AVOIDANCE AND MITIGATION OF PUBLIC HARM

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Introduction.

Historically, there are many episodes/cases of financial turmoil. The outcome of the troubled party ranges from complete failure/bankruptcy to full bailout/recovery.

- Firms.
  Bailout: GM, Chrysler
  Bankruptcy: Pan Am (1991), Daewoo (1999)
- Financial institutions.
  Bailout: LTCM (1998), Citigroup (2008)
  Bankruptcy: Lehman Brothers (2008)
  Washington Mutual (2008)
- Sovereign countries.
  1994 Mexico Tequila crisis
  1997 Asian financial crisis
  Current Euro area crisis

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Two conflicting views about bailout:

- Financial turmoil/failures often would generate too much negative externality, so bailout is beneficial and sometimes necessary ex-post. Too-big-to-fail is consistent with this view.
- Bailout creates moral hazard problem: institutions have less incentive to be diligent to reduce crisis incidence since they know that they will be bailed out.

A third view:

- The observed pattern of bailing out some troubled institutions, but not others, is consistent with the view that the optimal bailout policy is a mixed strategy that deals with both views above.

Research program

- Construct a schematic, non-cooperative, 2-player model
  - One agent takes costly, unobservable action to try to avert a crisis.
  - If the crisis occurs, both agents decide how much to contribute mitigating it.
- Characterize Nash equilibrium of the one-shot game: both bailout and no-bailout equilibria always exist.
- Consider an infinite repetition of the one-shot stage game
  - Study in particular equilibrium that minimizes expected, discounted total cost.
  - Is some equilibrium consistent with the third view?

The one-shot game

- Two agents.
  agent 1 — active
  agent 2 — passive
- Two periods.

Period 1:

- Agent 1 chooses $a \in A = \{0,\, 1\}$ (avoidance/no avoidance)
  The cost of avoidance is d.
- The state $\xi \in X = \{0,\, 1\}$ is realized.

$$Pr(\xi = 1 \,|\, a = 0) = 1$$
$$Pr(\xi = 1 \,|\, a = 1) = \varepsilon$$
$$Pr(\xi = 0 \,|\, a = 1) = 1 - \varepsilon$$

$\varepsilon \in (0, 1)$.

Period 2:

- If $\xi = 1$ (crisis state), the two agents play a mitigation game.
  Agent i contributes $m_i \in M = [0,\, 1]$

$$u_i(1,\, m_1,\, m_2) = \begin{cases} -m_i & \text{if } m_1 + m_2 \geq 1 \\ -m_i - c_i & \text{otherwise} \end{cases}$$

- If $\xi = 0$, no mitigation is necessary.

$$u_i(0,\, m_1,\, m_2) = -m_i$$

Assumption 1.

$$c_i \in (0, 1) \quad \text{for } i = 1, 2.$$
$$c_1 + c_2 > 1.$$

Nash equilibrium of the one-shot game

Period 2. Mitigation game.

Agent i's period-2 strategy $m_i(\xi)$, $m_i \colon X \to M$.

When $\xi = 0$, no need to contribute, $m_1^*(0) = m_2^*(0) = 0$.

When $\xi = 1$, two types of Nash equilibrium.

- No-bailout: neither agent contributes anything,
$$m_1^o(1) = m_2^o(1) = 0$$
$$u_i(1, m_1^o(1), m_2^o(1)) = -c_i.$$
- Bailout: jointly contribute 1 unit to mitigate
$$m_1^b(1) \in [1 - c_2, c_1], \quad m_2^b(1) = 1 - m_1^b(1)$$
$$u_i(1, m_1^b(1), m_2^b(1)) = -m_i^b(1)$$

Period 1. Agent 1's avoidance decision $a \in A$.

$v_i(a,\, m_1,\, m_2)$—the expected value of agent i in period 1 if

- agent 1 takes period-1 action a,
- two agents' strategy in period 2 is $(m_1(\xi),\, m_2(\xi))_{\xi \in X}$.

$$\begin{aligned} v_1(a,\, m_1,\, m_2) &= \sum_{\xi \in X} Pr(\xi|a) u_1(\xi,\, m_1(\xi),\, m_2(\xi)) - ad \\ v_2(a,\, m_1,\, m_2) &= \sum_{\xi \in X} Pr(\xi|a) u_2(\xi,\, m_1(\xi),\, m_2(\xi)) \end{aligned}$$

Agent 1's optimal period-1 action a depends on which of the period-2 equilibrium is to be played in case of crisis.

