Efficient partnership dissolution under buy-sell clauses

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Abstract

When a partnership comes to an end partners have to determine the terms of the dissolution. A well known way to do so is by enforcing a buy-sell clause. Under its rules one party offers a price for the partnership and the other party chooses whether to sell her share or buy her partner's share at this price. It is well known that in a model with private valuations this dissolution rule may generate inefficient allocations. However, we show that if partners negotiate for the advantage of being chooser, then buy-sell clauses result in an ex-post efficient outcome. This result explains why such clauses are so widely used in partnership contracts.

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1 Introduction

Partners in commercial relationships have to think about the possibility that the partnership might end. Disagreement about the future strategy of the commonly owned firm, or family circumstances of one of the partners, might lead to the inevitability of splitting-up. If the partnership is considered to be profitable it is more sensible for one partner to buy out the other and realize the profits than to liquidate. Assets may be less valuable to a third party than to either partner[1]. Being aware of this advantage, business attorneys recommend including buy-sell clauses (commonly referred to as shoot-out clauses) in the initial partnership agreement[2]. A buy-sell clause is a deadlock solution that works as follows: One party proposes a price and the other decides whether to buy or sell at that price. Real life examples of partnerships that have included buy-sell clauses in their partnership agreements abound[3].

Buy-sell clauses have also caught the attention of economic theorists. When partners' valuations are private information, reflecting what they know about their ability to run the business, buy-sell clauses are not recommended by economists to solve a deadlock (see McAfee[1992]). Contrary to legal advice, they have stressed that buy-sell clauses could result in inefficient dissolutions, i.e. the business may not end up in the hands of the partner who values it most[4]. When partners only know their own valuation, the proposer has to set a price based on her estimate for her partner's valuation. She offers a price above (below) her valuation if she believes that her partner's valua-

[1] Empirical evidence confirms that buy-outs take place very often. Hauswald and Hege[2003], from a sample of 193 two-parent US joint ventures, report that in 32% of them there was a buy-out and in 20% there was either a liquidation or sale to a third party, during the time period from 1985 to 2000.

[2] Buy-sell clauses are described in many law-books on partnership agreements, see, e.g., the book by J. Cadman [2004] or G. Stedman and J. Jones[1990]. The Guide to US Real State Investing issued by the Association of Foreign Investors in Real Estate (AFIRE), states that "it is common for joint venture agreements to provide a buy-sell or put-call mechanism by which the venture can be terminated. Such a clause is usually thought of as the ultimate mechanism for resolving disputes".

[3] The following partnerships all included buy-sell clauses: the partnership for the German TV-channel VOX between the media companies CLT-UFA and News Corporation; the home shopping partnership between Liberty and Comcast; the joint venture to run the New York-New York Hotel between Primadonna Resorts Inc. and MGM Grand Inc.; and the joint venture to own and operate "Original Levi's Stores" between Levi Strauss & Co. and Designs Inc.

[4] The importance of buy-sell agreements is now so broadly recognized that a lawyer's failure to recommend or include them in modern joint venture agreements is considered *malpractice* among legal scholars and practitioners (see Brooks and Spier[2004]). In contrast, McAfee[1992] concludes that "This result [of inefficiency] casts a shadow on the entire literature on cake-cutting type mechanisms [i.e. buy-sell provisions]".

tion is likely to be higher (lower) than her own[5]. Consequently, whenever the other party's valuation lies between hers and the price, she becomes a buyer, whereas efficiency requires her to sell. Similarly, when the other party's valuation lies between the price and hers, she inefficiently becomes the seller. Given this negative result, auctions have been suggested as a solution to the dissolution problem, see Cramton et al. [1987] and McAfee [1992][6]. Surprisingly, auctions are rarely considered as an alternative to buy-sell clauses in the literature on corporate law[7].

The economic and legal advice on how to dissolve a partnership seem to diverge. Nevertheless, the inefficiency argument against the buy-sell clause does not have bite if the *right* party is called to name the price. When both partners have low valuations (and would hence propose a price above their valuation) an efficient dissolution is guaranteed if the partner with the higher valuation proposes the price. Similarly, if both partners have high valuations, efficiency is guaranteed if the partner with the lower valuation proposes. Note that in either case the party with the valuation closer to the median valuation should propose. If one partner's valuation is below the median and the other's is above then either party proposing the price ensures efficiency. The determination of the proposer is then crucial for efficiency as it is from a legal point of view. Note that if a buy-sell clause exists there remains the question of who has to propose. Parties may disagree on this issue as the legal case Hotoyan v Jansezian illustrates. Two partners who had signed an agreement that included a buy-sell clause, went to the Ontario Court as they disputed over the matter of who had to go first[8]. An alternative way of settling the issue was taken by Comcast UK and Telewest, each 50% owners of the Cable London franchise. After several years of partnership, in February 1998 Comcast announced the intention to sell its cable interest to the NTL group. This announcement resulted in negotiations between Comcast UK and Telewest for the dissolution of their partnership. In August 1998 the Telewest spokesman announced that they had "solved an ambiguity in

[5]If a proposer perceives herself being more likely the seller, she is interested in a high price and would therefore optimally increase her price above her valuation. By contrast, if she expects to be buyer she optimally decreases her offer below her valuation.

[6]De Frutos [2000] analyzes properties of this auction in a model where partners have asymmetric prior beliefs. Kittsteiner [2003] shows the possibility of inefficiencies even in a model with symmetric priors but interdependent valuations. An overview on this literature is given in Moldovanu [2002].

[7]Hoberman[2001], when discussing alternatives to the buy-sell clause, does not even mention auctions. The same is true for books devoted to the drafting of shareholders' agreements (see e.g. Cadman[2004] and Stedman and Jones[1990]).

[8]This case was reported by the lawyer Bill Rosser (see "The Trusted Legal Advisor", March/April 2000).

the original ownership agreement". They agreed that by no later than 30 September 1999, Comcast (or NTL after the amalgamation with Comcast) would notify Telewest of a price at which Telewest would be required either to purchase or sell. The buy-out was completed in August 1999 with Comcast/NTL proposing a price of approx. £428 million to Telewest, who decided to buy. The agreement between Telewest and Comcast suggests that a buy-sell clause may be used after a negotiation stage to identity the proposer[9]. The particulars in this legal case impelled us to study the performance of buy-sell agreements that specify a buy-sell clause preceded by negotiations over the identity of the proposer.

