interpreted as an uncontingent one that gets renegotiated, this situation can also be interpreted as a case where renegotiation is (endogenously) costly. Any deviation from $p$ would lead to liquidation.

All three regions are non-trivial as illustrated in the diagram below, which illustrates these three equilibrium regions for a general $\nu$.

Figure 2: Equilibrium Regimes

## 3.4 Comparative statics on welfare

First note that when $r > \frac{2-\nu}{\nu}$ welfare is first best and independent of $r$ within this range. In the case where the optimal contract involves $p = 0$, welfare is equal to $(1 + y)$ and, again, within this range it is independent of $r$, raising $r$ to a level where either an interior $p$ is optimal (which requires $W_I > 1 + y$) or the full separation equilibrium is attained (and the first best level of welfare is achieved) trivially raises welfare.

The most interesting analysis is for parameters in the region with a single interior interim payment to the bank—that is, where $r \leq \frac{2-\nu}{\nu}$. Raising $r$ so that the equilibrium shifts to the fully separating case, which is first best, trivially raises welfare. We now consider how welfare varies with $r$ within this region.

Having obtained explicit characterizations of $l$ and $p$ in terms of the exogenous parameters of the model and noting that welfare in this region is given by Equation (6), we consider the comparative statics of welfare. It is of particular interest, to consider how welfare changes (and the channels through which it changes) as $r$, the exogenous rate of interest in the opaque sector, varies.

First note that in the range $r < \frac{2-\nu}{\nu}$ and for $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ and $W_I \geq 1 + y$.

$$\frac{dl}{dr} = \frac{4\nu(3+\nu)(1-\nu)(2+y)}{(6\nu + 8r\nu + \nu^2 + 1)^2} > 0. \tag{13}$$

In particular, this suggests that one source of inefficiency is reduced, since as $r$ increases $l$ rises and so fewer inefficient projects are conducted.

Note also that

$$\frac{dp}{dr} = -\frac{4\nu(1-\nu)^2(2+y)}{(6\nu+8r\nu+\nu^2+1)^2} < 0. \tag{14}$$

As the interim payment falls, and since the lowest type borrowing rises, then the amount of borrowing from the opaque sector falls; however, since the cost of borrowing from the opaque sector rises, the welfare consequences may be ambiguous. By examining welfare directly we can see that the first of these two effects always dominates, as shown in Proposition 6.

Note that the welfare as defined in Equation (6) does not take into account surplus gained by the alternative sector. Including this surplus into the welfare calculation would suggest that the only source of inefficiency would be inefficient liquidation and so only the first effect would apply. The qualitative results would be unchanged—welfare increases in $r$. The analysis here would still be of interest, inasmuch as Equation (6) captures consumer surplus.

**Proposition 6** *Welfare is non-decreasing in the hidden lender's rate ($\frac{dW}{dr} \geq 0$) and strictly increasing when the lender's rate is sufficiently low ($r < \frac{2-\nu}{\nu}$) and the proportion of inefficient projects is high ( $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > y$ and $W_I > 1+y$).*

**Proof.** See Appendix. ■

## 4 Partially Hidden Borrowing

We modify the model slightly to allow for a partially hidden lender. We introduce the possibility that the banking sector observes the level of hidden borrowing of the lender with some probability $(1-h)$. With probability $h$, borrowing from the non-banking sector remains hidden. A rationale for this modelling assumption is that the banking sector investigates each of its borrowers and obtains full information about the borrowing position of each of them with some probability $(1-h)$. Once a borrower is successfully investigated, its borrowing position with all possible alternative lenders is perfectly known by the whole banking sector. On the contrary, if a particular borrower is not investigated, the banking sector cannot observe any borrowing outside the pool of competitive creditors and is aware of the possibility of some additional lending.

If a borrower is investigated, we assume that the borrower is aware of it, and that she has the opportunity to repay the opaque sector immediately. Early repayment entails a cost $sd$, where $d$ is the amount borrowed from the opaque sector and $s < r$ since repayment is early.

If an agent is investigated and repays early, then we know that full separation holds. By the proof of Proposition 1, if the full separation contract is offered, then there are no incentives to imitate downwards. With observable payoffs there is no feasible way to imitate upwards as banks would take into account any inefficient borrowing and discount it when calculating the agent's true type. Given that the incentive compatibility constraints for the fully revealing equilibrium hold and that it achieves first best, this is the only equilibrium once an agent has been investigated.

We make the following assumption to guarantee that early repayment is the optimal strategy of the borrower once the alternative lender becomes transparent:

*Assumption 8* $s < r - \frac{1}{\nu}$.

**Lemma 5** *Once the hidden borrowing is observed, early repayment is the optimal strategy for the agent.*

**Proof.** Borrowing from a hidden source gives no concealment benefit, so the only benefit from that borrowing comes from either investing or consuming those funds. Investing them at the gross market interest rate of 1 or consuming them gives a (negative) expected utility of $1 - \nu r$. This loss has to be compared with the cost of early repayment $-\nu s$. Early repayment is therefore the optimal strategy as long as $-\nu s > 1 - \nu r$, which is guaranteed by Assumption 8. ■

The model with probabilistic observability of the hidden borrowing is therefore like a switching model in which, with probability $(1 - h)$ full separation is achieved for sure and with probability $h$ the model looks like that of the previous sections. In this latter case the only difference is that, from the point of view of the borrower, the costs and benefits of the hidden borrowing need to be recalculated, since with probability $(1 - h)$ hidden borrowing is useless and entails a cost $s$.