If no-bailout equilibrium $(m_1^o(1),\, m_2^o(1))$ is anticipated,

$$v_1(a, m_1^o, m_2^o) = \begin{cases} -c_1 & \text{if } a = 0 \\ -d - \varepsilon c_1 & \text{if } a = 1 \end{cases}$$

$$\text{the optimal action is} \quad a^o = \begin{cases} 1 & \text{if } c_1 \geq \frac{d}{1-\varepsilon} \\ 0 & \text{otherwise} \end{cases}$$

If the bailout equilibrium $(m_1^b(1),\, m_2^b(1))$ is anticipated,

$$v_1(a, m_1^b, m_2^b) = \begin{cases} -m_1 & \text{if } a = 0 \\ -d - \varepsilon m_1 & \text{if } a = 1 \end{cases}$$

$$\text{the optimal action is} \quad a^b = \begin{cases} 1 & \text{if } m_1 \geq \frac{d}{1-\varepsilon} \\ 0 & \text{otherwise} \end{cases}$$

Table 1. Equilibrium of the one-shot game

parameter range	a	$m_1(1)$	ex-ante cost
(1) $\frac{d}{1-\varepsilon} \leq 1 - c_2$	1	$[1-c_2, c_1]$	$d+\varepsilon$
	1	0	$d+\varepsilon(c_1+c_2)$
(2) $1 - c_2 < \frac{d}{1-\varepsilon} \leq c_1$	1	$[\frac{d}{1-\varepsilon}, c_1]$	$d+\varepsilon$
	0	$[1-c_2, \frac{d}{1-\varepsilon}]$	1
	1	0	$d+\varepsilon(c_1+c_2)$
(3) $c_1 < \frac{d}{1-\varepsilon}$	0	$[1-c_2, c_1]$	1
	0	0	c_1+c_2

- By Assumption 1, $0 < 1 - c_2 < c_1 < 1$.
- Regardless of the parameter region, both bailout and no-bailout equilibrium always exist.
- Any combination of avoidance and mitigation can occur.

Ex-ante expected total cost of (a, m_1, m_2)

$$= ad + (1 - a + a\varepsilon)[m_1 + m_2 + (c_1 + c_2)I_{\{m_1+m_2<1\}}]$$

An action profile (a, m_1, m_2) is said to *ex-ante dominate* another one if it has a lower expected total cost.

- Assumption 1 says that bailout dominates no-bailout ex-post ($c_1 + c_2 > 1$). In region (1) and (3), bailout also dominates ex-ante.
- In region (2), avoidance/bailout achieves the lowest ex-ante expected total cost among all equilibria. The ranking of the other two types of equilibrium is unclear.

The repeated game

Time is discrete, $t = 1, 2, \ldots$.

At each date t, the two-period one-shot game is played between the two players with discount factor $\delta \in (0, 1)$.

Public information.

- At t, $h_t = (\xi_t, m_{t1}, m_{t2}) \in H \equiv X \times M^2$.
- History of public information at the beginning of date t,

$$h^t = (h_1, \ldots, h_{t-1}) \in H^{t-1}$$

  $H^0 = \emptyset$.
- When agents decide (m_{t1}, m_{t2}), the public information is $(h^t, \xi_t) \in H^{t-1} \times X$.

Private information.

- Agent 1's avoidance decision $\{a_t\}_{t=1}^{\infty}$ is private and never revealed.

A strategy is *public* if it depends only on public history.

Without loss of generality, focus on perfect Bayesian equilibrium where both agents play public strategies.