In this paper we consider the buy-sell agreement as described above in the classical framework with independent private values. We model the negotiations as an ascending auction with parties bidding for the right to choose. It can be thought of as a simplified model of a negotiation procedure in which parties make alternating offers for the right-to-choose. If one of the parties is not willing to increase her offer further, she becomes proposer and receives the last offer made as a monetary payment[10]. Our main result shows that the information revealed during the negotiations leads to an efficient dissolution of the partnership. The rationale is as follows: If bidding strategies were silent, such that no information can be inferred from the bidding, then the value that partners assign to get the right to choose is a U-shaped function (in their valuation). If a buy-sell clause is enforced, partners with extreme valuations (i.e. either very high or very low) have a strong preference for becoming chooser as they can make large profits from exploiting their informational advantage if they choose. Note that the proposer's uncertainty about her partner's valuation makes her name a price close to the median valuation. By the same token, partners with valuations closer to the median cannot profit as much from choosing. We show that when partners take into account the information that will be revealed through their bidding, their values remain U-shaped. This ensures that the party with the valuation closer to the median valuation will propose and hence that an efficient dissolution will take place.

Our results may matter to academics as well as practitioners as the contractual arrangement we propose may avoid the cost of deadlocks, inefficien-

[9] Further evidence that parties negotiate over which partner has to propose comes from the cases *Damerow Ford Co. v. Bradshaw,* 876 P.2d 788 (1994), and *Rochez Bros., Inc. v. Rhoades,* 491 F.2d 402 (1974).

[10] In the press releases that Comcast and Telewest issued to announce that Comcast would be the proposer, they also state that Telewest agreed to acquire Birmingham Cable for £125 million. This supports our model as we perceive that these transfers incorporate the payment Telewest had to make for being chooser.

cies and/or lengthy court battles. We show that it is in the interest of the members of a partnership to write buy-sell agreements which contractually require parties to negotiate for the right to be chooser. This result not only reconciles the legal and economic view on buy-sell clauses, but it also provides practical advice on the drafting of buy-sell agreements and the triggering of buy-sell-clauses. In particular we recommend that parties lay out the rules that determine who sets the price.[11] It may be argued that a buy-sell clause specifies which party must propose: the party that seeks an exit strategy. Under this general specification, whenever both parties want to dissolve the partnership, they face a war of attrition as neither wants to be the one to make the buy-sell offer. We argue that there should be no contractual obligation for the party who initiates the dissolution to be compelled to name the price.

The economic literature provides other reasons why partners should write buy-sell clauses in their partnership agreements. In a complete information framework buy-sell clauses are recommended based on efficiency and fairness grounds. For a comprehensive study on this topic see Brams and Taylor[1996].

The use of contingent ownership structures in joint ventures as a response to double-moral hazard has received much attention in the academic literature. Nöldeke and Schmidt [1998] analyze the effect of contingent ownership structures on partners' investments in a joint venture. They show that an option contract giving one partner the right to buy the partnership at a predetermined price might lead to efficient investment prior to the dissolution. In a similar setup, Chemla et al [2003] consider the effect of option contracts on investments and on partners' incentives to withdraw assets (like know-how that is used for purposes that compete with the partnership) prior to the dissolution. They show that put and call options do not distort investments and can minimize the withdrawal of assets. Specifying the price in these option contracts typically requires determination of the value of the joint-venture, which is often delegated to an external accountant. Usually, this is problematic, since partners have a much better understanding of the business environment of their firm than outsiders and possess private information about the value of the firm. In addition the value of a partnership might differ between partners which makes it impossible to predetermine a suitable put- or call price. In such situations, partners are advised to include a price-finding rule, like a buy-sell clause in their partnership contract.

[11] When values are common and parties have asymmetric information (with one informed and one uninformed party) Brooks and Spier [2004] show that a mandatory buy-sell clause ensures that all potential gains from dissolution are realized.

Minehart and Neeman[1999] are the first who analyze incentives to invest under a buy-sell clause. They show that buy-sell clauses might outperform auctions in a framework where parties have to invest and exert effort in their partnership.

The paper is organized as follows. In section 2 we introduce the model and the buy-sell clauses. In section 3 we analyze the performance of buy-sell clauses with preceding negotiations to become the chooser and show its efficiency. In section 4 we show that if parties engage in a war of attrition waiting for the other to propose they will incur losses due to costly waiting. Nevertheless, the allocation of the property rights will still be efficient. Section 5 contains our conclusions. Finally, proofs are relegated to the Appendix.

2 Buy-Sell Clauses

We study partnership dissolution in the symmetric independent private values framework. Two risk neutral partners, 1 and 2, have valuations v_1 and v_2 respectively, for the sole ownership of the business[12]. Valuations are independently and identically distributed according to a cdf F with continuous density f and compact support on $[0,1]$. Even though the valuation v_i is private to partner i, reflecting what she knows about her ability to run the business, the model does not preclude partners from sharing information on a common value component of the business (as employee structure, assets, financial situation, etc). Each partner owns an equal share of the partnership which is due for dissolution.[13] An agent's utility is linear in payments and share, i.e. the utility of partner i who holds a share of α in the partnership and receives a payment m is given by $U_i = \alpha v_i + m$.

The buy-sell clause in this environment is from an economic viewpoint a mechanism where one party specifies a price (the proposer), and the other party decides whether to buy or sell at that price (the chooser). If p is the

[12] Throughout the paper we use the term *partnership*, but it should be clear that the model also applies to other legal entities. In particular to unincorporated joint ventures (partnerships), corporate joint ventures, strategic alliances, Societas Europaea and to some "dual-headed" structures. For the particulars of these legal entities see "Joint ventures and alliances: A guide to the legal issues" available from www.freshfields.com/practice/corporate/publications/pdfs/jointventure.

[13] In Hauswald and Hege's [2003] sample of joint ventures, the data show that about 80% are two-partner joint ventures. Further, about two-thirds of two-partner joint ventures have 50-50 equity allocations. Similarly, from a sample of 668 worldwide alliances, Veugelers and Kesteloot (1996) report that more than 90% of the alliances only involve two parties. Even though the data cover partnerships between unevenly sized firms, more than 50% exhibit 50-50 ownership.

price specified by the proposer, then the chooser selects either $p/2$ or the business, in which case she pays the proposer $p/2$. It can easily be verified that the chooser decides to take the money as long as the price p is larger than her valuation; otherwise, she decides to buy her partner's share.

The proposer's expected utility (or expected profit) if she proposes p is:

$$\begin{aligned} U^P(v_P, p) &= (v_P - p/2)\Pr(v_C \leq p) + p/2\Pr(v_C > p) \\ &= (v_P - p)F(p) + p/2, \end{aligned}$$

where subscripts P and C stand for proposer and chooser, respectively. Let us define the revenue maximizing price for the proposer by $p^*(v_P) = \arg\max_p U^P(v_P, p)$, and the derivative of U^P with respect to its second argument by U_2^P. It is important to note that the proposer's optimal strategy depends on the distribution of the chooser's valuation, whereas the chooser's optimal strategy depends only on the proposed price p and her own valuation v_C (it is therefore independent of any distributional assumptions). In what follows, we assume that the standard hazard rate conditions are satisfied:

$$\begin{aligned} \frac{d}{dx}\left(x + \frac{F(x)}{f(x)}\right) &\geq 0, \\ \frac{d}{dx}\left(x - \frac{1 - F(x)}{f(x)}\right) &\geq 0. \end{aligned} \tag{1}$$

The next proposition characterizes the equilibrium price set by the proposer and some of its properties. Let v^{med} be the median valuation.