In fact, once the alternative borrowing remains hidden, the rest of the model with probabilistic observation of the hidden borrowing can be fully solved by realizing that the cost of borrowing from the hidden source is now $\frac{hr+(1-h)s}{h}$ instead of just $r$. Borrowing one unit from the hidden source costs $r$ with probability $h$ and costs $s$ with probability $(1 - h)$. It only produces some concealment benefit to the borrower with probability $h$, so the whole cost has to be re-scaled by $h$.

We write $r_h = \frac{hr+(1-h)s}{h}$ as the effective interest rate when borrowing from the opaque sector remains hidden with probability $h$, the rate of interest is $r$ when borrowing remains hidden, and the cost of early repayment when the banking sector observes the borrowing is $s$. With this notation, we obtain the following results, which are similar to those in the fully hidden case:

**Proposition 7** *When the opaque sector lends at a sufficiently high effective interest rate ($r_h > \frac{2-\nu}{\nu}$), then there exists an equilibrium where consumers offer the menu $\{p(\alpha), q(\alpha)\}$ with $p(\alpha) = \alpha$ and $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$ and there is no borrowing from the hidden sector. When the opaque sector lends at a sufficiently low effective interest rate ($r_h \leq \frac{2-\nu}{\nu}$), all types that do not liquidate make the same interim payment $p$ and owe the same amount, $q$, to the bank in period 2.*

**Proof.** The proof is almost identical to the ones in Propositions 1 and 4 except that now borrowing from the hidden source entails higher costs, and so further details are omitted. ■

The functional form of the welfare equation and the incentive compatibility conditions are similar to those of the basic model, so similar results to those of Section 3.2.1 hold. In particular, if $r_h \leq \frac{2-\nu}{\nu}$ the only possible equilibrium is one of full pooling, and if the optimal solution is interior then the optimal first payment is:

$$p = \frac{\nu^2 + 1 - 4y\nu - 2\nu - 4yr_h\nu}{6\nu + 8r_h\nu + \nu^2 + 1} \tag{15}$$

and the type of borrower who is just indifferent between liquidating the project and continuing it is:

$$l = \frac{2\nu - 2y - 2y\nu + 4r_h + \nu^2 - 3}{6\nu + 8r_h\nu + \nu^2 + 1}. \tag{16}$$

Total welfare in this regime can be expressed as:

$$W = \begin{array}{l} h\left[\int\limits_{l}^{1}(2x+\upsilon B-D)dx-(\nu r-1)\int\limits_{l}^{p}(p-x)dx\right] \\ +(1-h)\left[\int\limits_{-\frac{y}{2}}^{1}(2x+\upsilon B-D)dx-\nu s\int\limits_{l}^{p}(p-x)dx\right]. \end{array} \tag{17}$$

The first term corresponds to the welfare when the hidden sector remains hidden, while the second one is related to when it becomes observable. The above expression can be rearranged as

$$W = h\left[\int\limits_{l}^{1}(2x+\upsilon B-D)dx-(\upsilon r_h-1)\int\limits_{l}^{p}(p-x)dx\right]+(1-h)\left[\int\limits_{-\frac{y}{2}}^{1}(2x+\upsilon B-D)dx\right]. \tag{18}$$

Note that the expression in the first bracket is identical to the expression (6) in Section 3 with a change of the social cost of borrowing from $r$ to $r_h$ and that the second bracket is constant in $p$ and $l$.

The welfare implications of the changes in the probability of the hidden sector becoming transparent $(1-h)$ are as follows: A higher $(1-h)$ implies higher welfare in a couple of ways. First is the automatic switching from the pooling equilibrium to the first-best full separation equilibrium whenever the banking sector observes the hidden lending. Second, increasing $(1-h)$ increases $r_h$, and so the results on welfare increasing in $r$ from Section 3 apply. Similarly an increase in $s$ raises $r_h$ and so also raises welfare.

Note that our analysis is related to the literature on the interactions between direct screening of lenders through active investigation and the indirect screening that can be achieved by offering them a menu of contracts, as in Manove et al. (2001). While in most models these are seen as substitutes, in our model they are complements. That is, an increase in $(1-h)$ leads to more information about some borrowers directly and also to a more informative equilibrium with respect to the other borrowers (who may have loans from the alternative sector that remain hidden).[17]

## 5 Conclusions

In this paper we have presented a model in which a banking sector and an alternative opaque source of lending coexist. The results show that if the alternative source of borrowing is sufficiently inefficient, banking contracts will achieve first-best. The optimal contract gives incentives to borrowers to reveal their intermediate cash flows perfectly by rewarding higher interim payments with lower future interest rates. However if the alternative source of borrowing is relatively efficient, then the fully contingent contract is not sustainable as agents may want to conceal their types by borrowing from the hidden source and repaying a larger part of their loans early. Here, the optimal contract is not contingent on the

[17] Even though so far we have considered $h$ as an exogenous parameter, endogenizing it seems relatively straightforward. We could allow banks to choose their monitoring effort $h$ at a cost. Higher transparency (lower $h$) would be more costly and competition among banks should equalize the marginal cost of additional monitoring (reducing $h$) with its marginal gain in terms of welfare in equilibrium.

interim payments of the loans. Assuming types are uniformly distributed allows us to fully characterize an equilibrium in which there is only one possible first payment and associated second payment. The contract fails to achieve first-best for two reasons. First, a number of inefficient projects are funded, and, second, some borrowers access the inefficient alternative sector. Note that for some parameters this first payment would be zero and all repayment would be in the final period. Thus the model can also be seen as characterising the timing of debt repayments.