Strategy profile $(\alpha, \sigma) = (\alpha, \sigma_1, \sigma_2) = (\alpha_t, \sigma_{t1}, \sigma_{t2})_{t=1}^{\infty}$

$$\alpha_1 \in \Delta(A), \quad \forall t > 1, \quad \alpha_t \colon H^{t-1} \to \Delta(A)$$

for $i = 1, 2$,

$$\sigma_{1i} \colon X \to M, \quad \forall t > 1, \quad \sigma_{ti} \colon H^{t-1} \times X \to M$$

Let Σ_i denote the set of agent i's public strategies.

Expected present discounted value of payoff stream induced by strategy profile (α, σ), $V(a, \sigma) = (V_1(\alpha, \sigma), V_2(a, \sigma))$,

$$V_i(\alpha, \sigma) = (1 - \delta)\mathsf{E}[\sum_{t=1}^{\infty} \delta^{t-1} \sum_{a \in A} \alpha_t(h^t)(a) v_i(a, \sigma_t(h^t, \xi_t))]$$

For any public history h^t, let $(\alpha|_{h^t}, \sigma|_{h^t})$ denote the strategy profile induced by (α, σ) after t periods of history.

Definition 1. A public strategy profile (α^*, σ^*) is a *perfect public equilibrium (PPE)* if $\forall t \geq 1$, $\forall h^t \in H^{t-1}$, $(\alpha^*|_{h^t}, \sigma^*|_{h^t})$ is a Nash equilibrium from t on, that is, for $i = 1, 2$, for any other public strategy $(\alpha, \sigma_1) \in \Sigma_1$, $\sigma_2 \in \Sigma_2$,

$$V_1(\alpha^*|_{h^t}, \sigma_1^*|_{h^t}, \sigma_2^*|_{h^t}) \geq V_1(\alpha|_{h^t}, \sigma_1|_{h^t}, \sigma_2^*|_{h^t})$$
$$V_2(\alpha^*|_{h^t}, \sigma_1^*|_{h^t}, \sigma_2^*|_{h^t}) \geq V_2(\alpha^*|_{h^t}, \sigma_1^*|_{h^t}, \sigma_2|_{h^t})$$

and $\forall \xi_t \in X$,

$$\begin{aligned}
&(1-\delta)u_1(\xi_t, \sigma_{t1}^*, \sigma_{t2}^*) + \delta V_1(\alpha^*|_{h^{(t+1)*}}, \sigma_1^*|_{h^{(t+1)*}}, \sigma_2^*|_{h^{(t+1)*}}) \\
&\quad \geq (1-\delta)u_1(\xi_t, \sigma_{t1}, \sigma_{t2}^*) + \delta V_1(\alpha|_{h^{(t+1)1}}, \sigma_1|_{h^{(t+1)1}}, \sigma_2^*|_{h^{(t+1)1}}) \\
&(1-\delta)u_2(\xi_t, \sigma_{t1}^*, \sigma_{t2}^*) + \delta V_2(\alpha^*|_{h^{(t+1)*}}, \sigma_1^*|_{h^{(t+1)*}}, \sigma_2^*|_{h^{(t+1)*}}) \\
&\quad \geq (1-\delta)u_2(\xi_t, \sigma_{t1}^*, \sigma_{t2}) + \delta V_1(\alpha^*|_{h^{(t+1)2}}, \sigma_1^*|_{h^{(t+1)2}}, \sigma_2|_{h^{(t+1)2}})
\end{aligned}$$

where

$$h^{(t+1)*} = (h^t, \xi_t, \sigma_{t1}^*, \sigma_{t2}^*)$$
$$h^{(t+1)1} = (h^t, \xi_t, \sigma_{t1}, \sigma_{t2}^*), \quad h^{(t+1)2} = (h^t, \xi_t, \sigma_{t1}^*, \sigma_{t2})$$

A PPE always exists: repetition of any static Nash equilibrium of the two-period stage game is a PPE.

Let $\mathcal{V}$ denote the set of PPE payoff vectors,

$$\mathcal{V} = \{V(\alpha, \sigma) \,|\, (\alpha, \sigma) \; is \; a \; PPE \; \}$$

$\mathcal{V} \neq \emptyset$.

Following APS (1990), find $\mathcal{V}$ through a self-generation procedure.