Proposition 1 *(McAfee [1992]) The optimal price* $p^*(v)$ *is the unique solution for* p *to* $U_2^P(v, p) = 0$. *It is non-decreasing and satisfies* $p^*(v) = v$ *if* $v = v^{med}$, $p^*(v) < v$ *if* $v > v^{med}$, *and* $p^*(v) > v$ *if* $v < v^{med}$.

The rationale behind the properties of the equilibrium price is clear. If a partner with a valuation above the median sets a price equal to her own valuation, she will more likely end up buying the business. She would hence improve her profits by reducing the buying price[14]. Similarly, if her valuation is below the median she is better off setting a price above her valuation, as she is more likely the selling partner.

A dissolution mechanism ensures an ex-post *efficient* allocation if the partner with the highest valuation gets the entire partnership. Using a buy-sell clause to dissolve a partnership may lead to inefficient allocations. The

[14] By lowering the price, she would also sell to partners with a valuation slightly below her own, therefore making a loss on these trades. This loss is of second order whereas the gain because of buying at a lower price is of first order.

inefficiency might arise when partners' valuations are either both below or both above the median valuation. However, in either case, inefficiencies only arise when the *wrong* partner is proposing. To make this point clear, consider first that partners' valuations are both below the median. As either partner will name a price larger than her valuation, efficiency requires that the partner with the largest valuation proposes. Similarly, if valuations are both above the median then an efficient allocation emerges whenever the partner with the smallest valuation proposes. This suggest that the partner with the lowest valuation should choose if both valuations are below the median, and propose if they are above the median. A natural question to ask is then whether an endogenous determination of the proposer can render efficient allocations.

3 Negotiating the right to choose

When a management deadlock comes forth, parties must decide on who proposes. Legal cases, as those referred to in the introduction, show that parties stuck in a deadlock have followed different routes, from going to Court to reaching agreement through negotiations on who will propose. We here study the performance of this latter exit strategy. We examine the outcome of a dissolution when partners must abide by the buy-sell clauses in their shareholder agreement, and they negotiate to determine the party entitled to choose.

We consider a dissolution procedure which consists of two stages. In the first stage, *the negotiation stage,* partners determine who becomes chooser and proposer. In the second stage, *the pricing stage,* they dissolve the partnership according to the rules of the buy-sell clause.[15] We will refer to this sequential game as the *dissolution game*. The negotiation stage, is modelled as an ascending auction. Both parties raise their bids continuously, and either party can drop out of the auction at any time. The party who drops out becomes proposer and receives a payment equal to the bid at which the auction ends. The ascending auction can be seen as a continuous version of an alternating-offer negotiation game.

Since the dissolution game is a sequential game with incomplete information, our equilibrium concept is Perfect Baysian Equilibrium. To convey the intuition, we first solve the pricing stage in which the identity of the proposer is already determined assuming U-shaped bidding functions in the

[15] The legal case Telewest/Comcast is consistent with this sequence of events. In Telewest's press release from 17/08/1998, it was announced that they had agreed on Comcast becoming the proposer. Furthermore it was announced that by no later than 30/09/1999 Comcast had to notify Telewest the price offer.

first stage. We then solve the entire game and confirm that the purported bidding is consistent with equilibrium behavior.

3.1 The Pricing Stage

An important aspect of the dissolution game is that information about partners' valuations is revealed by the strategies played in the first round. The proposing party hence updates her beliefs about the distribution of her partner's valuation. Information is only important for the proposer as the chooser's decision is belief independent. Note that the chooser only needs to compare his valuation with the proposed price.

As a starting point we assume that there is an equilibrium in which bidding strategies are silent, such that no inference about valuations can be made. If this were the case partners' willingness to pay for being entitled to choose would be U-shaped. This can be seen from Figure 1 for the uniform distribution, and is shown to hold for any cdf in Proposition 9 in the Appendix. The intuition for this result is that choosers benefit from the fact that the proposer has uncertainty about whether she will sell or buy. Because of this uncertainty, prices are set close to v^{med}, which favors choosing parties with "more extreme" valuations. In addition, when these types propose, they cannot set a price too close to v^{med}, as it would result in unprofitable trade with high probability. Consequently, they cannot take much advantage of their "extreme" valuations when they propose. These extreme types have hence a higher willingness to pay. The opposite holds for a type close to v^{med}. If this type proposes, she optimally names her valuation as she is equally likely to be buyer or seller. Her expected utility as proposer is hence half her valuation. As chooser she faces prices close to her valuation. If they are above she sells and gets an expected utility slightly larger than half her valuation. If they are below, she buys which also give her a expected utility slightly above half her valuation. The difference in expected profits is hence strictly positive but small.[16]

To solve the pricing stage, we assume that bidding strategies are indeed U-shaped. As such strategies reveal information, the behavior of the losing party at the pricing stage takes into account what she learns from the auction. In particular, a losing party who bid $b(v_P)$ knows that the other party was willing to bid higher. Because of the U-shaped form of the bidding function, she further knows that there may exist another valuation $\widetilde{v}_P$ that would have dropped out at the same bid, i.e. $b(v_P) = b(\widetilde{v}_P)$. Assuming that $\widetilde{v}_P > v_P$

[16]The preference to be the chooser also holds in a common values environment with incomplete information (see Morgan[2003] and Brooks and Spier[2004]).

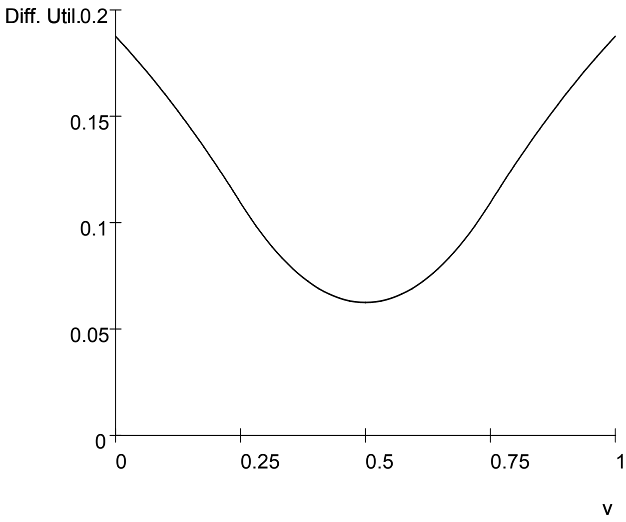

Figure 1: $U^C(v) - U^P(v)$ for uniform distribution.

the losing party concludes her partner's valuation must be either above $\widetilde{v}_P$ or below v_P. Figure 2, illustrates this argument. The precise updating is the content of the next lemma.