It is worth restating that the banks, even though they may be more efficient, cannot simply replicate the loans provided by the hidden sector. This follows since it is assumed that a bank is committed to share all information about its loans to other banks. In principle, one might think that the bank could offer two types of credit and commit not to act on the information revealed. However, even if the commitment was credible, this arrangement would lead to cross-subsidies among borrowers that would be observable by other banks, and so the bank offering this arrangement would suffer from cream-skimming and suffer losses.

We show that overall welfare increases if the cost of borrowing from the opaque sector is higher. This result is in contrast to some conventional wisdom in discussions of developing economies, which focuses on the role that the informal sector may play in alleviating the financing constraints. The informal sector lends to firms and households when the formal sector is rationing them.[18] However in our model, as the informal sector gets more efficient, the banking sector has to offer a less contingent contract, and total welfare falls. This approach, considering the interaction between formal and informal sectors from an informational point of view, provides a counter-argument to traditional literature by showing how an efficient informal sector may reduce welfare.

Relaxing the assumption that the alternative sector is not entirely opaque makes it less appealing for a borrower to use the costly alternative sector to disguise her type as this may turn out to be ineffective. In this case, the qualitative results outlined above carry through in this richer environment and moreover, welfare is decreasing in the opacity of the informal sector. These results suggest that as the informational transparency of the financial sector as a whole improves, banks are able to offer more sophisticated financial instruments.

While we presented a model of start-up financing for an investment project, the central mechanisms and in particular the interaction of different sources of borrowing and the implications for contractual form have wide applicability. For example, over the last twenty years the consumer sector in the UK has seen the development of more flexible mortgages integrated with checking accounts and credit lines. Intuitions developed through this paper suggest that this may be partially a consequence of financial institutions having better means of processing and transferring information about consumer credit histories.

Our results highlight that one of the possible reasons for long-term debt contracts being inflexible with respect to interim payments is that the information that long-term lenders would extract from

[18] Jain (1999), who also discusses related literature, provides an explanation for the observation of borrowers in both sectors based on a trade-off between the formal sector's lower opportunity cost of funds and the informal sector's better information. Instead we assume that the formal sector is unambiguously more efficient and the informal sector may or may not have an informational advantage.

these interim payments would be corrupted by additional borrowing from hidden sources of funds. They also suggest an explanation for simultaneous borrowing from different sources even when there is a clear pecking order among them and there is available borrowing from the cheaper one (for example mortgages and credit card borrowing). Finally, we also show that the existence of an alternative opaque source of borrowing may be welfare diminishing because it may distort the set of contracts that the competitive lending sector may offer.

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## 6 Appendix

**Proof of lemma 3**

**Proof.** Suppose that borrowers face the choice between two generic contracts $a$ and $b$ and without loss of generality, we label them so that $p_a > p_b$. There are a number of cases to consider:

(i) $\alpha > p_a > \beta > p_b > \gamma$

(ii) $p_a > p_b > \alpha > \beta > \gamma$

(iii) $\alpha > \beta > \gamma > p_a > p_b$

(iv) $\alpha > p_a > p_b > \beta > \gamma$

(v) $\alpha > \beta > p_a > p_b > \gamma$

We examine each in turn, noting first that in general an agent of type $x$ prefers the repayment schedule $a$ to schedule $b$ if and only if:

$$x - p_a + \nu(B + x - p_a - q_a - r(p_a - x)1_{p_a > x}) \geq x - p_b + \nu(B + x - p_b - q_b - r(p_b - x)1_{p_b > x}). \quad (19)$$

(i) $\alpha > p_a > \beta > p_b > \gamma$

In this case, substituting into Expression (19) and noting that $1_{p_a > \gamma} = 1_{p_b > \gamma} = 1$, an agent of type $\gamma$ would choose the repayment schedule $a$ over the schedule $b$ if and only if:

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu). \quad (20)$$

Similarly, $1_{p_a > \beta} = 1$ and $1_{p_b > \beta} = 0$, and so an agent of type $\beta$ will prefer the schedule $a$ over $b$ whenever

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu) + r\nu(p_a - \beta). \quad (21)$$

Note that the right hand side of this expression is strictly grater than $(p_a - p_b)(1 + \nu) + r\nu(p_a - p_b)$ since $\beta > p_b$. Therefore, since we maintain the assumption that a borrower of type $\gamma$ prefers schedule $a$ ensuring that inequality (20) holds, it must be true that the inequality (21) also holds and thus that an agent of type $\beta$ must also prefer schedule $a$ to schedule $b$.

