Labor-Market Volatility in the Search-and-Matching Model: The Role of Investment-Specific Technology Shocks

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Abstract

Shocks to investment-specific technology have been identified as a main source of U.S. aggregate output volatility. In this paper we assess the contribution of these shocks to the volatility of labor market variables, namely, unemployment, vacancies, tightness and the job-finding rate. Thus, our paper contributes to a recent body of literature assessing the ability of the search-and-matching model to account for the large volatility observed in labor market variables. To this aim, we solve a neoclassical economy with search and matching in the labor market, where neutral and investment-specific technologies are subject to shocks. The three key features of our model economy are: *i)* Firms are large, in the sense that they employ many workers. *ii)* Adjusting capital and labor is costly. *iii)* Wages are the outcome of an intra-firm Nash-bargaining problem between the firm and its workers. In our calibrated economy, we find that shocks to investment-specific technology explain 40 percent of the observed volatility in U.S. labor productivity. Moreover, these shocks generate relative volatilities in vacancies and the workers' job finding rate which match those observed in U.S. data. Relative volatilities in unemployment and labor market tightness are 55 and 75 percent of their empirical values, respectively.

Keywords: Business Cycles; Labor Market Fluctuations; Investment-Specific Technical Change; Search and Matching; Adjustment Costs; Wage Bargaining.

JEL Classification Numbers: E22; E24; E32; J41; J64; O33.

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1 Introduction

In the last few years, a large and active literature has emerged around the unemployment volatility puzzle. More precisely, this literature assesses the extent to which the search-and-matching model with Nash wage bargaining can account for the following three observations: 1) large fluctuations in labor market variables relative to the fluctuations of labor productivity; 2) low sensitivity of unemployment with respect to unemployment benefits, and 3) a high correlation between wages and productivity in new matches. (Admittedly, the extent of this latter correlation is still a subject of debate and more contributions will certainly be produced.)

Shimer (2005) and Costain and Reiter (2008) have shown that the textbook version of the Mortensen-Pissarides model is unable to generate the observed relative fluctuations in labor market variables in response to shocks to labor productivity. The failure of the model is to be found in the surplus' sharing rule implied by Nash bargaining. That is, wages absorb most of the increases in labor productivity, thus reducing the procyclicality of the firm's share of the surplus and so the incentive for vacancy creation. Costain and Reiter (2008) and Hagedorn and Manovskii (2008) note that a different calibration of the model can generate large fluctuations in labor market variables. Indeed, with high, acyclical non-market returns to workers and low workers' bargaining power, wages become relatively rigid and the firm's share of surplus more procyclical, restoring the firm's incentives to create vacancies. However, as pointed out by Costain and Reiter (2008) and by Pissarides (2008), this calibration strategy implies a counterfactually high sensitivity of unemployment to non-market returns. A different line of research has advocated for the replacement of continuous Nash wage bargaining by some stickier sharing rule. Gertler and Trigari (2008) propose wage stickiness à la Calvo. Hall (2005) argues in favor of efficient wage stickiness where wages do not react, or only partially, to high-frequency changes in labor productivity.[1] Objections to models with sticky wages have been raised by Pissarides (2008) and Haefke, Sonntag and van Rens (2008) by arguing that they fail to generate the high correlation between wages and productivity observed in new matches.

In this paper, we retain the assumption of continuous Nash renegotiation of wages and contribute to this literature by endogenizing labor productivity to the firm's investment and hiring policy. The mechanism we explore in this paper relies on the firm's adjustment of capital and labor to exogenous changes in technology. With this aim, we remove the assumption of employer-worker pairs producing without capital and assume instead the standard neoclassical firm that employs many workers and owns capital.[2] We then explore the ability of the model to amplify

[1] Different sources of wage rigidity in the search-and-matching model have been studied e.g. by Hall and Milgrom (2008), Kennan (2006), Menzio (2005), Moen and Rosen (2006) and Rudanko (2009).

[2] For early work on the search-and-matching model with large firms see Andolfato (1996) and Merz (1995).

the volatility of labor market variables after shocks to both neutral and investment-specific technology. We model this latter type of technology as in Greenwood, Hercowitz and Krusell (1997, 2000), which allows us to calibrate investment-specific technology shocks using the cyclical component of the relative price of new capital goods. In our model economy, job separations within the firm occur exogenously and capital depreciates at a constant rate. To hire workers the firm must open vacancies and then negotiate wages for new and continuing workers. Adjusting the level of capital and employment is costly and these costs are jointly determined by investment and hiring rates. An important consequence of the large firm assumption is the so-called intra-firm bargaining, that is, the fact that the firm anticipates the wage effects of its hiring and investment policy.[3] By virtue of intra-firm bargaining, the wage function in our model becomes increasing in the level of neutral technology and decreasing in investment-specific technology.

In our calibrated economy, shocks to investment-specific technology account for 40 percent of the observed volatility in U.S. labor productivity. Moreover, these shocks generate relative volatilities in vacancies and the workers' job finding rate which match those observed in U.S. data. Relative volatilities in unemployment and labor market tightness are 55 and 75 percent of their empirical values, respectively. In other words, from this quantitative exercise we conclude that 40% of the volatility in labor productivity explains 22% of the observed volatility in unemployment, 40% of the volatility in vacancies, 30% of the volatility in tightness and 40% of the volatility in the job finding rate. These numbers are about one order of magnitude higher than those obtained by Shimer (2005) within the textbook version of the Mortensen-Pissarides model.

The mechanism for amplification works through the costly adjustment of capital and labor and its effect on the intra-firm bargained wage. There are two main forces shaping the response of vacancies to investment-specific technology shocks in our model economy. First, a positive shock to investment-specific technology increases investment by making capital relatively cheap. Hence, convex adjustment costs create a tension between investment and hiring, leading to an initial contraction in hiring. This initial freeze in hiring is compensated by an increase in hours of work per employee when the intensive margin in labor is operative. The second force comes from the increased productivity of labor caused by investment and from the compressing effect of investment-specific technology shocks on wages, which increases the procyclicality of the firm's share of the surplus with respect to these shocks. This second force leads to a delayed increase in hiring after the positive shock to technology. The dynamics of factor adjustment just described matches recent empirical findings on the short-run response of labor market variables to investment-specific shocks. We will elaborate further on this in the main text of this paper.

[3]See Stole and Zwiebel (1996a, 1996b) for a general discussion of intra-firm bargaining. For studies of intra-firm bargaining within the search-and-matching model see, e.g., Cahuc and Wasmer (2001), Krause and Lubik (2007) and Rotemberg (2006).

The increased volatility in labor market variables in our model economy is not at the cost of a counterfactually high sensitivity of unemployment to unemployment benefits. Since we calibrate the model to match a replacement rate of 45 percent, non-market returns are low compared to returns from employment. In our benchmark economy, the semi-elasticity of unemployment with respect to benefits is slightly above one, which is in the lower bound of estimated values. On the other hand, the correlation between wages and labor productivity in our economy with investment-specific technology shocks is 0.9, a value which is in line with the correlation estimated in new matches.

The way we introduce investment-specific technological change in our model economy closely follows the original idea of technology embedded in new investment goods, which the firm can acquire by devoting resources to investment. As in Greenwood, Hercowitz and Krusell (1997, 2000), we allow the firm to continuously invest in new capital, in contrast to the Schumpeterian view where only newly created firms have access to new technologies, with all its implications in terms of employment reallocation. In this regard, the mechanism we explore in this paper to generate volatility in labor market variables after shocks to investment-specific technology is different from the ones put forward recently by Reiter (2008), Hornstein, Krusell and Violante (2007) and Michelacci and Lopez-Salido (2007). Reiter (2008) abstracts from capital and retains the assumption of employer-worker pairs. This author models embodied technical change by assuming that the exogenous process for labor productivity has a permanent match-specific component. Hornstein, Krusell and Violante (2007) introduce capital into the employer-worker pair but assume that firms in an existing match can never adjust their capital stock in response to a change in the environment. In these two works, amplification in unemployment and vacancies is at the cost of an excessive volatility in wages. Michelacci and Lopez-Salido (2007) are interested on the effects of neutral and investment-specific technical change on job flows, especially on job destruction. They assume that while newly created jobs embody the most advanced technologies, existing jobs fail to upgrade their capital. They find that advances in investment-specific technology reduce job destruction.[4]

Finally, the work of Cooper et al. (2007), Elsby and Michaels (2008) and Yashiv (2008) also use the assumption of large firms, but they abstract from capital and from investment-specific technology. These authors study labor market volatility by introducing idiosyncratic labor productivity shocks.

The remainder of the paper is organized as follows. In Section 2 we lay out our search-and-

[4]Other papers with investment-specific technology shocks are Silva and Toledo (2007) and De Bock (2007). These authors assume that the output produced by a job within the firm is independent of the number of workers in the firm. Their setting does not generate amplification in labor market variables in response to investment-specific shocks.

matching model with large firms, we define the search equilibrium and derive the wage function for the parameterized economy. In Section 3 we carry out our quantitative analysis and assess the amplification of unemployment, vacancies, tightness and the job finding rate after shocks to neutral and investment-specific technology. Section 4 studies two separate extensions of the basic framework: introduces an intensive margin in the labor market and assumes a stochastic process for job separation. This Section also addresses the sensitivity of unemployment to unemployment benefits. Section 5 concludes and Section 6 contains two Appendixes.

2 The Model

2.1 The labor market

There is a measure one of identical, risk-neutral workers and an equal measure of firms. Unemployed workers search for jobs and firms open vacancies in a frictional labor market. The total number of matches per period, $\mathcal{M}$, is given by an increasing, concave and homogeneous-of-degree-one matching function,

$$\mathcal{M} = \mathcal{M}(V, 1 - N),$$

where V denotes the total number of vacancies created by all firms and $1 - N$ is the number of unemployed workers.

The vacancy matching rate, μ, is thus given by $\mathcal{M}/V$, and the job-finding rate of an unemployed worker is $\mathcal{M}/(1 - N) = \theta\mu$, where θ denotes labor market tightness $V/(1 - N)$.