Lemma 2 *If the bidding strategies in the negotiation stage are U-shaped, a proposer who bid $\widehat{b}$ has updated beliefs given by*

$$\begin{aligned} F^C(x) &= \Pr\left(v_C \leq x \middle| \; v_C \in [0, v^*) \cup (v^{**}, 1]\right) \\ &= \begin{cases} \frac{F(x)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [0, v^*) \\ \frac{F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [v^*, v^{**}] \\ \frac{F(x)-F(v^{**})+F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in (v^{**}, 1], \end{cases} \end{aligned} \tag{2}$$

where $b(v^*) = b(v^{**}) = \widehat{b}$, and $v^* < v^{**}$.
If $b(v^*) = b(v^{**})$ does not hold for two different types, then the updated distribution is given by the formula above with $b(v^*) = \widehat{b}$.

The distribution F^C is a continuous, piecewise defined function, which has three regions, the inner one flat. Consequently, it is not strictly monotonic in valuations. This fact makes the maximization problem of the proposer non-concave which may result in multiplicity of optimal prices. In the Appendix we show that the optimal price in the two outer regions is unique. In the

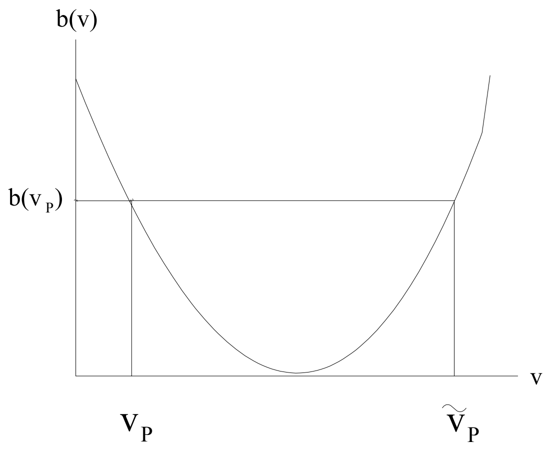

Figure 2: The proposer's inference after the negotiation stage.

inner region, either the optimal price is at one of the boundaries and therefore unique or any price in that region is optimal. Furthermore we show that if the global maximum is at the outer regions, then it is unique. If it lies in the inner region it is also unique unless $F^C(v^*) = \frac{1}{2}$. In the latter case any price that lies in the inner region is optimal.

Lemma 3 *If bidding functions at the negotiation stage are U-shaped and $F^C(v^*) \neq \frac{1}{2}$, there is a unique optimal price for the proposer in the pricing stage.*

3.2 The Negotiation Stage

We now focus on the bidding functions that will be optimal for the partners given the continuation pricing game described before. We first note that the overall utility of a partner in the dissolution game can be decomposed into her expected payoff in the pricing game, plus the payments she expects to receive/pay in the auction.

Equilibrium bidding strategies reflect parties' willingness to pay to become chooser, which in our two-stage game is given by the expected profits from the pricing game and the expected transfers from the auction. As argued in the last subsection, a plausible conjecture is that bidding functions are U-shaped. Furthermore, since a candidate type for having the minimal willingness to pay is v^{med}, it is arguable that the equilibrium bids are symmetric around this type. I.e., types v and $s(v)$, $v < v^{med} < s(v)$, bid the same amount $b(v)$, where $s(v) := F^{-1}(1 - F(v))$ is the complementary quantile of v.

To investigate whether this symmetry is consistent with equilibrium behavior consider the effects of a small (marginal) change in the bids of types v and $s(v)$. If this deviation had the same (marginal) effect on their expected profits in the pricing game, they would have the same (local) incentives to bid $b(v)$. To see that these incentives are indeed equal consider a type v partner who marginally increases her bid to $b(v - dv)$. We just have to look at the change in expected profits that results from trades with the marginal types v and $s(v)$. She now becomes chooser with respect to these types (her marginal gain), whereas before was proposing to those types (her marginal loss).

As chooser from a partner with valuation v who proposes a price of v she buys and hence gets $v - \frac{v}{2} = \frac{v}{2}$.[17] As chooser from a partner with valuation $s(v)$ she sells and gets $\frac{s(v)}{2}$. Since both events happen with "probability" $f(v)$, her marginal gain equals $\frac{v+s(v)}{2}f(v)$.

As proposer she would set a price of v. She would hence buy from another partner with valuation v getting $\frac{v}{2}$ with probability $f(v)$. She would sell to $s(v)$, receiving $\frac{v}{2}$with probability $f(v)$.[18] Therefore, her marginal loss is $vf(v)$.

Subtracting $\frac{v+s(v)}{2}f(v)$ from $vf(v)$ results in an overall marginal change in expected profits equal to $\frac{s(v)-v}{2}f(v)$.

Consider now a deviation by a type $s(v)$ who marginally increases her bid to $b(s(v - dv))$.[19] Her marginal gain from being chooser is $\left(\frac{3}{2}s(v) - \frac{v}{2}\right)f(v)$. Note that as chooser she buys from v and pays $\frac{v}{2}$ thus getting $s(v) - \frac{v}{2}$, and she sells to $s(v)$ receiving $\frac{s(v)}{2}$. Her marginal loss from deviating is $s(v)f(v)$. The difference in expected profits from being chooser instead of proposer (with respect to the marginal types v and $s(v)$) is $\frac{s(v)-v}{2}f(v)$. Thus, the marginal change in expected profits from the pricing stage are the same for v and $s(v)$.

In equilibrium this change in expected profits has to equal the change in transfers in the auction. The latter includes the direct effect of an increase in

[17] To ease the exposition we here assume that proposers with valuations in the interval $(w, s(w))$ set prices equal to their valuations. But as shown in the proof of lemma 3, they can optimally set any price in that interval. None of the arguments here depend on this assumption as long as proposers randomly draw the price from that interval according to a uniform distribution.

[18] Note that a decrease in v does increase $s(v)$ hence the probability that the partner faces a type $s(v)$ partner is given by $-\frac{d}{dv}F(s(v)) = -\frac{d}{dv}(1 - F(v)) = f(v)$.

[19] Deviations by decreasing the bid are computationally more involved and require an Envelope-Theorem argument. See the proof of Theorem 6 (in the Appendix) for details.

$b(v)$ on payments and on winning probabilities. By increasing the bid type v is more likely to win, she will hence have to pay the bid of the marginal type $b(v)$ whereas before she was receiving this amount as payment. This happens with probability $2f(v)$ which gives a marginal loss of $-4b(v)f(v)$. But with probability $2F(v)$ she remains loser and now gets a larger payment. Her marginal gain is then $2F(v)\frac{d}{dv}b(v)$. The difference in expected profits in the auction adds up to $-4b(v)f(v)-2F(v)\frac{d}{dv}b(v)$. In equilibrium, the marginal loss and the marginal gain from deviating must cancel out. Therefore the following differential equation has to be fulfilled:

$$2F(v)\frac{d}{dv}b(v)+4b(v)f(v)=\frac{s(v)-v}{2}f(v).$$

The next theorem provides an equilibrium bidding function $b(v)$ that solves this differential equation and is indeed U-shaped.