In the remaining cases, it can be easily verified that the conditions which ensure that the $\beta$ type prefers the $a$ schedule to the $b$ schedule are identical either to the condition for the $\alpha$ type (in case v), the $\gamma$ type (in case iv) or both (in cases ii and iii). In each of the cases, therefore, if the preferred schedule for an $\alpha$ and $\gamma$ agent is the $b$ schedule, it is also the preferred one for agent of type $\beta$. This proves Lemma 3. ■

**Proof of Lemma 4**

**Proof.** Consider the cases that appear in the Proof for Lemma 3 above. In cases (ii), (iii) and (v), the conditions for an agent of type $\alpha$ to prefer a repayment of schedule $a$ to one of type $b$ are identical to the conditions for an agent of type $\beta$. It remains to consider cases of type (i) and (iv).

In Case (i) an agent of type $\beta$ prefers schedule $a$ to schedule $b$ whenever Inequality (21) holds, and an agent of type $\alpha$ prefers schedule $b$ to schedule $a$ whenever the following condition is satisfied:

$$\alpha - p_b + \nu(B + \alpha - p_b - q_b) \geq \alpha - p_a + \nu(B + \alpha - p_a - q_a), \tag{22}$$

or equivalently $\nu(q_b - q_a) \leq (p_a - p_b)(1+\nu)$ which contradicts (21).

Finally, in Case (iv), the condition for a type $\alpha$ agent to prefer the $b$ schedule is $\nu(q_b - q_a) \leq (p_a - p_b)(1+\nu)$ and the condition for a type $\beta$ agent to prefer the $a$ schedule is that $\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu)$. These conditions are mutually incompatible.

In all cases, therefore it cannot be that an agent of type $\alpha > \beta$ strictly prefers the schedule with the first payment $p_b < p_a$ and the agent of type $\beta$ prefers the schedule with the first payment $p_a$. This completes the proof. ■

**Proof of Proposition** 2

**Proof.** To show that with $r < \frac{2-\nu}{\nu}$ there cannot be an equilibrium where a continuum of borrowers are able to separate we proceed in a similar fashion as with the proof of Proposition 1 and show that if two agents that are arbitrarily close to each other are able to separate we reach a contradiction.

We start by conjecturing an equilibrium menu that achieves the separation of some agents in a continuum. Then pick two arbitrarily close agents $\alpha$ and $\alpha'$ with $\alpha < \alpha'$ and $p(\alpha) \neq p(\alpha')$. The corresponding break even second payments $q(\alpha) = \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu}$ and $q(\alpha') = \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}$. We know by Lemma 4 that $p(\alpha) < p(\alpha')$. These payment schedules have to fulfill similar incentive compatibility conditions as the ones shown in Proposition 1.

In particular we can define the two conditions as:

IC1: no agent of a higher type ($\alpha'$) wants to imitate an agent of a lower type ($\alpha$).

IC2: No agent of a lower type ($\alpha$) wants to imitate an agent of a higher type ($\alpha'$).

If there is a continuum of agents that can individually separate at least one of the following situations must be true.

a) At least two arbitrarily close agents are neither consuming or borrowing at $t = 1$

b) At least two arbitrarily close agents are both consuming $t = 1$

c) At least two arbitrarily close agents are both borrowing at $t = 1$

We analyse each of this situations in turn.

a) Is part of the equilibrium characterized by Proposition 1, and we know that IC2 cannot hold in this situation if $r < \frac{2-\nu}{\nu}$.

b) Suppose that there is an agent $\alpha'$ that fully separates from the rest and is able to consume at $t = 1$ (that is $p(\alpha') < \alpha'$. Then there must be an agent $\alpha$, such that $\alpha < \alpha'$, that is also able to pay $p(\alpha')$ without borrowing. The utility of agent $\alpha$ of claiming his own type is $\nu(B - \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu} + \alpha) + (\alpha - p(\alpha))$ and the utility of imitating agent $\alpha'$ is $\nu(B - \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha'))$. The necessary and sufficient condition for IC2 to hold is therefore:

$$\nu(B - \frac{D - p(\alpha) - (1-\nu)\alpha}{\nu} + \alpha) + (\alpha - p(\alpha)) > \nu(B - \frac{D - p(\alpha') - (1-\nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha'))$$

Which simplifies to: $(1-\nu)(\alpha-\alpha') > 0$ which is always false, so we reach a contradiction.

c) In this case we start by exploring IC2.

An agent of a lower type would have a utility of $\nu((B-\frac{D-p(\alpha)-(1-\nu)\alpha}{\nu}+\alpha)-r(p(\alpha)-\alpha))$, while claiming to be a higher type agent would yield her a utility of $\nu(B-\frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}+\alpha-r(p(\alpha')-\alpha))$. Subtracting the first term to the second we get a condition that must be smaller than zero for IC2 to hold.

$$\nu(B-\frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}-r(p(\alpha')-\alpha)+\alpha)-\nu((B-\frac{D-p(\alpha)-(1-\nu)\alpha}{\nu})-r(p(\alpha)-\alpha)+\alpha))<0$$

Which can be simplified as $(1-\nu)(\alpha'-\alpha)+(1-r\nu)(p(\alpha')-p(\alpha)))<0$

However, in this case IC1 becomes:

$$\nu(B-\frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}-r(p(\alpha')-\alpha')+\alpha')-\nu((B-\frac{D-p(\alpha)-(1-\nu)\alpha}{\nu})-r(p(\alpha)-\alpha')+\alpha'))>0$$

This expresion simplifies to $(1-\nu)(\alpha'-\alpha)+(1-r\nu)(p(\alpha')-p(\alpha)))>0$ which is exactly the opposite condition to the one necessary for IC2. Therefore, when two arbitrarily close agents borrow and achieve separation IC1 and IC2 are mutually incompatible, which poses a contradiction. ■

**Proof of Proposition 4**

**Proof.** We prove by contradiction. Suppose that this result is false, then there must be at least two types that pay different amounts. We focus on the highest two payments (and by Lemma 4 these will correspond to the highest differing types). We will find that in equilibrium, the top two pools would rather be combined as a single pool. Then an induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract as there cannot be any top two pools.

