January 23, 2019

SHEET 2. UNCONSTRAINED OPTIMIZATION

(1) Find and classify the critical points of the following functions.
  (a) $f(x,y) = x^2 - y^2 + xy$.
  (b) $f(x,y) = x^2 + y^2 + 2xy$.
  (c) $f(x,y) = e^{x \cos y}$.
  (d) $f(x,y) = e^{1+x^2-y^2}$.
  (e) $f(x,y) = x \sin y$.
  (f) $f(x,y) = xe^{-x}(y^2 - 4y)$

(2) Find the critical points of the following functions. For which points the second derivative criterion does not give any information?
  (a) $f(x,y) = x^3 + y^3$.
  (b) $f(x,y) = \left((x-1)^2 + (y+2)^2\right)^{1/2}$.
  (c) $f(x,y) = x^3 + y^3 - 3x^2 + 6y^2 + 3x + 12y + 7$.
  (d) $f(x,y) = x^{2/3} + y^{2/3}$

(3) Let $f(x,y) = (3-x)(3-y)(x+y-3)$.
  (a) Find and classify the critical points.
  (b) Does f have absolute extrema? (hint: consider the line $y = x$)

(4) Find the values of the constants a, b and c so that the function $f(x,y) = ax^2y + bxy + 2xy^2 + c$ has a local minimum at the point $(2/3, 1/3)$ and the minimum value at that point is $-1/9$.

(5) The income function is $R(x,y) = x(100-6x) + y(192-4y)$ where x and y are the number of articles sold. If the cost function is $C(x,y) = 2x^2 + 2y^2 + 4xy - 8x + 20$ determine the maximum profit.

(6) A milk store produces x units of whole milk and y units of skim milk. The price for whole milk is $p(x) = 100 - x$ and the price for skim milk is $q(y) = 100 - y$. The cost of production is $C(x,y) = x^2 + xy + y^2$. How should the company choose x and y to maximize profits?

(7) A monopolist produces a good which is bought by two types of consumers. The consumers of type 1 are willing to pay $50 - 5q_1$ euros in order to purchase q_1 units of the good. The consumers of type 2 are willing to pay $100 - 10q_2$ euros in order to purchase q_2 units of the good. The cost function of the monopolist is $c(q) = 90 + 20q$ euros. How much should the

monopolist produce in each market?

(8) In the method of least squares of regression theory, the line $y = a + bx$ is fit to the data

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

by minimizing the quadratic errors

$$E(a, b) = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$$

by choice of the two parameters a (the intercept) and b (the slope).

(a) Determine the necessary conditions for minimizing $E(a, b)$ by choice of a and b. Show that the sufficient conditions are met.

(b) Find a and b.

(c) Apply the results to the data

$$(0, 0), (1, 1), (4, 2), (16, 4)$$

and draw a graph.