2.2 Firms

The production sector is described by a measure one of value-maximizing firms facing an infinite time horizon. Firms produce an identical, aggregate good with a production technology given by $F(z, k, n)$, where k denotes capital, n is the firm's level of employment and z is the level of neutral technology. Function F is assumed to be increasing, jointly concave and linearly homogeneous in capital and labor. Firms own the capital stock and thus both capital and labor are predetermined variables.

In a given period, the firm loses employment at the exogenous rate λ, and it opens vacancies to hire new workers. Newly hired workers start producing in the next period. Firms expect vacancies to be matched with workers at the rate $\mu(S)$, where S denotes the vector of aggregate state variables [we will use "small" s to denote the vector of firm-level state variables]. The cost to the firm of advertising v vacancies is given by the convex function $C(v)$. The evolution of employment within the firm is given by,

$$n' = \mu(S)v + (1 - \lambda)n.$$

The stock of capital depreciates at the constant rate δ. The assumption of investment-specific technical change implies that one unit of the aggregate good invested in capital increases its stock by q units. That is, the evolution of capital is,

$$k' = iq + (1 - \delta)k,$$

where i denotes gross investment. Thus, factor q represents the level of investment-specific technology, which is assumed to follow an exogenous, stochastic process. This particular modeling was first used by Greenwood, Hercowitz and Krusell (1997, 2000) in order to assess the role of investment-specific technical change in generating both postwar U.S. growth and U.S. aggregate fluctuations. Since $1/q$ is the relative price of capital, we will use de-trended price series to derive the volatility and persistence of the technology process q.

Accommodating μv new workers and iq units of new capital within the firm is costly. We assume that adjustment costs of labor and capital interact and represent total adjustment costs by the function $H(v, i, s, S)$, where $s = (z, q, k, n)$ and $S = (z, q, K, N)$ are the vectors of individual and aggregate state variables, respectively. Function H is convex with $H_v > 0$, $H_i > 0$, $H_k < 0$ and $H_n < 0$, for $k > 0$ and $n > 0$. The assumption of interrelation between labor and capital adjustment costs implies that $H_{kn} \neq 0$. We will provide evidence supporting this assumption below.

The firm's objective is the maximization of the present value of cash flows. First, the firm opens vacancies and invests in capital. Next, wages for new hires and existing workers are negotiated.[5] The standard Nash-bargaining solution is assumed. Since the firm is large —i.e. it employs a mass of workers— negotiated wages will depend on the firm's hiring and investment policy. Intra-firm bargaining implies that the firm anticipates the wage effects of its policy. Thus, if we denote by $\omega(v, i, s, S)$ the wage function that solves the wage bargaining problem of a firm that opened v vacancies and invested i in new capital, at individual and aggregate states s and

[5] This timing for investment is not relevant for our results. We carried out the analysis assuming that investment is decided upon simultaneously with wage negotiations and found no significant differences.

S, then the maximization problem of the firm is,

$$\Pi(s,S) = \max_{v,\,i} \Big\{ F(z,k,n) - \omega(v,i,s,S)n - i - C(v) - H(v,i,s,S) + \beta\,\mathbb{E}\left[\Pi(s',S')|z,q\right] \Big\}$$

$s.t.$

$$n' = \mu(S)v + (1-\lambda)n \tag{2.1}$$

$$k' = iq + (1-\delta)k \tag{2.2}$$

$$\ln z' = \rho_z \ln z + \epsilon_z \tag{2.3}$$

$$\ln q' = \rho_q \ln q + \epsilon_q \tag{2.4}$$

$$(\epsilon_z, \epsilon_q)^T \sim N(0, D), \quad \text{where } D \text{ is a matrix,} \tag{2.5}$$

where $\Pi(s,S)$ is the value of the firm and where aggregate state variables are expected to evolve according to some function of the current levels. The firm discounts future cash flows at the rate β, which is the same rate risk-neutral workers use to discount income. Parameters ρ_z and ρ_q denote persistence of the neutral and investment-specific technology processes, respectively. Vector $(\epsilon_z, \epsilon_q)^T$ denotes the respective innovations with variance-covariance matrix D.

It should be noted that the concavity of the production function and the convexity of the adjustment cost and vacancy cost functions do not guarantee the concavity of the firm's maximization problem. Since the firm foresees the wage function that will solve the Nash-bargaining problem with the workers, the maximization problem may be non-concave. We will argue below that in our benchmark economy first-order conditions are necessary and sufficient.

The first-order condition to vacancies can be written as,

$$\omega_v n + C_v + H_v = \mu\beta\,\mathbb{E}\,\Pi'_n, \tag{2.6}$$

where function arguments have been omitted as there is no risk of ambiguity. ω_v denotes the derivative of $\omega(v,i,s,S)$ with respect to vacancies; C_v is the derivative of the vacancy cost function; H_v is the derivate of the adjustment cost function with respect to v, and Π'_n is the derivative of the value function with respect to employment, evaluated at next-period values. The term $\omega_v n$ on the left-hand side of (2.6) is a consequence of the assumption of large firms conducting intra-firm bargaining. The effect of vacancies on wages, *via* adjustment costs, is internalized by the firm when opening vacancies. That is, the firm takes into account the change in total wage costs, $\omega_v n$, when determining its hiring policy.

The first-order condition to investment is given by,

$$\omega_i n + 1 + H_i = q\beta\,\mathbb{E}\,\Pi'_k. \tag{2.7}$$

As with vacancies, the firm also weighs the effect of investment on total wage costs, $\omega_i n$, when setting the level of investment. From the envelope condition, the value of capital for the firm,

Π_k, satisfies the following non-arbitrage condition,

$$\Pi_k = F_k - \omega_k n - H_k + (1-\delta)\beta\,\mathbb{E}\,\Pi_k', \tag{2.8}$$

which also embeds the effect of capital on the wage cost of labor.

Finally, the net value of employment for the firm, $J \equiv \Pi_n$, must satisfy the following non-arbitrage condition,

$$J = F_n - \omega_n n - \omega - H_n + (1-\lambda)\beta\,\mathbb{E}J', \tag{2.9}$$

where F_n is the marginal productivity of labor and $\omega_n n$ captures the effect of employment on the cost of labor.

2.3 Workers

Workers are risk-neutral and discount future consumption of the aggregate good at the rate β. A worker earns a wage, ω, if employed and receives income, b, if unemployed (this income is interpreted as unemployment benefits or home production). The change in employment status depends on job creation and job destruction. Each period, λn employed workers lose their job and $\theta\mu u$ of the unemployed are matched with a vacancy. When negotiating wages, workers take matching probabilities as given. The value of employment at the firm, W_E, is given by,

$$W_E = \omega + \beta\,\mathbb{E}\left[\lambda W_U' + (1-\lambda)W_E'\right], \tag{2.10}$$

and the value of unemployment, W_U, is,

$$W_U = b + \beta\,\mathbb{E}\left[(1-\theta\mu)W_U' + \theta\mu\hat{W}_E'\right], \tag{2.11}$$

where $\hat{W}_E'$ is the expected value of employment outside the firm next period.

Thus, denoting by W the worker's net value of employment at the firm, i.e. $W_E - W_U$, the following non-arbitrage condition must hold,

$$W = \omega - b + \beta\,\mathbb{E}\left[(1-\lambda)W' - \theta\mu\hat{W}'\right]. \tag{2.12}$$

From our assumption of identical firms it follows that $W' = \hat{W}'$ in equilibrium.

2.4 Wage bargaining

A firm negotiates wages with each of its workers. The Nash-bargaining solution maximizes the weighted product of the worker's and the firm's value of employment. We use γ to denote the bargaining power of the worker. Formally, the wage is the solution to the following problem,

$$\omega = \arg max\left\{W^{\gamma}J^{1-\gamma}\right\}. \tag{2.13}$$

The first-order condition to this maximization problem yields the standard sharing rule,

$$(1-\gamma)\,W = \gamma J. \tag{2.14}$$

Combining the equation above with (2.6), (2.9) and (2.12), and using the assumption of continuous wage renegotiation, we obtain

$$\omega(v,i,s,S) = \gamma\left[\,F_n(z,k,n) - \omega_n(v,i,s,S)n - H_n(v,i,s,S)\right] + (1-\gamma)\left[b + \ \theta\mu\beta\mathbb{E}\,\hat{W}'\right]. \tag{2.15}$$

Equation (2.15) is a differential equation in the unknown wage function $\omega(v,i,s,S)$. This equation embeds two important departures from the standard Mortensen-Pissarides model, where the production side is made up of employer-worker pairs (small firms) without capital. In our setting, the firm's flow value of the match is not solely pinned down by the marginal productivity of the worker. Here, the firm also takes into account the value of the worker's contribution to decreasing wages (the second term within the first brackets) as well as total adjustment costs (the third term within first brackets). The sharing of factor adjustment costs –including capital adjustment— between the firm and the workers is an important feature captured by this wage equation. Since new hires, μv, and new capital, iq, interact in the determination of total adjustment costs, the wage function also depends on the level of investment-specific technology q.

2.5 Equilibrium

A recursive search equilibrium with intra-firm bargaining can be defined by decisions rules for vacancies and investment, a wage function $\omega(v,i,s,S)$, a vacancy matching rate $\mu(S)$, labor market tightness $\theta(S)$, value functions and laws of motion for aggregate state variables such that:

i) Decision rules for vacancies and investment solve the firm's maximization problem, given the wage function, the vacancy matching rate and the laws of motion for aggregate variables.

ii) The wage function is the solution to the Nash-bargaining problem (2.15).

iii) Matching rates are given by the matching function evaluated at $V = v$ and $N = n$.

iv) Laws of motion for the aggregate states are consistent with individual behavior.

v) Value functions solve the firms' and workers' maximization problems.

2.6 The Wage Function in the Parameterized Economy

Functional forms for production, matching, adjustment costs and vacancy creation costs are now established. All of our functional forms are standard.

The production technology of the representative firm is represented by a Cobb-Douglas function with constant returns to scale in capital and labor,

$$F(z,k,n) = zAk^{\alpha}n^{1-\alpha}, \tag{2.16}$$

where $A > 0$ and $0 \leq \alpha \leq 1$ are parameters.