We continue by considering the top two pools of types that do not liquidate.

First note that if any type $\alpha$ strictly prefers not to liquidate then all types $\beta>\alpha$ would prefer to mimic $\alpha$ than to liquidate. Thus in restricting attention to the highest two payments $p_1<p_2$ and associated types (and by Lemma 4 we know that higher types are associated with higher payments) then we can be sure that there are some $\alpha_1\leq\alpha_2$ such that types $(\alpha_1,\alpha_2]$ pay $p_1$ in the first period (with the associated $q_1$) and $(\alpha_2,1]$ pay $p_2$ in the first period (with the associated $q_2$ in the second period).

The resulting contradiction is somewhat involved but the structure is as follows. First we highlight a number of possible cases. In each case we seek to determine the optimal choice of $\alpha_2$ (and associated $p_1$ and $p_2$) given that there are two pools in the range $(\alpha_1,1]$ while keeping $\alpha_1$ indifferent (and so all other types below may also remain with their existing contracts and there are no changes to equilibrium or welfare consequences from types below $\alpha_1$).[19]

By definition $p_2>p_1$. There are a number of cases to consider:

I $\quad p_2\geq 1$ and $p_1\geq\alpha_2$

II $\quad p_2\geq 1$ and $\alpha_2\geq p_1\geq\alpha_1$

III $\quad p_2\geq 1$ and $\alpha_1\geq p_1$

[19] While noting that sufficiently bizarre off-equilibrium beliefs could justify a wide range of equilibria, we focus on the most efficient equilibria (which would also be the one preferred by the borrowers).

IV $\quad 1 \geq p_2 \geq \alpha_2$ and $p_1 \geq \alpha_2 \geq \alpha_1$

V $\quad 1 \geq p_2 \geq \alpha_2$ and $\alpha_2 \geq p_1 \geq \alpha_1$

VI $\quad 1 \geq p_2 \geq \alpha_2$ and $\alpha_1 \geq p_1$

We focus on each case in turn and show that the optimum outcome in all cases pushes $\alpha_2$ into a corner. This necessarily implies that the equilibrium $\alpha_2$ that maximizes welfare is such that $\alpha_2 \in \{\alpha_1, 1, p_2\}$. The first two options $\alpha_2 \in \{\alpha_1, 1\}$ contradict the assumption that there are two distinct pools of borrowers. To complete the proof we finally suppose that $\alpha_2 = p_2$ and show that this too leads to a contradiction. So all the possible cases lead to a contradiction. By induction, if the top two pools cannot exist, the only possible equilibrium with a finite number of pools is one with only one pool. ■

**Lemma 6** *In the conjectured equilibrium* $\alpha_2 = p_1$

**Proof.** Note, first, that in equilibrium, the break even conditions imply that $q_1 = \frac{D-p_1-(1-\nu)\frac{\alpha_1+\alpha_2}{2}}{\nu}$ and $q_2 = \frac{D-p_2-(1-\nu)\frac{1+\alpha_2}{2}}{\nu}$.

We proceed by examining each of the cases highlighted above in turn.

**Case I** $p_2 \geq 1$ and $p_1 \geq \alpha_2$

The incentive compatibility condition for an agent of type $\alpha_2$ which ensures that she prefers the schedule $(p_2, q_2)$ to $(p_1, q_1)$ is:

$$\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) \geq \nu(B + \alpha_2 - q_1 - r(p_1 - \alpha_2)), \tag{23}$$

which yields $q_1 - q_2 \geq rp_2 - rp_1$. Substituting in for $q_1$ and $q_2$ and simplifying yields $p_2 \geq \frac{1-\nu}{\nu r-1}\frac{1-\alpha_1}{2} + p_1$. Note that since borrowing is inefficient, in equilibrium $p_2$ will be as low as possible and in particular, the constraint will bind and these conditions will hold with equality. In particular, note that this implies that $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2}$.

The relevant constraint for an agent of type $\alpha_1$ is that the agent is kept at a given level of utility, which we arbitrarily label $k$:

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k. \tag{24}$$

We can rearrange Equation (24) to obtain the following expression for $p_1$:

$$p_1 = \frac{1}{\nu r - 1}(-k + \nu B - D + (1-\nu)\frac{\alpha_1 + \alpha_2}{2} + \nu(1+r)\alpha_1). \tag{25}$$

It follows that $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$.