Define expected payoff of action profile (ϕ, m_1, m_2) if continuation value is $w\colon X \times M^2 \to \Re^2$, for $i = 1, 2$,

$$\begin{aligned}
g_i(\phi, m_1, m_2, w) \;\; \equiv \;\; & \sum_{a \in A} \phi(a) \Big[(1-\delta) v_i(a, m_1, m_2) \\
& \qquad + \delta \sum_{\xi \in X} Pr(\xi|a) w_i(\xi, m_1(\xi), m_2(\xi)) \Big]
\end{aligned}$$

Definition 2. For any $W \subset \Re^2$, an action profile (ϕ, m_1, m_2) together with payoff function $w\colon X \times M^2 \to \Re^2$ is *admissible* with respect to W if

(1) $\forall \xi \in X,\ w(\xi, m_1(\xi), m_2(\xi)) \in W$.
(2) $(\phi, m_1) = arg\max_{\phi' \in \Delta(A), \{m_1'(\xi) \in M\}_{\xi \in X}} g_1(\phi', m_1', m_2, w)$
(3) For any $\xi \in X$, for any m_1' and m_2',

$$\begin{aligned}
&(1-\delta)u_1(\xi, m_1(\xi), m_2(\xi)) + \delta w_1(\xi, m_1(\xi), m_2(\xi)) \\
&\quad \geq (1-\delta)u_1(\xi, m_1'(\xi), m_2(\xi)) + \delta w_1(\xi, m_1'(\xi), m_2(\xi)) \\
&(1-\delta)u_2(\xi, m_1(\xi), m_2(\xi)) + \delta w_2(\xi, m_1(\xi), m_2(\xi)) \\
&\quad \geq (1-\delta)u_2(\xi, m_1(\xi), m_2'(\xi)) + \delta w_2(\xi, m_1(\xi), m_2'(\xi))
\end{aligned}$$

For any $W \subset \Re^2$, define

$$B(W) = \Big\{ r \mid \exists(\phi, m_1, m_2, w) \text{ admissible w.r.t. W} \\ \text{such that } r = g(\phi, m_1, m_2, w) \Big\}$$

Then $B(\mathcal{V}) = \mathcal{V}$.

The set of PPE payoff vectors $\mathcal{V}$ can be obtained numerically by starting from some initial set $W^0 \subset \Re^2$,

$$B^t(W^0) \to \mathcal{V} \quad \text{as} \quad t \to \infty$$

PPE that minimizes the expected discounted total cost

Case 1. $\frac{d}{1-\varepsilon} \leq 1 - c_2$

- Repetition of avoidance/bailout at every date,

  $$a_t^* = 1, \quad \sigma_{t1}^*(m_1) = m_1, \quad \sigma_{t2}^*(m_1) = 1 - m_1$$

  where $m_1 \in [1 - c_2, c_1]$.

Case 2. $1 - c_2 < \frac{d}{1-\varepsilon} \leq c_1$

- Repetition of avoidance/bailout at every date,

  $$a_t^* = 1, \quad \sigma_{t1}^*(m_1) = m_1, \quad \sigma_{t2}^*(m_1) = 1 - m_1$$

  where $m_1 \in [\frac{d}{1-\varepsilon}, c_1]$.

In both cases, (a^*, σ^*) is a PPE since it is a repetition of the static Nash equilibrium of the stage game.

Case 3. $c_1 < \frac{d}{1-\varepsilon}$

- At any static Nash equilibrium of the stage game, agent 1 chooses no avoidance.

Assumption 2. $d + \varepsilon < 1$.

That is, avoidance/bailout yields the lowest one-period ex-ante expected total cost.

Question 1: Can avoidance/bailout be sustained at some PPE of the repeated game?

An example of a simple mechanism

Assume $c_1 < \frac{d}{1-\varepsilon}$.

Two-state automaton, $\{S,\, \mu^0,\, (f_1,\, f_2),\, \pi\}$

- The set of states $S = \{0,\, 1\}$.
- Distribution of initial state $\mu^0 \in \Delta(S)$.
- Decision rule $f_1\colon S \to A \times M,\;\; f_2\colon S \to M$

  $f_{11}(0) = f_{11}(1) = 1$
  $f_{12}(0) = m_1^0, \quad f_2(0) = 1 - m_1^0$
  $f_{12}(1) = m_1^1, \quad f_2(1) = 1 - m_1^1$

  That is, avoidance/bailout is imposed.
  Assume that $m_1^1 \geq m_1^0$.