The matching technology is represented by a constant-returns-to-scale Cobb-Douglas function, which is the standard functional form in the literature of frictional labor markets,

$$\mathcal{M}(V, 1-N) = MV^{1-\eta}(1-N)^{\eta}, \tag{2.17}$$

where $M > 0$ is the matching-efficiency parameter and $0 \leq \eta \leq 1$ is the elasticitiy of matches with respect to unemployment.

The cost of adjusting labor and capital is represented by the following quadratic adjustment cost function,

$$H(\mu v, iq, n, k) = a_1(iq/k)^2 + a_2\frac{iq}{k}\frac{\mu v}{n} \qquad \text{for } \; n > 0 \; \text{ and } \; k > 0 \tag{2.18}$$

and

$$H(\mu v, iq, n, k) = 0 \qquad \text{for } \; n = 0 \; \text{ or } \; k = 0, \tag{2.19}$$

where $a_1 > 0$ and $a_2 > 0$ are parameters. This adjustment cost function is a particular case of one recently estimated by Merz and Yashiv (2007). As explained above, an important feature of this function is the interaction between employment and investment rates in the determination of total adjustment costs, which is captured by the last term in equation (2.18). Merz and Yashiv (2007) find that this interaction term is key in accounting for the market value of U.S. firms. Further support for this interaction can be found in the empirical literature on employment and capital adjustment decisions [see, i.e., Sakellaris (2004), Letterie, Pfann and Polder (2004) and Contreras (2006a, 2006b).] For example, Contreras (2006a) finds, using data from the Colombian Annual Census of Manufacturing, that it is more costly for the firm to adjust capital and employment at the same time rather than sequently. Indirect evidence can also be found in the work of Letterie, Pfann and Polder (2004) who analyze data on Dutch plants in the manufacturing sector and find that only 20% of the positive employment spikes occur in the same period as an investment spike.

We show in this paper that the interaction between labor and capital in the adjustment cost function is also key to generate volatility in labor market variables.

Finally, the cost of opening vacancies is assumed to be linear in the number of vacancies, $C(v) = cv$, where $c > 0$ is a parameter.

For this parameterization, the wage function that solves the Nash-bargaining problem — differential equation (2.15)— can be found analytically. The next proposition presents the general solution to the differential equation.

Proposition: The general solution to the differential equation characterizing the symmetric Nash-bargaining problem is given by,

$$\omega = \psi n^{-2} + \frac{1-\alpha}{2-\alpha} z A k^{\alpha} n^{-\alpha} + a_2 \frac{iq}{k} \frac{\mu v}{n} \frac{\ln(n)}{n} + \frac{1}{2}\left[b + \theta\mu\beta\mathbb{E}\,\hat{W}'\right], \tag{2.20}$$

where ψ is an arbitrary constant.

Proof: See Appendix I.

We will impose $\psi = 0$ to focus our attention on the particular solution yielding a wage bill equal to zero at $n = 0$. Further, we will also make sure that the bargaining set is non-empty and wages are bounded along the equilibrium path of our baseline economy. The last term within brackets in equation (2.20) is the worker's value of unemployment which, in equilibrium, is given by $b + \theta[\omega_v n + C_v + H_v]$. It should be noted that all the derivatives of the wage function assume the worker's reservation value as given and independent of firm-level variables.

It is apparent from a simple inspection of the wage function in (2.20) that the levels of neutral and investment-specific technology affect wages differently. On one hand, neutral technology, z, has a positive, direct effect on wages, which leads to a perfect, positive correlation between labor productivity and wages. As discussed in Shimer (2005), it is the perfect correlation between productivity and wages generated by the Mortensen-Pissarides model that lies at the heart of its failure to account for the observed volatilities in labor market variables. On the other hand, investment-specific technology, q, enters with a negative sign in the wage function (notice that the firm's level of employment is bounded above by one, and, therefore, $\ln(n)$ is a negative number). Thus, investment-specific technology shocks are bound to reduce the contemporaneous correlation between labor productivity and wages and, as a consequence, to generate amplification in labor market fluctuations.

The concavity of the firm's maximization problem for the parameterized economy can now be assessed. Since the firm foresees the wage function (2.20) when choosing its hiring and investment policy, it is straightforward to show that the return function $F(z, k, n) - \omega(v, i, s, S)n - i - C(v) - H(v, i, s, S)$ is concave in the controls but not in the state variables. Indeed, the term $-\omega(v, i, s, S)n$ is convex in n. This class of dynamic maximization problems have been studied in Skiba (1978) and Ladrón-de-Guevara, Ortigueira and Santos (1999), and conditions for optimality have been established. In short, if parameter values are such that there exists a unique, interior, saddle-path stable, steady-state equilibrium with real roots, then first-order conditions are necessary and sufficient. This is the condition we will check in our baseline economy below.

2.7 Parameter Values

We now assign values to all parameters of the model in order to assess the quantitative effects of different sources of volatility on labor market variables. Parameter values are set so that the steady-state equilibrium of our baseline economy matches some key averages of the 1951-2003 U.S. economy. Steady-state equations are shown in Appendix II.

A time period in our model is set to one month. Values of the constant in the production function, A, and of the bargaining power parameter, γ, are set arbitrarily. We normalize the value of A to one, and set γ equal to 1/2. The assumption of symmetric bargaining is standard in the literature. The rate of job separation is set equal to 0.034, which corresponds to the probability that a worker loses his job within an average month, as estimated by Abowd and Zelner (1985). The elasticity of matches with respect to unemployment is set at 0.6, which is the midpoint value of the range estimated by Petrongolo and Pissarides (2001). The value of the discount factor, β, is set to 0.995, which yields a monthly interest rate of 0.5 percent. The depreciation rate of capital, δ, is 0.011 and the value of α in the production function is set at 0.3. The value chosen for δ yields a yearly rate of depreciation of the order of 13 percent, which is the value for the depreciation rate of equipment capital in the U.S. economy.

Income during unemployment b, the match-efficiency parameter M, the marginal cost of vacancy creation c, and the two parameters in the adjustment cost function a_1 and a_2 are set so that the steady-state equilibrium of the model yields: *i)* A vacancy-filling rate of 0.9 per month. This is the average rate for vacancies in the U.S. economy [see, for instance, Fujita and Ramey (2007)]. *ii)* Income during unemployment represents 45 percent of employment income. This replacement rate is similar to the one chosen by Shimer (2005). *iii)* The total cost of adjusting capital and labor represents 2.4 percent of output. This is the value estimated by Merz and Yashiv (2007). *iv)* The sum of the marginal costs of adjusting capital and labor, i.e., $H_i k + H_v n$, amounts to 3.5 percent of output, also as estimated by Merz and Yashiv (2007). *v)* Vacancy creation costs represent one percent of output. This is the value used by Blanchard and Gali (2006).

It must be noted that our baseline economy has been pinned down without targeting the unemployment rate and the workers' job-finding rate. Yet, the values implied by the baseline economy for these two variables are fairly close to U.S. average values. Thus, the steady-state equilibrium yields an unemployment rate of 6.2% and a job-finding rate of 0.5, whereas U.S. average values for the period 1951-2003 are 5.7% and 0.45, respectively.

Our baseline economy is presented in Table 1 below.

Table 1: Baseline Economy

Efficiency of matching	M	0.64
Matching-unemployment elasticity	η	0.6
Average job destruction rate	λ	0.034
Vacancy creation costs	c	0.9
Parameter in adjustment cost function	a_1	180
Parameter in adjustment cost function	a_2	99
Workers' bargaining power	γ	0.5
Elasticity of output w.r.t. capital	α	0.30
Depreciation rate of capital	δ	0.011
Discount factor	β	0.995
Unemployment benefits	b	1

We close this section by confirming that our baseline economy satisfies the condition for optimality stated above. A log linearization around the unique, interior, steady-state equilibrium yields real eigenvalues, two of them lying outside the unit circle, which is the condition for saddle-path stability in our model economy.

3 Model Evaluation

In this section we review a number of labor market stylized facts and then assess the ability of our model to account for these facts. Tables 2 and 3 below present a selection of business cycle statistics which have guided most of the recent research in the macro labor literature. Standard deviations and correlations reported in these two tables are taken from Shimer (2005) and Horstein, Krusell and Violante (2007) and correspond to the cyclical components of logged variables detrended with a Hodrick-Prescott filter with smoothing parameter 10^5. Data consist of quarterly observations for the U.S. economy from 1951 to 2003.

The most salient features of fluctuations in U.S. labor market variables are the large volatilities of unemployment (u), vacancies (v), tightness (v/u) and the workers' job-finding rate ($\theta\mu$), relative to the volatility of labor productivity (p). As shown in the second row of Table 2, unemployment fluctuates 9.5 times more than labor productivity, vacancies fluctuate 10 times more; fluctuations in labor market tightness are almost 20 times larger than those in productivity and the job-finding rate fluctuates almost 12 times more than productivity. On the contrary, the volatility of wages is close to the volatility in labor productivity. Another important feature of these data is the low-to-moderate correlation between fluctuations in labor market variables and fluctuations in labor productivity (see fourth row of Table 2). Unemployment is counter-cyclical with a correlation coefficient of -0.408. Vacancies, tightness and the job-finding rate are pro-cyclical, with a coefficient of correlation of about 0.4. Wages are also pro-cyclical with a correlation of 0.65. In terms of autocorrelation, all variables except wages have an autocorrelation coefficient of about 0.9. Wages have a first-order autocorrelation coefficient of 0.78.

Table 2. 1951-2003 Quarterly U.S. Labor Market

	u	v	v/u	$\theta\mu$	ω	p
Standard Deviation(%)	19.00	20.20	38.20	11.80	2.200	2.000
Std. Dev. relative to p	9.500	10.10	19.10	5.900	1.100	1
Auto-Correlation	0.936	0.940	0.941	0.908	0.781	0.878
Correlation with p	-0.408	0.364	0.396	0.396	0.655	1

Notes. Standard Deviations and correlations in this table correspond to quarterly series, detrended using a Hodrick-Prescott filter with smoothing parameter 10^5, as calculated by Shimer (2005).

Cross correlations in fluctuations of labor market variables are presented in Table 3 below. It is worth noting the strong negative correlation between fluctuations in unemployment and vacancies of -0.89. This is the slope of the Beveridge curve.