Given these expressions for $p_1$ and $p_2$, we can substitute them into the expression for welfare and maximize welfare with respect to $\alpha_2$ to determine its equilibrium value. Specifically, consider overall welfare

$$W = \int_{\alpha_1}^{1}(\nu B - D + 2x)dx - (\nu r - 1)\int_{\alpha_1}^{\alpha_2}(p_1 - x)dx - (\nu r - 1)\int_{\alpha_2}^{1}(p_2 - x)dx + cst, \tag{26}$$

where the final constant term arises from the welfare of those types $\alpha < \alpha_1$. It can be determined that

$$\frac{dW}{d\alpha_2} = (\nu r - 1)(p_2 - p_1) = \frac{1-\nu}{2}(1-\alpha_1)). \tag{27}$$

Note $\frac{dW}{d\alpha_2}$ is independent of $\alpha_2$ and so it is either always positive or always negative, and so either $\alpha_2 = \alpha_1$, which violates the assumption that there are two distinct pools or else $\alpha_2 = p_1$.

**Case II** $p_2 \geq 1$ and $\alpha_2 \geq p_1 \geq \alpha_1$

The incentive compatibility condition for an agent of type $\alpha_2$, as above will bind and is given by:

$$\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1). \tag{28}$$

Substituting for $q_1$ and $q_2$ and rearranging yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$. In particular, note that $\frac{dp_2}{d\alpha_2} = 1$.

The constraint for an agent of type $\alpha_1$ is given by

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k. \tag{29}$$

Substituting for $q_1$ and rearranging yields $p_1 = \frac{1}{\nu r-1}(-k + \nu B - D + (1-\nu)\frac{\alpha_1+\alpha_2}{2} + \nu(1+r)\alpha_1)$ and note in particular that $\frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)} > 0$.

Next we turn to welfare and maximizing with respect to $\alpha_2$ allows us to characterize the equilibrium level of $\alpha_2$. Overall welfare is given by:

$$W = \int_{\alpha_1}^{1} (\nu B - D + 2x)dx - (\nu r - 1)\int_{\alpha_1}^{p_1} (p_1 - x)dx - (\nu r - 1)\int_{\alpha_2}^{1} (p_2 - x)dx + cst, \tag{30}$$

and, in particular,

$$\frac{dW}{d\alpha_2} = (\nu r - 1)(p_2 - 1 - \frac{1-\nu}{2(\nu r - 1)}(p_1 - \alpha_1)), \tag{31}$$

where $p_2$ and $p_1$ are functions of $\alpha_2$ and defined above.

We must verify that the appropriate second order condition holds, and to this end note that

$$\frac{d^2W}{d\alpha_2^2} = (\nu r - 1)(1 - (\frac{1-\nu}{2(\nu r - 1)})^2). \tag{32}$$

It follows that the first order condition $\frac{dW}{d\alpha_2} = 0$ defines a maximum if and only if $1 < (\frac{1-\nu}{2(\nu r-1)})^2$ or, equivalently, $r < \frac{3-\nu}{2\nu}$. In particular, when $r > \frac{3-\nu}{2\nu}$ then $1 > \frac{1-\nu}{2(\nu r-1)}$ and so setting $\frac{dW}{d\alpha_2} = 0$ defines a minimum and so the maximum must be at a corner, that is the equilibrium value of $\alpha_2$ is either 1, which violates the assumption of two distinct pools, or $p_1$.[20]

In the subcase when $r < \frac{3-\nu}{2\nu}$ the first order condition does indeed define a maximum rather than a minimum; however, we argue that this case is vacuous: Recall that $p_1 < \alpha_2 < 1$, we also know, following Equation (29), that $\alpha_1$ satisfies $p_1 = \frac{1}{\nu r-1}(-k + \nu B - D + (1-\nu)\frac{\alpha_1+\alpha_2}{2} + \nu(1+r)\alpha_1)$. Rearranging this

[20] Note that it is possible to have full separation with $\alpha_1 = 1$ only in the case that the top pool is infinitesimally thin—the full separation case, but with a finite number of pools such an outcome is ruled out.

last expression, we obtain that:

$$p_1 - \frac{1}{\nu r - 1}(-k + \nu B - D) - \frac{1-\nu}{(\nu r - 1)}\frac{\alpha_2}{2} = \alpha_1 \frac{2\nu r + 1 + \nu}{2(\nu r - 1)}. \tag{33}$$

Further $p_1 < \alpha_2$ (by assumption in this Case), $(\nu B - D) < 0$ and $k \geq 0$, since bringing together constraint (29) for an agent of type $\alpha_1$ and the assumption that she prefers to continue than to liquidate. Bringing together these observations with Equation (33), it follows that $\alpha_1 \frac{2\nu r+1+\nu}{2(\nu r-1)} < \alpha_2 - \frac{1-\nu}{(\nu r-1)}\frac{\alpha_2}{2}$, rearranging we obtain $\alpha_1 < (\frac{2(\nu r-1)-1+\nu}{1+\nu+2\nu r})\alpha_2$. Given that $2(\nu r - 1) - 1 + \nu < 0$ when $r < \frac{3-\nu}{2\nu}$ then $\alpha_1 < 0$ which is impossible and so the sub-case where $r < \frac{3-\nu}{2\nu}$ is indeed vacuous.

**Case III** $p_2 \geq 1$ and $\alpha_1 \geq p_1$

The incentive compatibility condition for an agent of type $\alpha_2$ is given by Equation (28) which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$.