- Transition probability $\pi\colon S \times X \to \Delta(S)$,

  $\pi(0,0) = 1 - \varepsilon\theta_0, \quad \pi(0,1) = \varepsilon\theta_0$
  $\pi(1,0) = (1-\varepsilon)(1-\theta_1) + \varepsilon(1-\theta_2)$
  $\pi(1,0) = (1-\varepsilon)\theta_1 + \varepsilon\theta_2$

  where

$$\theta_0 = \text{Prob}(s' = 1 \,|\, s = 0,\; \xi = 1) \in [0,1]$$
$$\theta_1 = \text{Prob}(s' = 1 \,|\, s = 1,\; \xi = 0) \in [0,1]$$
$$\theta_2 = \text{Prob}(s' = 1 \,|\, s = 1,\; \xi = 1) \in [0,1]$$

Question 2: Can this automaton, in particular, the decision rule $(f_1,\, f_2)$ be supported as a PPE?

- If the answer is yes, then the answer to question 1 is affirmative. That is, avoidance/bailout can be sustained as a PPE of the repeated game.

Claim. No, the automaton can not be supported as a PPE.

- The automaton has an ergodic distribution:
$$\bar{\mu}(1) = \frac{\varepsilon\theta_0}{1 + \varepsilon\theta_0 - [(1-\varepsilon)\theta_1 + \varepsilon\theta_2]}$$
$\bar{\mu}(0) = 1 - \mu(1)$. Assume that $\mu^0 = \bar{\mu}$.
- Calculate the expected discounted value for agent 1, (V_1^0, V_1^1)
- To support f_1 as agent 1's decision rule, (V_1^0, V_1^1) has to satisfies some IC constraints. The one for $f_{11}(s) = 1$ is
$$\delta\theta_0(V_1^0 - V_1^1) \geq (1-\delta)\Big[\frac{d}{1-\varepsilon} - m_1^0\Big]$$
which is equivalent to
$$\psi m_1^1 + (1-\psi)m_1^0 \geq \frac{d}{1-\varepsilon} \qquad (**)$$
where
$$\psi = \frac{\varepsilon\theta_0\delta}{1 + \delta\varepsilon\theta_0 - \delta[(1-\varepsilon)\theta_1 + \varepsilon\theta_2]} \leq \bar{\mu}(1)$$

The expected discounted total cost of the automaton to agent 1 is
$$\begin{aligned} & (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\Big(d + \varepsilon(\bar{\mu}(1)m_1^1 + \bar{\mu}(0)m_1^0)\Big) \\ = \;& d + \varepsilon\big(\bar{\mu}(1)m_1^1 + \bar{\mu}(0)m_1^0\big) \\ \geq \;& d + \varepsilon\big(\psi m_1^1 + (1-\psi)m_1^0\big) \\ \geq \;& d + \varepsilon\frac{d}{1-\varepsilon} \qquad \text{(by } (**)\text{)} \\ = \;& \frac{d}{1-\varepsilon} \; > \; c_1 \end{aligned}$$
The expected discounted total cost of no-avoidance/no-bailout for agent 1 is c_1 which is his minmax value of the game.

So $a = 1$ at every date is not incentive compatible for agent 1, and hence can not be an equilibrium strategy.

Conjecture 1. Assume that $c_1 < \frac{d}{1-\varepsilon}$. Avoidance/bailout regardless history can not be supported as a PPE.

- To show this, I will show that any given PPE payoff $v \in \mathcal{V}$ can be achieved with an appropriately programmed two-state automaton.

Conjecture 2. Assume that $c_1 < \frac{d}{1-\varepsilon}$. A modified two-state automaton with randomized decision rule, in particular, $f_1 \colon S \to \Delta(A) \times M$, may be supported as a PPE.

- If this is true, at such a PPE, the incidence of crisis is higher than ε, and higher punishment for agent 1, jointly governed by m_1^1, $\theta_0, \theta_1, \theta_2$, may be necessary.