Table 3. 1951-2003 Quarterly U.S. Labor Market

	u	v	v/u	$\theta\mu$	p
u	1	-0.894	-0.971	-0.949	-0.408
v	-	1	0.975	0.897	0.364
v/u	-	-	1	0.948	0.396
$\theta\mu$	-	-	-	1	0.396
p	-	-	-	-	1

Notes. Cross Correlations of detrended U.S. labor market variables.

3.1 Labor Market Volatility in the Baseline Economy

In order to assess the ability of the model to amplify and propagate shocks, we adopt a step-by-step strategy and introduce each of the shocks separately. We first consider neutral technology shocks, i.e. shocks that affect the production of the consumption and investment good equally. Secondly, we study the response of labor market variables to investment-specific shocks, i.e. to shocks that affect only the production of the capital good. In the next section we combine these two shocks along with shocks to the rate of job destruction.

3.1.1 Neutral Technology Shocks

The standard approach used to assess the volatility properties of the Mortensen-Pissarides model consists in assuming a reduced-form, stochastic, exogenous process for labor productivity and then deriving the implied fluctuations in unemployment, vacancies, tightness, the job-finding rate and wages. In our extended version of the model, however, labor productivity is endogenous to the firm's investment and hiring policies. Therefore, the obvious counterpart for our model of the above approach is to shock labor productivity —defined as output (net of adjustment and vacancy costs) per worker— by introducing neutral technology shocks, i.e., shocks to the technology to produce the aggregate good. In this section, we carry out this exercise and calibrate the neutral technology process by following the traditional approach of the business cycle literature. As specified in equation (2.3), neutral technology, z, follows the law of motion,

$$\ln z' = \rho_z \ln z + \epsilon_z, \quad \text{where } \ \epsilon_z \sim N(0, \sigma_{\epsilon_z}). \tag{3.1}$$

We set σ_{ϵ_z} equal to 0.0078 in order to match the quarterly standard deviation of U.S. labor productivity of 2%. The persistence parameter, ρ_z, is set equal to 0.95. In this section, the level of investment-specific technology is assumed to remain constant at its average value.

Our results, presented in Tables 4 and 5 below, are in accordance with those found by Shimer (2005) within the standard Mortensen-Pissarides model with small firms (employer-worker pairs) and no capital. Shocks to the level of technology in the production of the aggregate good fail to generate enough amplification in labor market variables. Neutral technology shocks that generate the observed volatility in labor productivity of 2% account for only 6% percent of the observed volatility in unemployment; for less than 12% percent of the volatility in vacancies and for 9% of the volatility in tightness. The second row of Table 4 shows the generated volatilities in labor market variables relative to the volatility of labor productivity. (Relative volatilities are the standard statistic reported in this literature to assess amplification.) Unemployment and the job-finding rate fluctuate relatively less than productivity. Vacancies and labor market tightness fluctuate only 1.23 and 1.81 times more than productivity, respectively. On the other hand, the relative volatility of wages is slightly below that observed in U.S. data.

Shocks to neutral technology also fail to account for the moderate contemporaneous correlation of labor market variables with labor productivity. In particular, labor market tightness in the model is almost perfectly correlated with productivity, while this correlation is only 0.4 in the data. The model has no propagation of neutral technology shocks.

The explanation for the model's limited amplification of neutral technology shocks is to be found, as in the framework of Shimer (2005), in the high sensitivity of wages to these shocks. In our baseline economy, the contemporaneous correlation between labor productivity and wages is 0.998 (see fourth row of Table 4). As formulated by Shimer (2005), the increase in wages after a positive technology shock absorbs most of the productivity increase and therefore reduces the incentive for vacancy creation. Hence, equilibrium unemployment, vacancies and the job-finding rate do no respond much to neutral technology shocks.

Table 4. Baseline Economy with Neutral Technology Shocks

	u	v	v/u	$\theta\mu$	ω	p
Standard Deviation(%)	1.307	2.478	3.634	1.450	1.929	2.000
Std. Dev. relative to p	0.653	1.239	1.815	0.725	0.964	1
Auto-Correlation	0.886	0.641	0.803	0.803	0.848	0.857
Correlation with p	-0.940	0.899	0.952	0.952	0.998	1

Notes. Baseline economy with neutral technology shocks: Standard deviations and correlations in this table correspond to detrended quarterly averages of the monthly generated series. A H-P filter with smoothing parameter 10^5 has been used to obtain the trend.

As for cross correlations, the baseline economy succeeds at generating the strong correlations between unemployment, vacancies, tightness and the job-finding rate observed in the U.S. economy.

Table 5. Baseline Economy with Neutral Technology Shocks

	u	v	v/u	$\theta\mu$	p
u	1	-0.821	-0.920	-0.920	-0.940
v	-	1	0.978	0.978	0.899
v/u	-	-	1	0.999	0.952
$\theta\mu$	-	-	-	1	0.952
p	-	-	-	-	1

Notes. Neutral Technology Shocks: Cross correlations of simulated variables.

We find it convenient to close this section by taking a further look at the high sensitivity of wages to neutral technology shocks from the dynamics of the firm's flow value of employment [the first expression within the brackets of equation (2.15)] and of the worker's value of unemployment [the second expression within the brackets of equation (2.15)]. Under symmetric Nash-bargaining the wage is the average of these two values. Figure 1 below shows the gross flow value of the match for the firm (upper series) and the worker's value of unemployment (lower series) when the model is run for one thousand months. Cyclical components of these values are not only strongly correlated (the correlation coefficient is 0.9944) but show also a strong correlation with the level of neutral technology (the correlation coefficient between the firm's flow value of a match and z is 0.9902, and the coefficient for the workers' value of unemployment and z is 0.9982.)

3.1.2 Investment-specific Technology Shocks

We now study business cycle fluctuations in labor market variables in the economy with investment-specific technology shocks and assess the ability of the model to generate amplification. In this section, we shut down shocks to neutral technology and leave shocks to investment-specific technology as the only source of volatility. As explained above, the volatility and persistence of the investment-specific technology process can be estimated from the capital price series (see Greenwood, Hercowitz and Krusell (2000) and Fisher (2006) for details). Thus, unlike neutral technology, whose volatility was set to match the observed volatility in labor productivity, the calibration of investment-specific technology is guided by observed capital price series. Consequently, the question addressed in this section is twofold. First, how much of the observed volatility in labor productivity can be explained by shocks to investment-specific technology? Second, do shocks to investment-specific technology generate the observed amplification in labor market variables? That is, does the model generate relative volatilities in unemployment, vacancies, tightness and the job-finding rate as those observed in the U.S. economy?

The two parameters to be calibrated in the investment-specific technology process,

$$\ln q' = \rho_q \ln q + \epsilon_q, \tag{3.2}$$

are the volatility of ϵ_q and the persistence parameter ρ_q. The volatility of shocks to investment-specific technology, σ_{ϵ_q}, is set at 0.0095 in order to match the 2.6% quarterly standard deviation of detrended capital prices of the U.S. economy. The baseline value for the persistence parameter is set at 0.98. Capital prices show high persistence and we have carried out a sensitivity analysis with respect to this parameter. Our results are robust to changes in this parameter.

The results of our exercise are shown in Tables 6 and 7 below. The answer to our first question is in the first row of Table 6: investment-specific technology shocks yield a volatility in labor productivity of 0.8%, thus accounting for 40% of its empirical value. This number agrees with the estimated contribution of investment-specific shocks to output volatility found by Greenwood, Hercowitz and Krusell (2000) and Fisher (2006). These authors use a standard neoclassical model with a Walrasian labor market and find that shocks to investment-specific technology explain about one third of U.S. output volatility.

Our question concerning the extent of labor market volatility generated by shocks to investment-specific technology is also addressed in Table 6. The volatility of unemployment is 4.15%, which amounts to 22% of the volatility observed in the U.S. economy. The volatility of vacancies is 8.09%, which is 40% of its observed value. As for labor market tightness and the job-finding rate, generated volatilities represent 30% and 40% of the observed values, respectively. The second row of Table 6 presents volatilities relative to the volatility of labor productivity. Relative volatilities are close to the ones observed in U.S. data. For instance, vacancies in our model economy fluctuate 10.05 times more than labor productivity and the job-finding rate fluctuates 5.79 times more. In the U.S. economy these two numbers are 10.1 and 5.9, respectively. Unemployment in the model fluctuates 5.15 times more than productivity, while this number is 9.5 in the U.S. economy. The relative volatility of labor market tightness is 14.4, against 19.1 in the data. Finally, the model matches the observed relative volatility of wages of 1.1.

Correlations of unemployment, vacancies, tightness and the job-finding rate with labor productivity are also in line with the ones observed in U.S. data. In the model, the contemporaneous correlation coefficient between unemployment and labor productivity is -0.42, against a -0.4 in the data. For the other three variables, the model yields coefficients of around 0.48, against correlations of around 0.4 in the data.

Table 6. Baseline Economy with Investment-specific Technology Shocks

	u	v	v/u	$\theta\mu$	ω	p
Standard Deviation(%)	4.150	8.095	11.61	4.668	0.947	0.805
Std. Dev. relative to p	5.155	10.05	14.42	5.798	1.176	1
Auto-Correlation	0.854	0.564	0.754	0.753	0.767	0.978
Correlation with p	-0.423	0.472	0.481	0.481	0.901	1

Notes. Baseline economy with investment-specific technology shocks: Standard deviations and correlations in this table correspond to detrended quarterly averages of the monthly generated series. A H-P filter with smoothing parameter 10^5 has been used to obtain the trend.

The explanation as to why investment-specific technology shocks amplify the volatility of labor market variables stems from the dynamics of firm's investment and hiring in the presence of costs of adjustment. Unlike neutral technology shocks, a positive shock to investment-specific technology introduces a bias in favor of capital by lowering its relative price. Hence, an increase in q yields an immediate increase in investment with a consequent increase in adjustment costs. In order to spread out these (convex) costs over time, the firm's optimal policy calls for an initial hiring freeze followed by a delayed increase in hiring after the increase in q. That is, the firm first builds up the capital stock, taking advantage of its lower price, and then increases employment. This sequential adjustment of investment and hiring after a shock to investment-specific technology amplifies the volatility of labor market variables. As mentioned above, the sequential timing of investment and hiring is in line with the findings of Contreras (2006a).