The participation constraint for an agent of type $\alpha_1$ is:

$$\alpha_1 - p_1 + \nu(B + \alpha_1 - q_1) = k. \tag{34}$$

Substituting for $q_1$, we obtain

$$\alpha_1 + \nu B - D + (1-\nu)\frac{\alpha_1 + \alpha_2}{2} = k. \tag{35}$$

Note that $p_1$ does not appear in this expression, neither does it appear (neither implicitly through $p_2$ nor explicitly) in the expression for welfare, which in this case is given by:

$$W = \int_{\alpha_1}^{1} (\nu B - D + 2x)dx - (\nu r - 1)\int_{\alpha_2}^{1}(p_2 - x)dx + cst. \tag{36}$$

Thus $p_1$ is unconstrained and does not affect welfare, without loss of generality therefore, we can take the limiting case that $p_1 = \alpha_1$.

**Case IV** $1 \geq p_2 \geq \alpha_2$ and $p_1 \geq \alpha_2 \geq \alpha_1$

The incentive compatibility condition for an agent of type $\alpha_2$ is just as in Case I, as given by Equation (23) and in particular $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2}$.

The participation constraint for an agent of type $\alpha_1$ is as in Equation (24) and so, in particular, $\frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$.

Noting that $\frac{dp_2}{d\alpha_2} = \frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$, and that welfare in this case is given by

$$W = \int_{\alpha_1}^{1} (\nu B - D + 2x)dx - (\nu r - 1)\int_{\alpha_1}^{\alpha_2}(p_1 - x)dx - (\nu r - 1)\int_{\alpha_2}^{p_2}(p_2 - x)dx + cst, \tag{37}$$

we can write

$$\frac{dW}{d\alpha_2} = -(\nu r - 1)p_1 - \frac{1-\nu}{2(\nu r - 1)}\alpha_1 + (\nu r - 1)p_2(2 - \frac{1-\nu}{2(\nu r - 1)}). \tag{38}$$

Since this expression is independent of $\alpha_2$, welfare is monotonic in $\alpha_2$ and takes its maximal value at an extremal value for $\alpha_2$, that is either at $\alpha_2 = \alpha_1$, or at $\alpha_2 = \min\{p_1, p_2\}$. In the latter case that $\alpha_2 = \min\{p_1, p_2\}$, the analysis of Case V applies.

**Case V** $\quad 1 \geq p_2 \geq \alpha_2$ and $\alpha_2 \geq p_1 \geq \alpha_1$

The incentive compatibility condition for an agent of type $\alpha_2$ is just as in Case I, as given by Condition (28) and in particular $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$ and $\frac{dp_2}{d\alpha_2} = 1$.

The participation constraint for an agent of type $\alpha_1$ is as in Equation (24) and so $\frac{dp_1}{d\alpha_2} = \frac{1-\nu}{2(\nu r-1)}$.

Welfare in this case is given by

$$W = \int_{\alpha_1}^{1} (\nu B - D + 2x)dx - (\nu r - 1)\int_{\alpha_1}^{p_1} (p_1 - x)dx - (\nu r - 1)\int_{\alpha_2}^{p_2} (p_2 - x)dx + cst, \tag{39}$$

and so, using the expressions derived above for $\frac{dp_2}{d\alpha_2}$ and $\frac{dp_1}{d\alpha_2}$, we can obtain:

$$\frac{dW}{d\alpha_2} = -\frac{1-\nu}{2}(p_1 - \alpha_1) < 0, \tag{40}$$

and so welfare is optimized (and the equilibrium value of $\alpha_2$ is chosen) where $\alpha_2$ is as low as possible that is where $\alpha_2 = p_1$.

**Case VI** $\quad 1 \geq p_2 \geq \alpha_2$ and $\alpha_1 \geq p_1$

The incentive compatibility condition for an agent of type $\alpha_2$ is given by Equation (28) which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$.

The participation constraint for an agent of type $\alpha_1$ is identical to that in Case III and just as in that case $p_1$ does not appear in this expression, neither does it appear (neither implicitly through $p_2$ nor explicitly) in the expression for welfare. Without loss of generality therefore, we can take the limiting case that $p_1 = \alpha_1$.

**Completing the proof**

By Lemma 6 $\alpha_2 = p_1$, there are two possibilities to consider either $p_2 > 1$ or $1 > p_2 > \alpha_2$, in both cases $p_2 > \alpha_2$. We show that this is inconsistent with the maintained assumption that $1 > \alpha_2 > \alpha_1$.

When $p_2 > \alpha_2 > \alpha_1$, the incentive compatibility condition for an agent of type $\alpha_2$ is given by Equation (28) which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$.

The constraint for an agent of type $\alpha_1$

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k. \tag{41}$$

Substituting for $q_1$ and since $p_1 = \alpha_2$, we obtain:

$$\alpha_2 = \frac{2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1}{\nu + 2r\nu - 3}. \tag{42}$$

The case is not degenerate, that is in equilibrium these top two pools do not collapse into one, as long as $\alpha_2$ is interior. In particular, it must be that both $\alpha_2 > \alpha_1$ and $1 > \alpha_2$. Specifically, substituting from (42) and rearranging $\alpha_2 > \alpha_1$ if and only if $\alpha_1 > \frac{1}{2}(1 + k)$. Similarly, $\alpha_2 < 1$ if and only if

$\alpha_1 < \frac{\nu+2r\nu-1+2k}{(1+\nu+2r\nu)}$. For an interior solution $1 > \alpha_2 > \alpha_1$ both condition must hold and in particular:

$$\frac{\nu + 2r\nu - 1 + 2k}{(1 + \nu + 2r\nu)} > \frac{1}{2}(1 + k), \tag{43}$$

rearranging this is true if and only if $k > 1$. This is impossible—the highest possible utility for a borrower is for the best possible type (type 1) to be recognized as such and in this case her expected utility would be $\nu B - D + 1 + 1 = 1$ and so it cannot be that $k$, which is the expected utility for the $\alpha_1$ type, is greater than 1.

