# Public Economics <br> Tax distorsions 

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## Distorsions

- AGZ 2.1. and Stiglitz. pg 488, and notes in Aula Global


## Distorsions

- Why do taxes create inefficiencies?
- Competitive Equilibrium -> Eficient allocation of resources (FFTWE).
- Consumers and produces -> same prices
- Tax-> consumer's price diiferent from producer's price


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- Tax function: $T(x, y, I)$
- The agent solves:

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\begin{array}{cc} 
& \operatorname{Max}_{x, y, I} \cup(x, y, 1-I) \\
\text { s.t. } & p_{x} I+p_{y} y=w l-T(x, y, l)
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- Comment: Where is the tax revenue?
- We obtain the same results if such revenue is included


## Basic model

- FFTWE -> Pareto efficient conditions (draw a figure to see it)

$$
\begin{align*}
R M S_{x y} & =\frac{p_{x}}{p_{y}}=R M T_{x y}  \tag{1}\\
R M S_{h x} & =\frac{w}{p_{x}}=R M T_{h x}  \tag{2}\\
R M S_{h y} & =\frac{w}{p_{y}}=R M T_{h y} \tag{3}
\end{align*}
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- This is an efficient tax.


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R M S_{x y} & =\frac{p_{x}\left(1+t_{x}\right)}{p_{y}} \neq \frac{p_{x}}{p_{y}}=R M T_{x y} \\
R M S_{h x} & =\frac{w}{p_{x}\left(1+t_{x}\right)} \neq \frac{w}{p_{x}}=R M T_{h x}
\end{aligned}
$$

## Excise tax

- It doesn't affect the marginal cost of production. It creates a distorsion on consumption
- The consumer will reduce $x$ and increase $y$ and $h$.


## Tax on consumption

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- Leisure is not taxed -> taxation creates a welfare inefficiency


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- Only labor income; no savings $->t_{c}$ is equivalent to $t_{w}=t_{c} /\left(1+t_{c}\right)$


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- Two production sectors $x$ and $y$.
- $L_{i}, K_{i}$ : labor and capital in sector $i, i=x, y$.
- Perfect competiton without taxation: factor price= marginal productivity
- The equilibrium condition with efficient allocation of capital and labor:

$$
R M T S_{K L}^{x}=\frac{r}{w}=R M T S_{K L}^{y}
$$

## Production inefficiencies

- Suppose a tax on wages on sector $x, t_{w x}$, (the tax is paid by the firms in sector $x$ )

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- -> ineffecient allocation
- What about a general tax on wages?
- No inefficienies in production (make sure you see it).
- But we still have distorsions in comsumption


## Notes in "aula global"

- Two consumption goods, $x$ and $y$.
- One production factor $I$.
- Two firms

$$
\begin{aligned}
& X^{s}=F_{k}\left(I_{x}\right) \\
& Y^{s}=F_{y}\left(I_{y}\right)
\end{aligned}
$$

- Fixed labor supplu $I^{*}$
- Consumer $U(x, y)$
- Frontier of possibilities of production: $y=T(x)$
- MRT $(x, y)$
- $\operatorname{MRS}(x, y)$


## Notes in "aula global"

- One can prove the following


## Theorem

If the allocation $\left\{\left(x^{D}, y^{D}\right),\left(x^{S}, I_{x}\right),\left(y^{S}, I_{y}\right)\right\}$ is effcient, then

$$
\begin{aligned}
I_{x}+I_{y} & =I^{*} \\
x^{D} & =x^{S} \\
y^{D} & =y^{S}
\end{aligned}
$$

$\operatorname{MRS}_{x, y}\left(x^{D}, y^{D}\right)=M R T_{x, y}\left(x^{S}, y^{S}\right)$

## Theorem

If $\left[\left(p_{x}, p_{y}, w\right),\left\{\left(x^{D}, y^{D}\right),\left(x^{S}, I_{x}\right),\left(y^{S}, I_{y}\right)\right\}\right]$ is a competitive equilibrium, then the allocation $\left\{\left(x^{D}, y^{D}\right),\left(x^{S}, I_{x}\right),\left(y^{S}, I_{y}\right)\right\}$ is effcient.