We will make use of impulse-responses to a positive shock to investment-specific technology, shown in Figure 2, to illustrate our results. The top chart of Figure 2 shows the responses of investment and vacancies. After the initial hiring freeze, vacancies are hump-shaped, showing a prolonged increase before they start decreasing to their steady-state value. The delayed increase in hiring is fostered by the dampening effect of q on wages. Intra-firm wage bargaining implies that the firm shares adjustment costs with the workers and thus, investment-specific technology shocks do not create a perfect correlation between labor productivity and wages (see fourth row of Table 6).[6] Even though this correlation coefficient is higher than its empirical value, 0.9 against 0.65, it represents a substantial decrease in the sensitivity of wages to labor productivity, as compared to the case of neutral technology shocks. Therefore, unemployment and vacancies respond relatively more to investment-specific shocks than they do to neutral shocks.

The responses of unemployment and the job-finding rate to the positive shock in investment-specific technology are shown in the bottom chart of Figure 2. Unemployment increases on impact to the positive shock and then shows a pronounced decrease before reaching the steady-state value. The job-finding rate falls initially and then increases. These responses match the impulse-responses estimated by Balleer (2009) for the U.S. economy. Using quarterly data for the 1955-2004 U.S. economy she finds that unemployment increases after a positive shock to investment-specific technology, and that the job-finding rate decreases.

Conditional correlations in our model are also in line with the observed cross correlations between labor market variables. The slope of the Beveridge curve is slightly below the U.S. value. The model yields a contemporaneous correlation coefficient between unemployment and vacancies of -0.77, against a -0.89 in the data.

Table 7. Baseline Economy with Investment-specific Technology Shocks

	u	v	v/u	$\theta\mu$	p
u	1	-0.772	-0.896	-0.895	-0.423
v	-	1	0.974	0.974	0.472
v/u	-	-	1	0.999	0.481
$\theta\mu$	-	-	-	1	0.481
p	-	-	-	-	1

Notes. Investment-specific technology shocks: Cross correlations of simulated variables.

[6]The contribution of intra-firm bargaining to amplification in labor market variables is sizable. We solved the model with and without intra-firm bargaining and found that it increases volatility by about 15%, with respect to the economy where firms do not anticipate the wage effects of its investment and hiring policy.

We close this section by showing the dynamics of the firm's and workers' values determining the wage (see Figure 3 below). Unlike the case of neutral technology shocks, the cyclical components of these two values are not perfectly correlated (the correlation coefficient is 0.6375). Moreover, the correlation of the cyclical component of each of these values with the level of investment-specific technology is also substantially lower than under neutral-technology shocks. Thus, the correlation coefficient between the firm's flow value of a match and q is 0.4696 and between the worker's value of unemployment and q is only -0.1727. This latter value implies a low feedback from the value of unemployment to the current wage, contrary to what we observe in the economy with neutral-technology shocks.

4 Extensions

4.1 Endogenous hours of work per employee

In this section we extend our previous framework by introducing an intensive margin of adjustment in the labor market. That is, we endogenize the number of hours worked per employee. The purpose of this exercise is twofold: on the one hand, it shows the robustness of our previous results on labor market volatility to variable hours of work. On the other hand, we gain new insights on firm's factor adjustment after technology shocks when both the extensive and the intensive margins in the labor market are available.

To study the model with endogenous working hours, we assume the following utility function on consumption and hours worked for each worker

$$u(c,h) = c - \psi \frac{h^{1+\frac{1}{\varrho}}}{1+\frac{1}{\varrho}}, \tag{4.1}$$

where ψ and ϱ are parameters. Parameter ϱ is the Frisch intertemporal elasticity of substitution. Firm's labor input is now defined as nh. Two different approaches to determining h can be found in the literature. A first approach is to use efficient bargaining, where the firm and each worker are assumed to negotiate the number of hours of work jointly with the wage. A second approach, proposed first by Trigari (2006) in a study of inflation dynamics, is the so-called right to manage, where firms choose hours to maximize profits. While it is yet unknown which of these two approaches is more appropriate for macro modeling, Rotemberg (2008) claims that there is no much evidence supporting efficient bargaining in hours and argues in favor of the right-to-manage model. In this paper, we adopt a version of this latter approach where firms choose hours to maximize profits foreseeing the wage that will result from the Nash-bargaining problem. The wage per hour of work is now given by,

$$\omega = \frac{1-\alpha}{2-\alpha} zAk^{\alpha}(nh)^{-\alpha} + a_2 \frac{iq}{k}\frac{\mu v}{n}\frac{\ln(n)}{nh} + \frac{1}{2h}\left[b + \psi\frac{h^{1+\frac{1}{\varrho}}}{1+\frac{1}{\varrho}} + \theta\mu\beta\mathbb{E}\,\hat{W}'\right]. \tag{4.2}$$

The profit-maximizing number of hours per worker chosen by the firm is given by,

$$F_h(k, nh) = \omega n + \omega_h nh, \tag{4.3}$$

where ω_h is the derivative of the wage function shown above with respect to hours. This condition equalizes the marginal product of hours —the left-hand side of the equation— to the marginal cost, calculated as the sum of direct wages plus the implied change in the wage bill.

In order to carry out the quantitative analysis of labor market volatility in this version of the model, the two new parameters ψ and ϱ need to be calibrated. Our calibration strategy is the following. The value of ϱ is set at 0.33, which is in the range of estimated values for the U.S. economy. (See Domeij and Floden (2006) for a recent discussion on the estimation of the Frisch intertemporal elasticity of labor supply.) Then, the value of ψ is set at 2.22 so that hours per employee in the steady-state equilibrium is equal to one. This is a normalization of average hours of work. Finally, and in order to restore a replacement rate of 45%, we decrease the value of b (unemployment benefits) to 0.445. Note that in the economy with endogenous hours the income value of unemployment includes a utility gain from being out of work, as expressed by the term $\psi\frac{h^{1+\frac{1}{\varrho}}}{1+\frac{1}{\varrho}}$ in the wage equation above. The economy thus calibrated has the same steady-state equilibrium as the economy of the previous version.

In this extended model, the volatility of labor market variables generated by investment-specific technology shocks is presented in Table 8 below. The process for technology is the same as the one used in the previous section.

Table 8. Baseline Economy with Endogenous Hours: Investment-specific Technology Shocks

	u	v	v/u	$\theta\mu$	ω	h	p
Stand. Dev. (%)	4.414	8.499	12.303	4.943	0.944	0.254	0.830
Std. Dev. rlt. to p	5.313	10.23	14.82	5.955	1.137	0.306	1
Auto-Corr.	0.860	0.590	0.765	0.765	0.756	0.985	0.975
Corr. with p	−0.443	0.497	0.503	0.503	0.902	0.905	1

Notes. Baseline economy with endogenous hours of work: Investment-specific technology shocks. Standard deviations and correlations in this table correspond to detrended quarterly averages of the monthly generated series. A H-P filter with smoothing parameter 10^5 has been used to obtain the trend.

The amplification of investment-specific technology shocks is slightly higher than the one found in the model with a fixed number of working hours per employee. For instance, the relative volatility of unemployment is 5.313, compared to the previous value of 5.155, and for labor market tightness the new value is 14.82, compared to 14.42.

Our impulse-response analysis illustrates new features of the economy's adjustment to a positive shock to investment-specific technology. First, the response of vacancies, unemployment and the job-finding rate is similar to that found in the model of the previous section (see Figure 4). The adjustment of the new endogenous variables is as follows. Hours per worker jump up on impact with the shock (see bottom chart of Figure 5). Due to the initial hiring freeze caused by adjustment costs, the firm relies on the intensive margin and increases the number of hours of work per employee. On the other hand, total hours worked show, after a slight initial decline, a prolonged increase (see bottom chart of Figure 5). The initial decline in total hours worked is consistent with recent empirical findings on the short-run effect of investment-specific technology improvements on hours worked [see e.g. Fisher (2006) and Basu, Fernald and Kimball (2006)].

The response of labor productivity to a positive investment-specific shock is shown in the top chart of Figure 5. Labor productivity decreases initially and then shows an increasing, hump-shaped profile. The initial drop in productivity at the time of the increase in investment-specific technology is consistent with both plant level and aggregate observations. For instance, Sakellaris (1994) uses data on U.S. manufacturing plants from the Annual Survey of Manufactures and studies plant productivity after factor adjustments. He finds that Total Factor Productivity falls in periods of investment spikes. His hypothesis for this fall is that the investment spike involves the introduction in the plant of new technology embodied in the installed equipment, which may be operated inefficiently in the short run. On the other hand, Hornstein and Krusell (1996) and Greenwood and Yorukoglu (1997) argue that the mid-70's productivity slowdown was caused by the increase in investment-specific technology of the mid 70's. They point to adoption and learning as the main mechanisms for the productivity slowdown. [See also Fisher (2006) and Balleer (2009) for a detailed exposition of the short-run effect of investment-specific technology shocks on productivity.]

Finally, correlations between labor market variables, conditional on investment-specific technology shocks, are shown in Tale 9.

Table 9. Baseline Economy with Endogenous Hours: Investment-specific Technology Shocks

	u	v	v/u	$\theta\mu$	h	p
u	1	-0.789	-0.904	-0.903	-0.036	-0.443
v	-	1	0.975	0.975	0.170	0.497
v/u	-	-	1	0.999	0.131	0.503
$\theta\mu$	-	-	-	1	0.131	0.503
h	-	-	-	-	1	0.905
p	-	-	-	-	-	1

Notes. Baseline economy with endogenous hours of work: Investment-specific technology shocks. Cross correlations of simulated variables.

4.2 Shocks to Neutral, Investment-specific Technology and Job Separation

In this section we study labor market dynamics in our model economy with three sources of volatility: two technology shocks —neutral and investment-specific— and shocks to the rate of job separation, λ. Fluctuations in job separation in the U.S. economy are large and their contribution to labor market volatility has been recently reassessed in the literature. Elsby et al.(2009), using data from the Current Population Suvery, find that countercyclical fluctuations in the job separation rate play an important role in unemployment dynamics. Fujita and Ramey (2008) also find that fluctuations in job separation are countercyclical, and estimate contemporaneous fluctuations in the separation rate explain 40% of the fluctuations in unemployment.