This final contradiction completes the proof. ■

**Lemma 7** *The condition $W_I \geq 1 + y$ is more likely to hold the smaller is $y$.*

**Proof.** First note $W_I \geq 1 + y$ if and only if

$$A = \begin{array}{c} -(-2y + 2\nu - 2y\nu - 4ry\nu + \nu^2 - 3)(6\nu + 8r\nu + \nu^2 + 1)2y \\ -2(-2y + 2\nu - 2y\nu - 4ry\nu + \nu^2 - 3)^2 \\ -(\nu r - 1)(-4y\nu - 2\nu - 4ry\nu + \nu^2 + 1 - (-2y + 2\nu - 2y\nu - 4ry\nu + \nu^2 - 3))^2 \end{array} \geq 0 \tag{44}$$

Taking the derivative with respect to $y$ yields

$$\frac{dA}{dy} = -\begin{array}{c} 8\nu(1-\nu) + 16r\nu(1-\nu)^2 - 8y\nu - 8ry\nu - 12\nu^2 + 8\nu^3 \\ +2\nu^4 - 48y\nu^2 - 8y\nu^3 - 112ry\nu^2 - 8ry\nu^3 - 64r^2y\nu^2 + 2 \end{array} \tag{45}$$

Note that this expression is linear in $y$ and in $r$ and so $\frac{dA}{dy}$takes its maximal value when $y = 0$ then and $r = \frac{1}{\nu}$ when this value is

$$\frac{dA}{dy} = -8\nu^3 + 4\nu^2 + 24\nu - 2\nu^4 - 18, \tag{46}$$

which it can be easily verified is non-positive in the range $\nu \in (0, 1)$. ■

**Proof of Proposition 6**

**Proof.** We begin by considering the parameter range $r < \frac{2-\nu}{\nu}$, $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)} > y$ and $W_I > 1 + y$. In this range $W = W_I$.

Note that $p - l = \frac{2(1-\nu)(2+y)}{6\nu+8r\nu+\nu^2+1}$ and so $\frac{d(p-l)}{dr} = -\frac{16\nu(1-\nu)(2+y)}{(6\nu+8r\nu+\nu^2+1)^2}$. Next, by taking the derivative of $W$ with respect to $r$ from Equation 6, we obtain:

$$\frac{dW}{dr} = \frac{dl}{dr}(-y - 2l) - \frac{1}{2}\nu(p - l)^2 - (\nu r - 1)(p - l)\frac{d(p - l)}{dr}. \tag{47}$$

Substituting in for $\frac{dl}{dr}$ and $\frac{d(p-l)}{dr}$ and simplifying:

$$\frac{dW}{dr} = \frac{4\nu + 16\nu^2 - 40\nu^3 + 16\nu^4 + 4\nu^5 + 32r\nu^2 - 64r\nu^3 + 32r\nu^4}{2(6\nu + 8r\nu + \nu^2 + 1)^3}(2 + y). \tag{48}$$

The denominator of this expression is positive and $(2+y)>0$ and so $\frac{dW}{dr}$ has the same sign as the numerator of the fraction. Specifically,

$$\begin{aligned} sign(\tfrac{dW}{dr}) &= sign(4\nu+16\nu^2-40\nu^3+16\nu^4+4\nu^5+32r\nu^2-64r\nu^3+32r\nu^4) \\ &= sign(1+4\nu-10\nu^2+4\nu^3+\nu^4+8r\nu-16r\nu^2+8r\nu^3) \end{aligned} , \tag{49}$$

where the second equality holds, since the sign of the factor $(4\nu)$ is positive. It follows that $\frac{dW}{dr}>0$ if and only if $1+4\nu-10\nu^2+4\nu^3+\nu^4+8r\nu-16r\nu^2+8r\nu^3>0$, which is true if and only if:

$$\frac{1+4\nu-10\nu^2+4\nu^3+\nu^4}{16\nu^2-8\nu-8\nu^3}>r. \tag{50}$$

Note that $\frac{2-\nu}{\nu}>r\geq\frac{1}{\nu}$ and so $\frac{dW}{dr}>0$ requires

$$\frac{1+4\nu-10\nu^2+4\nu^3+\nu^4}{16\nu^2-8\nu-8\nu^3}>\frac{1}{\nu}, \tag{51}$$

or equivalently,

$$4\nu^3-2\nu^2-12\nu+\nu^4+9>0, \tag{52}$$

which is always true for $\nu$ in the range $(0,1)$.

Outside of the parameter range $r<\frac{2-\nu}{\nu}$, $-\frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)}>y$ and $W_I>1+y$, $\frac{dW}{dr}=0$ trivially since $W$ is independent of $r$. ■