Theorem 4 *The following strategies constitute an equilibrium of the dissolution game:*

- In the first stage, both partners bid according to the following bidding function

$$b(v)=\begin{cases}\frac{\frac{1}{2}\int_0^v(s(t)-t)F(t)f(t)dt}{2F^2(v)} & \text{if} \quad v\leq v^{med}\\ \frac{\frac{1}{2}\int_v^1(t-s(t))(1-F(t))f(t)dt}{2(1-F(v))^2} & \text{if} \quad v>v^{med},\end{cases}\tag{3}$$

where

$$s(v):=F^{-1}(1-F(v)).$$

- In the second stage, the proposer sets a price equal to her valuation, i.e. $p=v_P$.

The equilibrium bidding functions are strictly decreasing for valuations below the median v^{med} and strictly increasing for $v>v^{med}$. In addition we have that $b(v)=b(s(v))$, i.e. for any v we have that the mass of valuations that submit a higher bid is equally distributed on valuations smaller than v and valuations that are larger than v. To further emphasize that the result does not depend on the cdf being symmetric, we illustrate the bidding function for the cdf $F(v)=v^2$.

Example 5 *Assume that valuations are distributed according to* $F(v)=v^2$, $v\in[0,1]$. *The following bidding function* $b_B(v)$ *(see Figure 3) is part of an ex-post efficient equilibrium:*

$$b_B(v)=\begin{cases}\frac{\frac{1}{4}\sqrt{1-v^2}\left(\frac{2}{5}v^4-\frac{2}{15}v^2-\frac{4}{15}\right)-\frac{1}{10}v^5+\frac{1}{15}}{v^4} & \text{if } 0\leq v\leq\frac{1}{\sqrt{2}}\\ \frac{\frac{1}{10}v^5-\frac{1}{6}v^3-\frac{1}{4}\sqrt{1-v^2}\left(\frac{2}{5}v^4-\frac{4}{5}v^2+\frac{2}{5}\right)+\frac{1}{15}}{(v^2-1)^2} & \text{if } \frac{1}{\sqrt{2}}\leq v\leq 1.\end{cases}$$

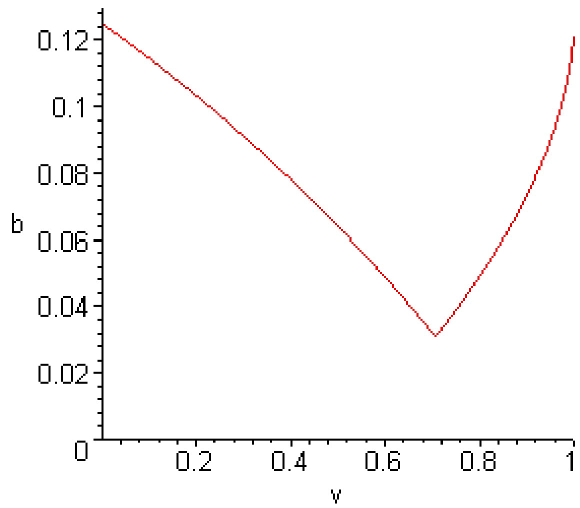

Figure 3: The bidding function b_B.

The most important property of the equilibrium in Theorem 6 is that it renders an efficient dissolution of the partnership. Note that what is needed for efficiency is the symmetry of the U-shaped bidding functions. This guarantees that the losing party assigns the same probability to her partner having a valuation larger than hers as she does to her partner having a valuation smaller than hers.[20].

Corollary 6 *The equilibrium is ex-post efficient.*

4 Waiting for the other to propose

We have shown in the previous section that gains from trade can be realized if parties negotiate the right to choose. In this subsection we provide another argument that favors this procedure. In the absence of negotiations stipulated in the termination agreement, either party may end up waiting for the other to name a price, as neither wants to propose.[21] The time it takes to reach an agreement might incur further cost to the parties as it may

[20] As argued above, it is also an equilibrium to bid according to (3) in the first stage and in the second stage to choose a price from the optimal set of prices (given the bidding behavior), provided that each price is named with the same probability. Note that setting the price equal to the own valuation is the simplest rule of thumb.

[21] In most statutes, a court does not have the ability to dissolve the entity based on a simple disagreement, see, e.g., section 32 of the Uniform Partnership Act.

prevent them from either getting involved in other ventures or from running the business. This costly and time consuming route can be modelled as a war of attrition. The losses associated with the war of attrition measure the resources that parties can save if they properly draft the implementation of the buy-sell clause.

We concentrate on a situation where both parties incur the same constant marginal cost of waiting, which is, per unit of time, normalized to 1.[22] As partners with extreme valuations are more reluctant to name a price, they will wait longer. Relying on arguments similar to the ones given above it can be shown that symmetric U-shaped bidding functions are consistent with equilibrium behavior. Let $b_W(v)$ denote the time at which a party with valuation v quits the war of attrition, given the other party has not quit. If the management deadlock occurs at time zero, $b_W(v)$ reflects the cost incurred by both parties when the first to quit has valuation v. Note that in the war of attrition both parties pay the cost of waiting, whereas in the negotiations any payment is a transfer from one party to the other. Due to this, in equilibrium, the expected profit from a marginal increase in the fighting time becomes

$$-2F(v)\frac{d}{dv}b_W(v) - \frac{s(v)-v}{2}f(v) = 0.$$

The last term reflects the marginal change in profits accruing from the pricing stage. It coincides with that under negotiations, as the information revealed by either mechanism is the same. The first term reflects the marginal increase in payments from continuing the fight. The equilibrium behavior of the war of attrition is obtained by solving the differential equation above with the boundary condition $b_W\left(v^{med}\right) = 0$.

Theorem 7 *The following strategies constitute an equilibrium of the war of attrition game:*

- In the war of attrition both partners quit according to the following function

$$b_W(v) = \begin{cases} \int_v^{v^{med}} \frac{(s(t)-t)f(t)}{4F(t)}dt & \text{if} \quad v \leq v^{med} \\ \int_{v^{med}}^{v} \frac{(t-s(t))f(t)}{4(1-F(t))}dt & \text{if} \quad v > v^{med}, \end{cases}$$

where

$$s(v) := F^{-1}(1-F(v)).$$

- In the second stage, the proposer sets a price equal to her valuation, i.e. $p = v_P$.

[22] This is not a crucial assumption. As long as both partners have an opportunity cost of waiting given by the same strictly increasing function in time, our analysis carries through.