To carry out this exercise we consider —in addition to the two technology processes introduced above— an exogenous, stochastic process for the rate of job separation. We adopt the process used by Ramey (2008), which is,

$$\ln \lambda' = (1 - \rho_\lambda) \ln \hat{\lambda} + \rho_\lambda \ln \lambda + \epsilon_\lambda, \tag{4.4}$$

where $\hat{\lambda}$ is the average rate of job destruction, ρ_λ is the persistence parameter and $\epsilon_\lambda \sim N(0, \sigma_{\epsilon_\lambda})$. First, we calibrate the two parameters ρ_λ and $\sigma_{\epsilon_\lambda}$ in the job separation process so that it matches the persistence and volatility of job separation in the 1951-2003 U.S. economy. For this period, the cyclical component of job separation yields a first-order autocorrelation coefficient of 0.733, and a volatility of 7.5% [see Shimer (2005)]. As for the investment-specific technology process we use the same calibration as in the previous subsection. Finally, we recalibrate the volatility of the neutral technology shock so that the standard deviation of labor productivity equals 2% in the model economy. That is, while the volatilities of the innovations to λ and q are set to match the observed volatilities in job separation and in capital prices, respectively, the volatility of innovations to z is chosen as the residual to match the volatility of U.S. labor productivity. Following this procedure, the values obtained for ρ_λ, $\sigma_{\epsilon_\lambda}$ and σ_{ϵ_z} are 0.85, 0.042 and 0.006, respectively. Finally, and in order to match the reported correlations between the two technology shocks and between job separation and labor productivity, we assume the following correlations between the three innovations. The correlation coefficient between ϵ_z and ϵ_q is set at -0.31, which is the value estimated by Lindquist (2004). The coefficient of correlation between neutral technology shocks and job separation shocks is set at -0.3, so that the model yields a correlation between labor productivity and job separation of -0.25.

Tables 10 and 11 present volatilities and correlations of labor market variables in our baseline economy with the two technology shocks and the job separation shock. As shown in the first row of Table 10, the model explains a substantial fraction of observed labor market volatility. In particular, the model accounts for 43% of the observed volatility in unemployment, 78% of the volatility in vacancies, 45% of the volatility in tightness and 61% of the volatility in the job-finding rate. The volatility of wages in the model is 97% of its empirical value. The model also accounts for the mild correlation between labor productivity and unemployment, vacancies, tightness and the job-finding rate.

Table 10. Baseline Economy with Three Sources of Volatility

	u	v	v/u	$\theta\mu$	ω	p
Stand. Dev. (%)	8.100	15.795	16.912	7.281	2.136	2.000
Std. Dev. rlt. to p	4.050	7.897	8.456	3.640	1.068	1
Auto-Corr.	0.764	0.346	0.536	0.519	0.654	0.874
Corr. with p	-0.302	0.284	0.401	0.391	0.890	1

Notes. Baseline economy with two technology shocks and a job separation shock: Standard deviations and correlations in this table correspond to detrended quarterly averages of the monthly generated series. A H-P filter with smoothing parameter 10^5 has been used to obtain the trend.

Cross correlations are presented in Table 11. As already shown by many authors —e.g. Cole and Rogerson (1999), Shimer (2005) and Ramey (2008)— shocks to the rate of job separation generate a counterfactual upward-sloping Beveridge curve. In our model economy, this slope is 0.172. It should be noted that the positive unemployment-vacancy correlation is not a result specific to our model nor to the assumption that separation is exogenous. In fact, Ramey (2008) solves a version of the search-and-matching model with small firms and shocks to the exogenous rate of job separation and obtains a correlation between unemployment and vacancies of 0.75. In a specification of his model where separation is endogenous the value for this correlation is as high as 0.92.

Table 11. Baseline Economy with Three Sources of Volatility

	u	v	v/u	$\theta\mu$	p
u	1	0.172	-0.332	-0.304	-0.302
v	-	1	0.879	0.880	0.284
v/u	-	-	1	0.991	0.401
$\theta\mu$	-	-	-	1	0.391
p	-	-	-	-	1

Notes. Baseline economy with two technology shocks and a job separation shock: Cross correlations of simulated variables.

In light of the results in this paper and in some recent papers in this literature, it may be said that the puzzle on labor market fluctuations is more about correlations than about volatilities. In other words, the question is not so much whether the search-and-matching model generates amplification in labor market variables as whether it generates amplification with the right correlations. A better understanding of the flow into unemployment seems to be crucial to answer this latter question.

The Sensitivity of Long-term Unemployment to Labor Market Policy. We now turn to the sensitivity of unemployment with respect to unemployment benefits. As pointed out recently by Costain and Reiter (2008), the unemployment puzzle is not only about volatilities but also about the sensitivity of unemployment to policy, particularly to unemployment benefits. Besides citing a large body of empirical studies aimed at estimating this sensitivity, these authors also conduct their own estimation using low-frequency cross-country data from 1960 to 1999. They find that the long-run semi-elasticity of unemployment with respect to benefits ranges from 1.33 to 2.45, depending upon whether controls for country or time effects are included or not.

In our benchmark economy, the long-run semi-elasticity of unemployment with respect to the replacement rate, $\frac{\triangle u}{\triangle b}\frac{1}{u}$, is equal to 1.1. This number is in the lower bound of empirical estimates, implying that in our calibration there is still room to increase unemployment benefits, which will allow us to generate even more volatility in labor market variables with investment-specific technology shocks, without creating an excessive sensitivity of unemployment to benefits.

5 Conclusions

In the vast majority of the literature on search-and-matching models of the labor market, production takes place in small firms (employer-worker pairs) —the Mortensen-Pissarides model[7].

[7]See Mortensen and Pissarides (1994) and Pissarides (2000).

The productivity of the worker is assumed to follow an exogenous process and capital is typically left out of the analysis. We depart from this framework by assuming large firms, which invest in capital, create vacancies to hire new workers and pay adjustment costs. We assume that wages are the outcome of a Nash-bargaining problem between the firm and the workers. Our main focus is on the business-cycle fluctuations of labor market variables. To this aim, we assume that the levels of neutral and investment-specific technologies are subject to shocks. A key feature of our model is that investment and hiring rates interact in the determination of total adjustment costs. Empirical support for this interaction has been recently offered by Merz and Yashiv (2007) using U.S. data.

While neutral technology shocks have only a small impact on labor market variables, shocks to investment-specific technology generate sizable fluctuations in unemployment, vacancies, labor market tightness and the worker's job finding rate. In our economy, fluctuations in labor market variables are not associated with an implausible elasticity of unemployment to unemployment benefits.

By bringing the neoclassical firm to the center stage of the search-and-matching model and by stressing the role played by costly factor adjustment, we show that volatility in labor market variables can be amplified without abandoning the assumption of continuous Nash-wage bargaining. Even though the model we explored in this paper is rather stylized, it allows us to gain new insights on the effect of investment-specific technology shocks on labor market volatility.

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6 Appendix

6.1 Appendix I

Proof of the Proposition: Equation (2.15) is a linear, first-order differential equation and its solution can be found analytically. Since the worker's outside value is independent of the firm's level of employment, n, it follows that if $\hat{\omega}$ is the general solution to,

$$\omega(v,i,s,S) = \frac{1}{2}\left[\, F_n - \omega_n(v,i,s,S)n - H_n(v,i,s,S)\right], \tag{6.1}$$

then, $\omega = \hat{\omega} + \frac{1}{2}\left[b + \theta\mu\beta E\,\hat{W}'\right]$ is the general solution to (2.15).

To solve equation (6.1), we re-arrange terms and write it as,

$$\omega_n = -\frac{2}{n}\,\omega + \frac{F_n}{n} - \frac{H_n}{n}. \tag{6.2}$$

Therefore, $\hat{\omega}$ is the solution to (6.2) if and only if,

$$\hat{\omega}_n e^{\int (2/n)dn} + (2/n)\hat{\omega} e^{\int (2/n)dn} = \left(\frac{F_n}{n} - \frac{H_n}{n}\right) e^{\int (2/n)dn} \tag{6.3}$$

that is, if and only if,

$$\frac{d}{dn}\left(\hat{\omega} e^{\int (2/n)dn}\right) = \left(\frac{F_n}{n} - \frac{H_n}{n}\right) e^{\int (2/n)dn}. \tag{6.4}$$

Multiplying through by dn and integrating, we get,

$$\hat{\omega} = \psi e^{-\int (2/n)dn} + e^{-\int (2/n)dn} \int \left(\frac{F_n}{n} - \frac{H_n}{n}\right) e^{\int (2/n)dn} dn, \tag{6.5}$$

where ψ is a constant of integration. For our parameterized economy, the integral in the last term of (6.5) can be solved. First, $e^{\int (2/n)dn} = e^{2\ln(n)} = n^2$. Then, using the functional forms specified in Section 2.6 we have that,

$$e^{-\int (2/n)dn} \int \frac{F_n}{n}\, e^{\int (2/n)dn} dn = n^{-2} \int (1-\alpha) z A k^{\alpha} n^{1-\alpha} dn = \frac{1-\alpha}{2-\alpha} z A k^{\alpha} n^{-\alpha} \tag{6.6}$$

and

$$-e^{-\int (2/n)dn} \int \frac{H_n}{n}\, e^{\int (2/n)dn} dn = n^{-2} \int a_2 \frac{iq}{k} \frac{\mu v}{n}\, dn = a_2 \frac{iq}{k} \frac{\mu v}{n} \frac{\ln(n)}{n}. \tag{6.7}$$

After plugging the last two expressions back into (6.5) we obtain $\hat{\omega}$. Finally, adding the worker's reservation wage to this solution we obtain the wage function shown in the Proposition in the text.

