UC3M
Optimization for Economics
17 May 2019

Niu: _____ Group: _____

Name: _____

Question:	1	2	3	4	Total
Points:	12	12	18	18	60
Score:					

1

Consider the function $f(x,y) = \sqrt{x+2y}$ defined on the set

$$A = \{(x,y) \,:\, x^2 + y^2 \geq 1, 0 \leq y \leq x, x \leq 2\}.$$

(a) (6 points) Draw the set A and discuss whether the function f and the set A satisfy the assumptions of the Weierstrass Theorem.

(b) (6 points) Draw the level curves of f on the set A, showing the directions in which f increases/decreases and determine (if they exist) the global extrema of f on A.

2

A firm sells two goods in amounts x and y, respectively.

(a) (6 points) Suppose that the profit function of the firm is

$$B(x,y) = -4x^2 - 24y^2 + 800x + 960y - 500.$$

Compute the critical points and study if they are global maximum.

(b) (6 points) Now suppose that the firm is unsure about its profit function. Precisely, the firm estimates that the profit function is

$$B(x,y) = -4x^2 - 24y^2 + 4axy + 800x + 960y - 500, \quad a \in \mathbb{R}.$$

Determine the interval(s) I of values of a, for which the profit function is strictly concave.

3

Consider the function $f(x,y) = 12x\sqrt{y}$ on the set $\{(x,y) \,:\, 3x + 4y = 12, y > 0\}$.

(a) (6 points) Obtain the Lagrange equations and find the critical points.

(b) (6 points) Classify the critical points found in part (a).

(c) (6 points) Suppose that the equality constraint changes to $3x + 4y = 13$. Without solving the problem, give an estimate of the new optimal value of $f(x,y)$.

4

Consider the function $f(x,y) = x^2 + 2y^2 + y - 1$ defined on the set

$$A = \{(x,y) \,:\, x^2 + y^2 \leq 1\}.$$

(a) (5 points) Find the Kuhn–Tucker necessary optimality conditions to the problem

$$\max f(x,y) \quad \text{subject to } (x,y) \in A.$$

(b) (13 points) Find all the solutions of the Kuhn–Tucker conditions established in part (a) and find the maximum of f on A.