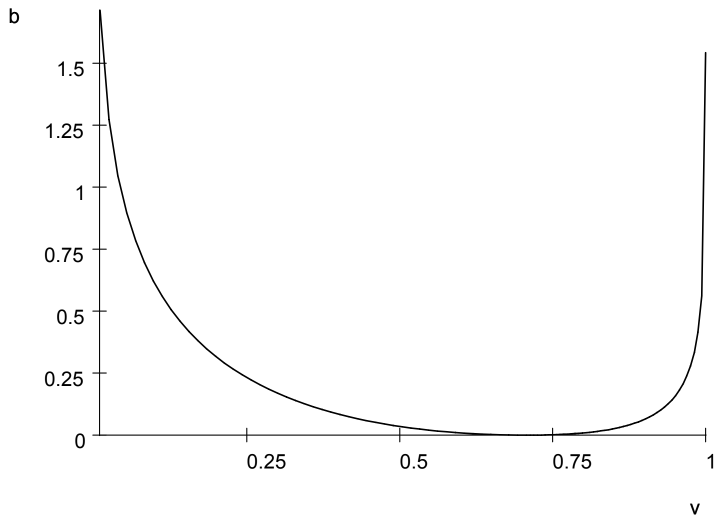

Figure 4: The strategy $b_W(v)$.

Example 8 *Assume that valuations are distributed according to $F(v) = v^2$, $v \in [0,1]$. The following strategy $b_W(v)$ (see Figure 4) is part of an ex-post efficient equilibrium:*

$$b_W(v) = \begin{cases} \frac{1}{2}v - \frac{1}{2}\sqrt{(1-v^2)} + \frac{1}{2}\operatorname{arctanh}\frac{1}{\sqrt{(1-v^2)}} - \frac{1}{2}\operatorname{arctanh}\sqrt{2} & \text{if } 0 \leq v \leq \frac{1}{\sqrt{2}} \\ \frac{1}{2}\sqrt{1-v^2} - \frac{1}{2}v + \frac{1}{2}\operatorname{arctanh}\frac{1}{v} - \frac{1}{2}\operatorname{arctanh}\sqrt{2} & \text{if } \frac{1}{\sqrt{2}} \leq v \leq 1. \end{cases}$$

The war of attrition achieves allocative efficiency but it generates inefficient costs of waiting. Hence, if both parties wait for the other to propose, the buy-sell clause remains renegotiation-proof. However, the possibility of high costs of waiting might lead parties to seek for outside assistance to settle the dispute, as the case Hotoyan v Jansezian mentioned in the introduction shows.

It can be shown that a partner, independently of her valuation, prefers to negotiate about the right to choose instead of initiating a war of attrition[23]. Hence one could always propose to partners to engage in negotiations (instead of initiating a war of attrition) and they will anonymously agree to do so, without revealing any information about their valuation.

[23] This is indeed true for any point of time during the war of attrition.

5 Conclusion

In this paper we have analyzed buy-sell clauses for the most prevalent partnership structure, 50-50 ownership. Buy-sell clauses are perhaps the most common dissolution arrangement for these partnerships. Nevertheless, despite its broad use in practice, the economic theory literature has not recommended this clause for efficiency. We have here argued that the possibility of efficient dissolution is related to how partners decide on which party should choose. In particular, we have shown that if partners negotiate over which party is entitled to choose, then an efficient dissolution always takes place. The information revealed during the negotiations leads to the losing party naming a price such that her partner only buys if she has a higher valuation than hers.

There are strong reasons why partners may want to write Termination agreements that achieve efficiency. Parties sign these agreements ex-ante, i.e., before they have learned the value they assign to run the business on their own. At this stage, they want to sign an agreement that they will not regret ex-post, that will hence be renegotiation proof. While our analysis has shown that a buy-sell clause preceded by negotiations is a tool to ensure efficiency, other contractual arrangements may perform better in other setups. For instance, if valuations are asymmetrically distributed, efficiency will not be ensured with the procedure analyzed. In this case, auctions are not efficient either. It is not clear which mechanism will perform better in this environment.[24]

The results of this paper may have normative implications. It is advisable that any partnership that recognizes the benefits of buy-sell clauses agrees to include in their termination agreement that parties will negotiate the right to choose. Waiting for the other party to name the price (war of attrition) or appealing to court to resolve the issue, consumes resources and/or may result in an inefficient allocation of the property rights. Accordingly, drafting workable buy-sell clauses is essential to any termination agreement.

[24]The asymmetric case is hard to analyze since no close-form solutions for the bidding functions exist. Any comparison must rely on particular distributional specifications, see de Frutos [2000].

6 Appendix

Proposition 9 *The chooser has an interim utility strictly greater than the proposer. Further, the difference in expected utility between being chooser and proposer is U-shaped and it has a minimum at v^{med}.*

Proof.
The expected utility of partner i if she proposes equals

$$\begin{aligned} U^P(v_i) &= (v_i - p^*(v_i))F(p^*(v_i)) + p^*(v_i)/2 \\ &= U^P(0) + \int_0^{v_i} F(p^*(z))dz. \end{aligned}$$

If she chooses it equals

$$U^C(v_i) = E_{v_j}\left[\max(v_i - \frac{p^*(v_j)}{2}, \frac{p^*(v_j)}{2})\right],$$

which can be written as:

$$U^C(v_i) = \begin{cases} \frac{1}{2}E_{v_j}(p^*(v_j)) & \text{if} \quad v_i \le p^*(0) \\ \begin{array}{c} v_i F((p^*)^{-1}(v_i)) + \frac{1}{2}E_{v_j}(p^*(v_j)) \\ - \int_0^{(p^*)^{-1}(v_i)} p^*(z)f(z)dz \end{array} & \text{if} \quad p^*(0) < v_i \le p^*(1) \\ v_i - \frac{1}{2}E_{v_j}(p^*(v_j)) & \text{if} \quad p^*(1) \le v_i. \end{cases}$$

If $v_i \le p^*(0)$, the difference between these expected utilities $(U^C(v_i) - U^P(v_i))$ becomes

$$\frac{1}{2}E_{v_j}(p^*(v_j)) - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

If $p^*(0) < v_i \le p^*(1)$, then it equals

$$v_i F((p^*)^{-1}(v_i)) + \frac{1}{2}E_{v_j}(p^*(v_j)) - \int_0^{(p^*)^{-1}(v_i)} p^*(z)f(z)dz - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

And, if $p^*(1) \le v_i$, then it equals

$$v_i - \frac{1}{2}E_{v_j}(p^*(v_j)) - U^p(0) - \int_0^{v_i} F(p^*(z))dz.$$

We must hence distinguish three cases:

1. If $v_i \leq p^*(0)$ then it is straightforward to see that $U^C(v_i) - U^P(v_i)$ is strictly decreasing in v_i.

2. If $p^*(0) < v_i \leq p^*(1)$ then $\frac{\partial\left(U^C(v_i) - U^P(v_i)\right)}{\partial v_i} = F((p^*)^{-1}(v_i)) - F(p^*(v_i))$. This is negative for $v_i \leq v^{med}$ and positive otherwise.