6.2 Appendix II

In this appendix we present the equations characterizing the steady-state equilibrium of our economy of Section 2. Using equilibrium conditions $K = k$, $N = n$, and $V = v$, steady-state equations are as follows. If we set the steady-state level of investment-specific technology equal to one, the law of motion for capital gives,

$$i = \delta k. \tag{6.8}$$

In the steady-state equilibrium, the flows into and out of unemployment must be the same so that unemployment is constant,

$$\lambda n = \mu v. \tag{6.9}$$

The non-arbitrage condition to capital in steady state, after using the first-order condition to investment, becomes,

$$\omega_i n + 1 + H_i = \frac{\beta}{1-\beta(1-\delta)}\left(F_k - \omega_k n - H_k \right). \tag{6.10}$$

Likewise, the job creation equation in steady state is,

$$\frac{C_v + \omega_v n + H_v}{\mu} = \frac{\beta}{1-\beta(1-\lambda)}\left(F_n - \omega_n n - \omega - H_n \right). \tag{6.11}$$

The wage equation is

$$\omega = \frac{1-\alpha}{2-\alpha} zAk^{\alpha}n^{-\alpha} + a_2 \frac{i}{k}\frac{\mu v}{n}\frac{\ln(n)}{n} + \frac{1}{2}\left[b + \theta(\omega_v n + C_v + H_v) \right], \tag{6.12}$$

where, as explained in Section 2.6, the derivatives of the wage function take the worker's reservation value as given. Finally, the vacancy-filling rate is given by the matching function,

$$\mu = Mv^{-\eta}(1-n)^{\eta}. \tag{6.13}$$

The system formed by the six non-linear equations (6.8)-(6.13) can be solved for the six unknowns k, n, i, v, ω and μ.

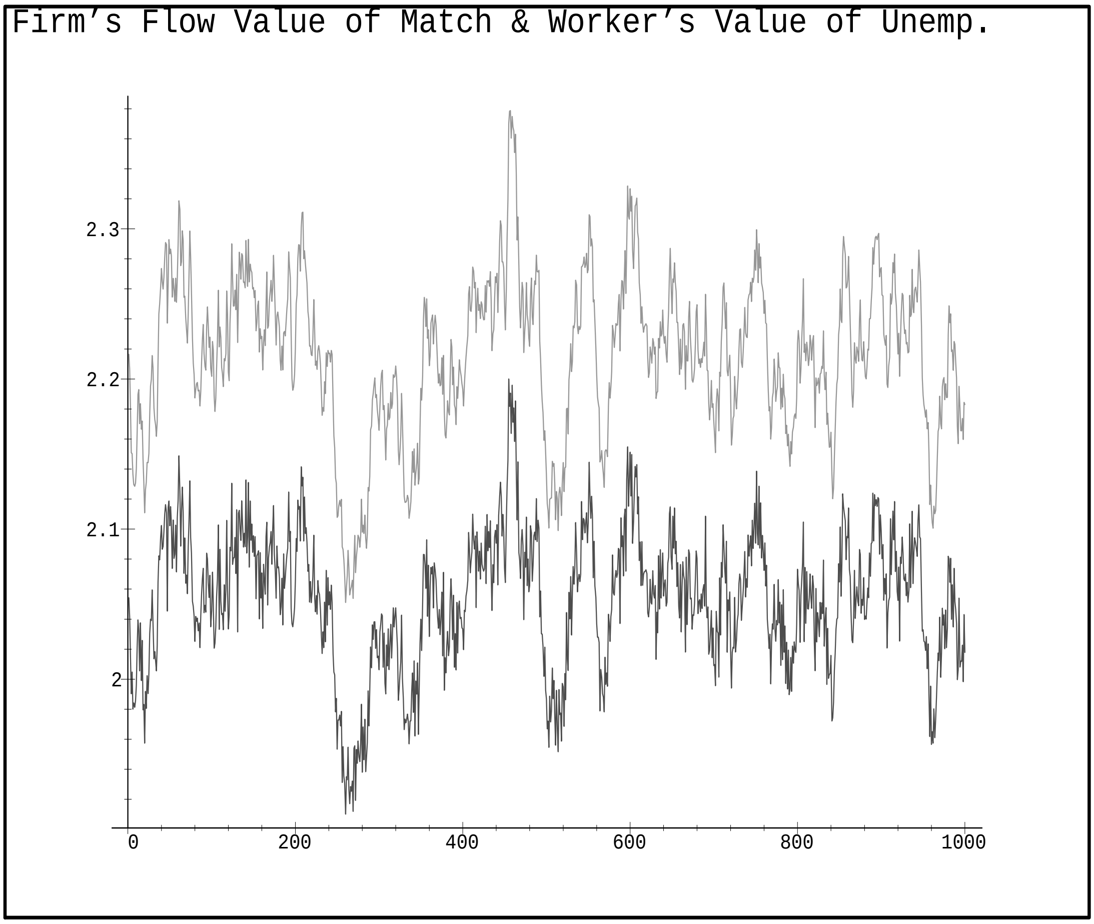


Figure 1: Figure 1 plots the firm’s flow value of a match (upper series) and the worker’s value of unemployment in our baseline economy under neutral technology shocks.

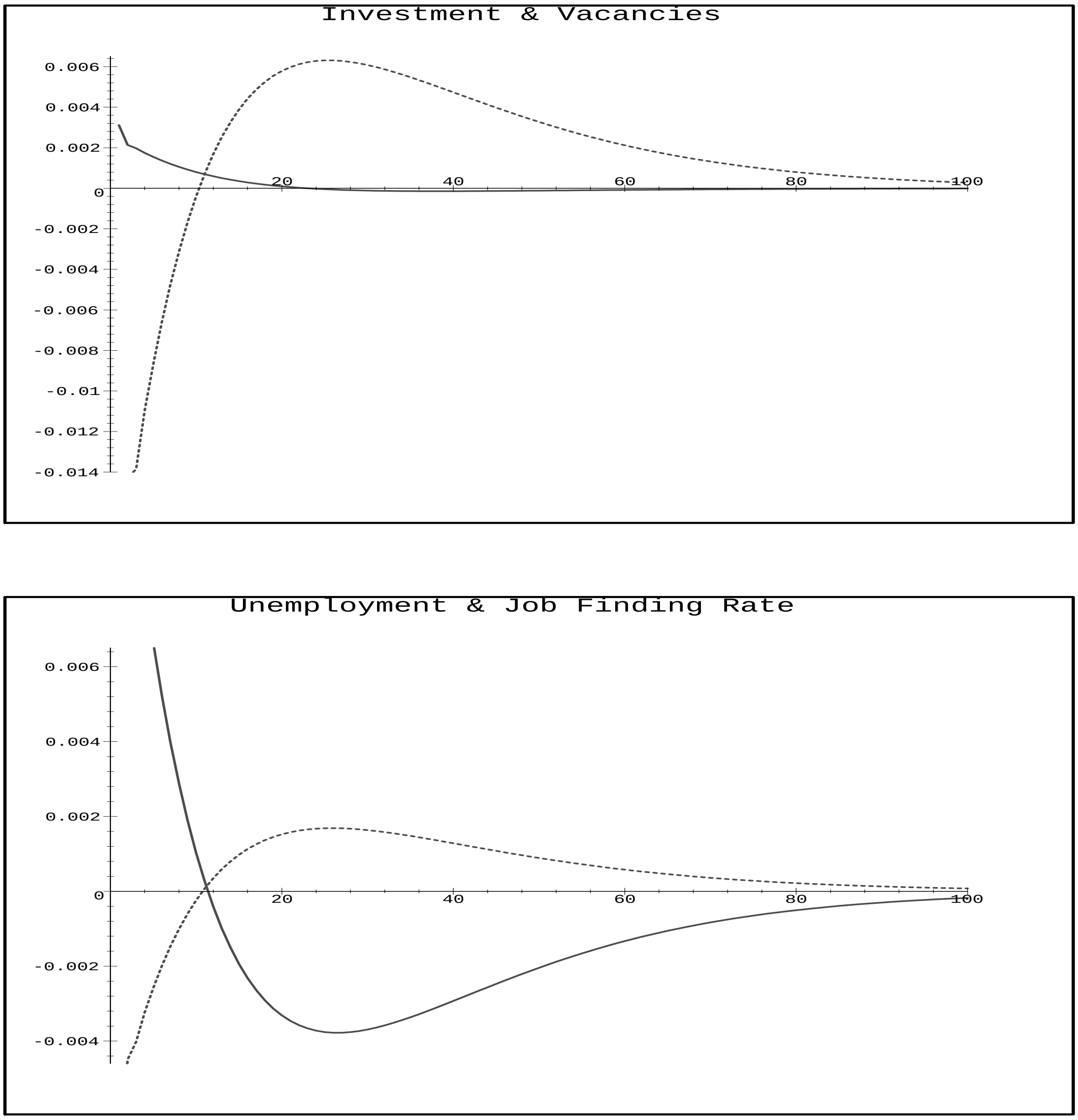


Figure 2: Impulse-Responses to positive Investment-specific Technology Shock. Top chart of Figure 2 plots the response of investment (solid line) and vacancies (broken line). Bottom chart plots the response of unemployment (solid line) and of the job-finding rate (broken line).

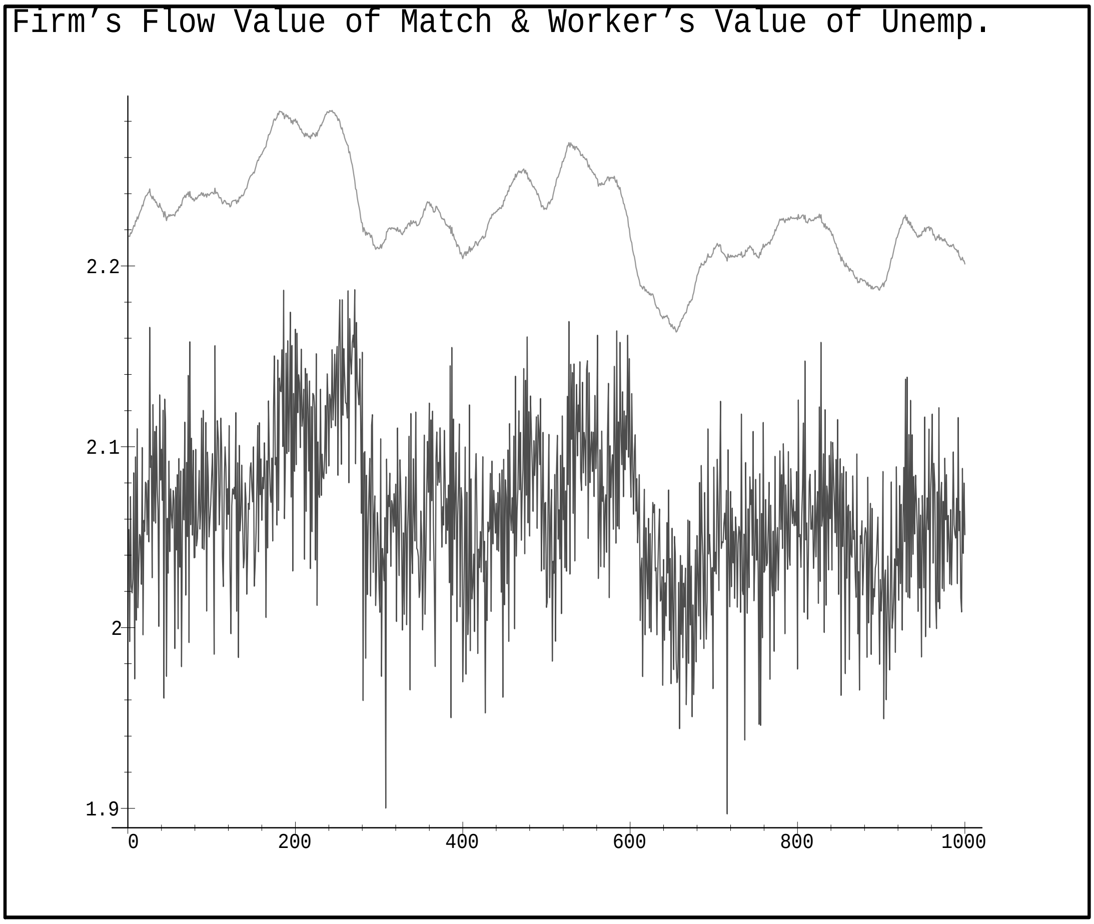
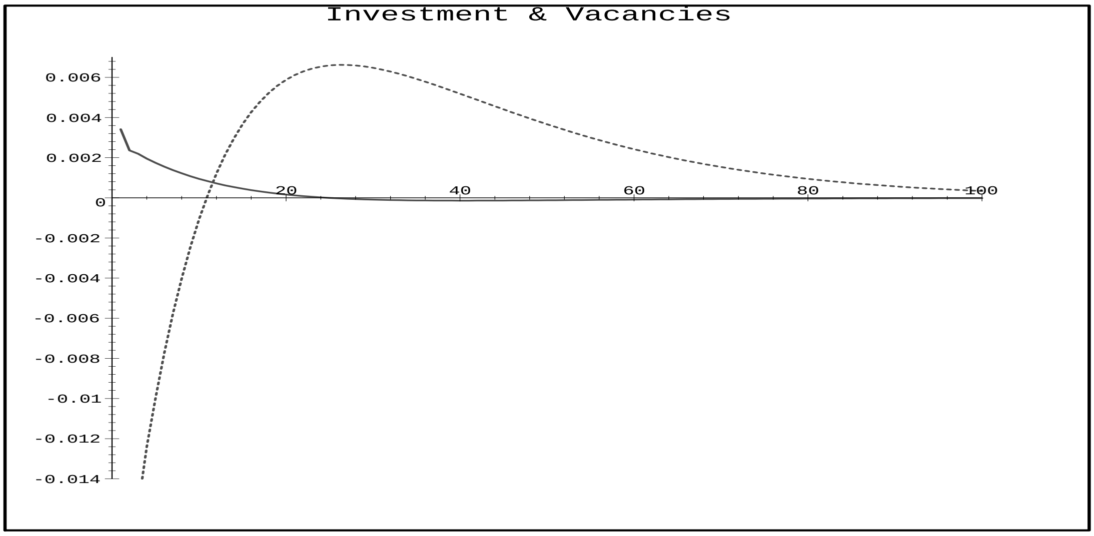


Figure 3: Figure 3 plots the firm’s flow value of a match (upper series) and the worker’s value of unemployment in our baseline economy under investment-specific technology shocks.



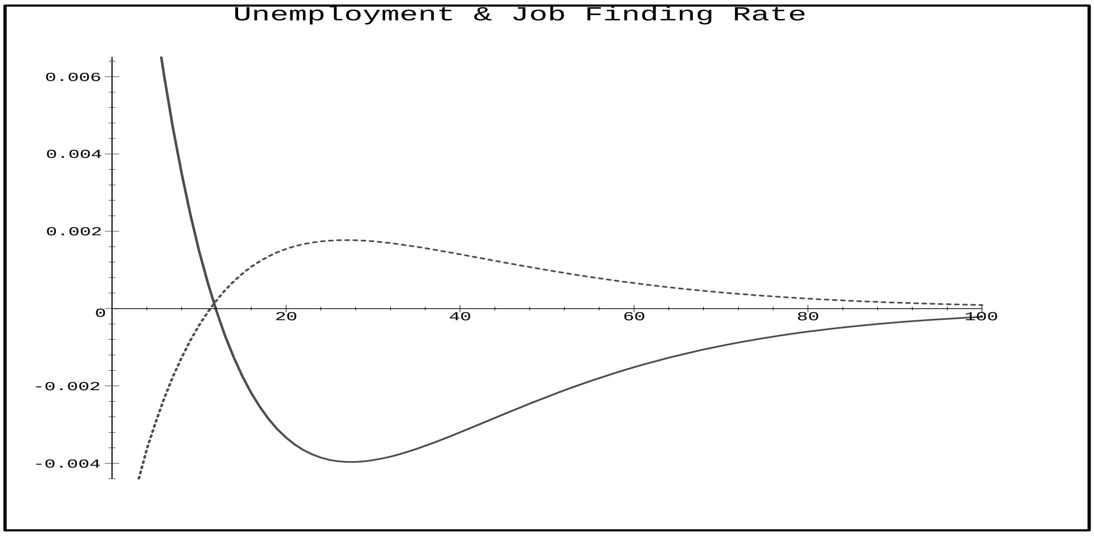


Figure 4: Impulse-responses to a positive shock to investment-specific technology in the model with endogenous hours. Top chart of Figure 4 plots the response of investment (solid line) and of vacancies (broken line). Bottom chart plots the response of unemployment (solid line) and the job-finding rate (broken line).

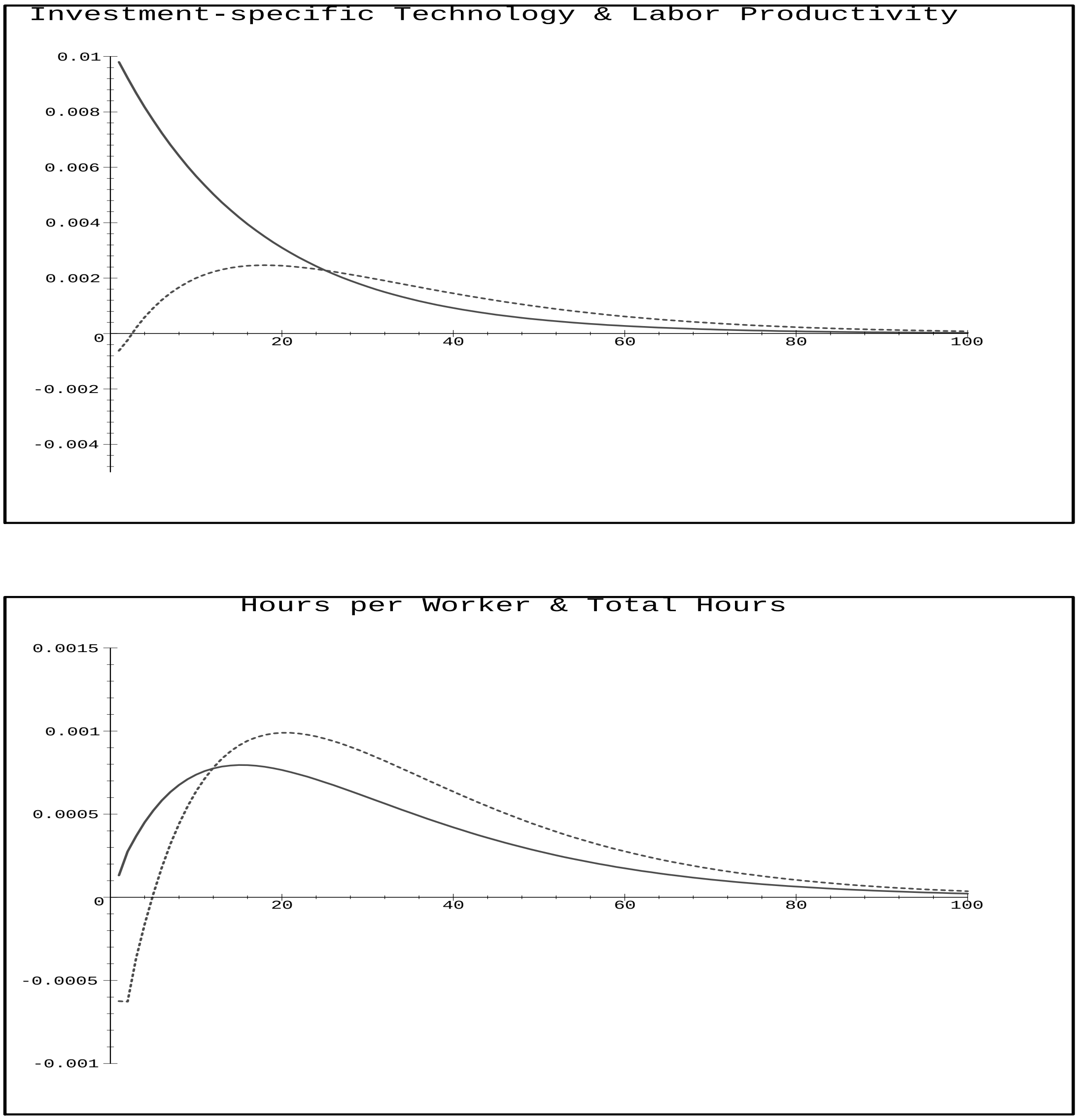


Figure 5: Impulse-responses to a positive shock to investment-specific technology in the model with endogenous hours. Top chart of Figure 5 plots the level of investment-specific technology (solid line) and labor productivity (broken line). Bottom chart plots the response of hours of work per employee (solid line) and of total hours worked (broken line).