3. If $p(1) \leq v \leq 1$ then $\frac{\partial\left(U^C(v) - U^P(v)\right)}{\partial v} = 1 - F(p(z)) \geq 0$.

Hence the difference in expected utility is U-shaped and it has a minimum at $v = v^{med}$. ■

Proof of Lemma 2:
It is straightforward and it is hence omitted. ■

Proof of Lemma 3:
If the bidding strategies in the negotiation stage are U-shaped, a proposer who bid $\widehat{b}$ has updated beliefs given by (2)

The hazard rate conditions (1) imply that for any $K \in [0, 1]$

$$\frac{d}{dx}\left(x + \frac{F(x) - K}{f(x)}\right) \geq 0.$$

It is then easy to verify that $F^C(x)$ also satisfies the standard hazard rate conditions (1) for any $x \in [0, v^*] \cup [v^{**}, 1]$. The proposer's utility, if she sets a price of p is given by:

$$U^P(v_P, p) = \left(v_P - \frac{p}{2}\right) F^C(p) + \frac{p}{2}\left(1 - F^C(p)\right).$$

The first order condition for $p \notin [v^*, v^{**}]$ becomes

$$\begin{aligned} U_2^P(v_P, p) &= (v_P - p) f^C(p) - F^C(p) + \frac{1}{2} \\ &= f^C(p)\left[v_p - p - \frac{F^C(p) - \frac{1}{2}}{f^C(p)}\right]. \end{aligned}$$

In addition, if $U_2^P(v_P, p) = 0$ holds then:

$$\frac{d^2}{dp^2} U^P(v_P, p) = f^C(p)\left[-\frac{d}{dp}\left(p + \frac{F^C(p) - \frac{1}{2}}{f^C(p)}\right)\right] < 0.$$

This shows that there is at most one optimal price in either outer region, i.e in $[0, v^*]$ and in $[v^{**}, 1]$.

Consider now the inner part. Since the utility of the proposer in this region is either strictly increasing or strictly decreasing in p, the optimal price in this region is always at the boundaries, as $F^C(v^*) \neq \frac{1}{2}$.

To show uniqueness and existence of a global maximum it is necessary to compare the local maxima in the intervals $(0, v^*)$ and $(v^{**}, 1)$ (given by the first order conditions above) with the value of the profit function at both v^* and v^{**}. This tedious mechanical exercise is here omitted but can be sent upon request. ■

Proof of Theorem 6:
Consider a bidder with valuation v smaller than v^{med} who bids $b(\widehat{v})$ when the other bidder bids according to (3). Let $U(v, \widehat{v})$ denote the interim utility (in the dissolution game) of this bidder. By imitating a bidder of type $\widehat{v}$ she will be a chooser in the pricing stage if the other agent's valuation is within the interval $[\widehat{v}, s(\widehat{v})]$ and a proposer otherwise. $U(v, \widehat{v})$ can be decomposed in the expected payoff from being chooser (denoted by $U^C(v, \widehat{v})$) and proposer (denoted by $U^P(v, \widehat{v})$) in the pricing stage, and the payments she expects to receive/ pay in the auction. Her expected utility is then

$$U(v, \widehat{v}) = U^P(v, \widehat{v}) + U^C(v, \widehat{v}) - \int_{\widehat{v}}^{s(\widehat{v})} b(x) f(x) \, dx + 2F(\widehat{v}) b(\widehat{v}). \quad (4)$$

Differentiating this overall expected utility with respect to its second argument we have

$$\begin{aligned} U_2(v, \widehat{v}) &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) - b(s(\widehat{v})) f(s(\widehat{v})) \frac{ds(\widehat{v})}{d\hat{v}} \\ &\quad + 3b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}} \\ &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) + 4b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}}, \end{aligned}$$

where the second equality follows from the symmetry of the bidding function with $b(\hat{v}) = b(s(\hat{v}))$ and from the definition of $s(\cdot)$ which gives $f(s(\widehat{v}))\frac{ds(\widehat{v})}{d\hat{v}} = -f(\widehat{v})$.

For $b(\cdot)$ to be an equilibrium strategy, it must be optimal for type v to bid $b(v)$, which provides the following necessary condition

$$U_2(v, \hat{v}) \mid_{\hat{v}=v} = 0 \text{ for all } v \in [0, v^{med}]. \quad (5)$$

We must show that $\max_{\widehat{v}} U(v, \widehat{v}) = U(v, v)$. Note that we only need to show this for $\widehat{v} \leq v^{med}$ by the symmetry of $b(\cdot)$. Since the probability of winning, the payments and the information revealed is exactly the same when bidding

$b(\hat{v})$ and $b(s(\hat{v}))$, a deviation to a bid $b(\widehat{v})$ with $\widehat{v} > v^{med}$ is equivalent to deviate to a bid $b(s(\tilde{v}))$ for some $\tilde{v} \leq v^{med}$.
In order to derive $U_2(v, \hat{v})$ we first compute $U_2^P(v, \hat{v})$. A losing bidder who bids $b(\widehat{v})$ correctly infers that the other partner's valuation is either smaller than $\hat{v}$ or larger than $s(\hat{v})$. She uses this information to update her beliefs. Consequently, she proposes a price p which maximizes

$$U_{\widehat{v}}(v, p) = \left(v - \frac{p}{2}\right) F_{\widehat{v}}^C(p) + \frac{p}{2}\left(1 - F_{\widehat{v}}^C(p)\right),$$

where

$$F_{\widehat{v}}^C(x) = \begin{cases} \frac{F(x)}{2F(\widehat{v})} & \text{if} \quad x \in [0, \widehat{v}] \\ \frac{F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [\widehat{v}, s(\widehat{v})] \\ \frac{F(x) - F(s(\widehat{v})) + F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [s(\widehat{v}), 1]. \end{cases}$$

Differentiating proposer's utility gives

$$\frac{d}{dp} U_{\widehat{v}}(v, p) = \begin{cases} (v - p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \leq \widehat{v}, \\ 0 & \text{if} \quad p \in (\widehat{v}, s(\widehat{v})) \\ (v - p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p) - 1 + 2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \geq s(\widehat{v}). \end{cases}$$

It is easy to see from this expression that the optimal price depends on $\hat{v}$.

Two cases have to be distinguished. If $\widehat{v} \leq v$ then the following two inequalities hold:

$$\begin{array}{rl} (v - p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} > 0 & \text{for} \quad p \leq \widehat{v}, \text{ and} \\ (v - p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p) - 1 + 2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 & \text{for} \quad p \geq s(\widehat{v}). \end{array}$$

Hence, setting a price in the interval $[\widehat{v}, s(\widehat{v})]$ is optimal, resulting in a utility as proposer equal to

$$U^P(v, \widehat{v}) = (v - \frac{1}{2}p^{opt})F(\widehat{v}) + \frac{1}{2}p^{opt}(1 - F(s(\widehat{v}))) = vF(\widehat{v}).$$

Since $\widehat{v} \leq v$, we further obtain

$$\lim_{\widehat{v} \nearrow v} U_2^P(v, \widehat{v}) = vf(v).$$

Consider now that $v^{med} \geq \widehat{v} \geq v$. In this case we have

$$\begin{array}{c} (v - \widehat{v})\frac{f(\widehat{v})}{2F(\widehat{v})} - \frac{F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} \leq 0 \text{ and} \\ (v - p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p) - 1 + 2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 \text{ for all } p \geq s(\widehat{v}). \end{array}$$

The optimal price must hence satisfy $p^{opt} \leq \widehat{v}$. Consequently, the utility as proposer is now equal to

$$\begin{aligned} U^P(v,\widehat{v}) &= (v-\frac{1}{2}p^{opt})F\left(p^{opt}\right)+\frac{1}{2}p^{opt}\left[F\left(\widehat{v}\right)-F\left(p^{opt}\right)+1-F\left(s(\widetilde{v})\right)\right] \\ &= p^{opt}\left(F\left(\widehat{v}\right)-F\left(p^{opt}\right)\right)+vF\left(p^{opt}\right), \end{aligned}$$

where p^{opt} satisfies the following FOC

$$\left(v-p^{opt}\right)f\left(p^{opt}\right)-F\left(p^{opt}\right)+F\left(\widehat{v}\right)=0. \tag{6}$$

Since $\lim_{\widehat{v}\searrow v} p^{opt}=v$, we obtain[25]

$$\begin{aligned} \lim_{\widehat{v}\searrow v} U_2^P(v,\widehat{v}) &= \lim_{\widehat{v}\searrow v}\left[F\left(\widehat{v}\right)-F\left(p^{opt}\right)-p^{opt}f\left(p^{opt}\right)+vf\left(p^{opt}\right)\right]\frac{d}{d\widehat{v}}p^{opt} \\ &\quad +p^{opt}f\left(\widehat{v}\right) \\ &= \lim_{\widehat{v}\searrow v}\ p^{opt}f\left(\widehat{v}\right) \\ &= vf\left(v\right). \end{aligned}$$

Analysis above ensures that $U_2^P(v,\hat{v})\mid_{\hat{v}=v}= vf(v)$ for all $\hat{v}\leq v^{med}$.

We now compute $U_2^C(v,\hat{v})$. As the proposer always sets a price equal to her valuation, the expected utility as chooser will be

$$U^C(v,\widehat{v})=\begin{cases} \int_{\widehat{v}}^{v}\left(v-\frac{x}{2}\right)f\left(x\right)dx+\int_{v}^{s(\widehat{v})}\frac{x}{2}f\left(x\right)dx & \text{if}\quad \widehat{v}\leq v \\ \int_{\widehat{v}}^{s(\widehat{v})}\frac{x}{2}f\left(x\right)dx & \text{if}\quad \widehat{v}\geq v. \end{cases}$$

Differentiating the chooser's utility with respect to $\hat{v}$ yields

$$U_2^C(v,\widehat{v})=\begin{cases} \left(\frac{\widehat{v}-s(\widehat{v})-2v}{2}\right)f(\widehat{v}) & \text{if}\quad \widehat{v}\leq v \\ -\left(\frac{\widehat{v}+s(\widehat{v})}{2}\right)f(\widehat{v}) & \text{if}\quad \widehat{v}\geq v, \end{cases}$$

Evaluating $U_2^C(v,\widehat{v})$ at $v=\widehat{v}$ gives

$$U_2^C(v,v)=-\left(\frac{s\left(v\right)+v}{2}\right)f\left(v\right).$$

Putting these results together the first order condition (5) becomes

$$-\left(\frac{s\left(v\right)-v}{2}\right)f\left(v\right)+2F\left(v\right)\frac{d}{dv}b\left(v\right)+4b\left(v\right)f\left(v\right)=0.$$

[25] To make sure that p^{opt} is uniquely defined (given v and $\widehat{v}$) we need the hazard rate condition to hold. For the complete argument see McAfee [1992].

For (5) to hold at $v=0$ we need that $b(0)=\frac{1}{8}$. Thus a differentiable equilibrium has to be a solution of the boundary value problem:

$$\begin{aligned} b(0) &= \frac{1}{8}, \\ -\frac{s(v)-v}{2}f(v)+2F(v)\frac{d}{dv}b(v)+4b(v)f(v) &= 0 \text{ for } v\leq v^{med}. \end{aligned}$$

The differential equation can be written as

$$\frac{s(v)-v}{2}F(v)f(v)=\frac{d}{dv}\left(2b(v)F^2(v)\right),$$

and then integrated to obtain (3).
We next show that bidding $b(v)$ indeed does not result in a lower payoff (for a bidder with valuation v) than bidding $b(\widehat{v})$ with $\widehat{v}\leq v^{med}$. Observe first that for $\widehat{v}\leq v$ we have that

$$U_2(v,\widehat{v})=-\frac{s(\widehat{v})-\widehat{v}}{2}f(\widehat{v})+2F(\widehat{v})\frac{d}{dv}b(\widehat{v})+4f(\widehat{v})b(\widehat{v})$$

with $U_{2,1}(v,\widehat{v})=0$, and therefore a bid of $b(\widehat{v})$ does not give larger payoffs than a bid of $b(v)$. Assume next that $\widehat{v}\geq v$. To show the optimality of $b(v)$ in this case, it is sufficient to show that $U_{2,1}(v,\widehat{v})\geq 0$ for all $v\leq\widehat{v}\leq v^{med}$ (see McAfee [1992]). Using the abbreviation $p_v:=\frac{\partial}{\partial v}p^{opt}$ we have that

$$\begin{aligned} U_{2,1}(v,\widehat{v}) &= U^P_{2,1}(v,\widehat{v}) \\ &= p_v f(\widehat{v}). \end{aligned}$$

From (6) we obtain that

$$p_v=-\frac{f(p^{opt})}{(v-p^{opt})f'(p^{opt})-2f(p^{opt})},$$

which shows that

$$\begin{aligned} U_{2,1}(v,\widehat{v}) &= \frac{f(p^{opt})f(\widehat{v})}{2f(p^{opt})-(v-p^{opt})f'(p^{opt})} \\ &= \frac{f(p^{opt})f(\widehat{v})}{2f(p^{opt})-\frac{F(p^{opt})-F(\widehat{v})}{f(p^{opt})}f'(p^{opt})} \\ &= \frac{f(\widehat{v})}{\left.\frac{d}{dp}\right|_{p=p^{opt}}\left(p+\frac{F(p)-F(\widehat{v})}{f(p)}\right)}\geq 0. \end{aligned}$$

The second equality follows from the optimality of p^{opt} (recall (6)) and the last inequality from the hazard rate conditions (1).
We can hence conclude that the candidate equilibrium bid maximizes the expected utility in (4).

The case $v > v^{med}$ can be shown similarly and it is hence omitted. Finally, bids outside the set of equilibrium bids are always dominated by either the lowest or highest bid in the range of equilibrium bids. ■

Proof of Theorem 7:
The proof uses arguments similar to those in the proof of theorem 6 and is hence omitted. ■

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