UC3M
Mathematical Optimization for Economics
Final Exam, 27 June 2019

Niu: ________________ Group: ______

Name: ____________________________________

Question:	1	2	3	4	Total
Points:	12	12	18	18	60
Score:					

Instructions:

- **DURATION OF THE EXAM: 90'**.
- Calculators are **NOT** allowed. **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 4 questions, for a total of 60 points.
- Justify all your answers.
- If you run out of room for an answer, you can use the last exam page.

1

Consider the function $f(x, y) = (x - 2y)^2$ defined on the set

$$A = \{(x, y) : x + y \geq 0, y \leq 3, x \leq 3\}.$$

(a) (6 points) Draw the set A and discuss whether the function f and the set A satisfy the assumptions of the Weierstrass Theorem.

(b) (6 points) Draw the level curves of f on the set A, showing the directions in which f increases/decreases and determine (if they exist) the global extrema of f on A.

2

A firm sells two goods A and B in amounts x and y, respectively. The revenue of the firm is

$$R(x, y) = 800x + 960y - 2x^2 - 12y^2 + 4axy,$$

where a is an unknown parameter. The cost of producing x units of good A and y units of good B is

$$C(x, y) = 2x^2 + 12y^2.$$

(a) (6 points) Find the profit function $B(x, y)$ of the firm and find all values of a for which $B(x, y)$ is strictly concave.

(b) (6 points) The firm knows that $a = 2$. Find the critical points of the profit function $B(x, y)$ and calculate the output levels that maximize profits, if they exist.

3

Let $M = \{(x, y, z) : x + y + z = 1\}$ and $f(x, y, z) = 3x^2 + 3y^2 + z^2 + 2yz$.

(a) (6 points) Obtain the Lagrange equations and find the critical points of the problem of minimizing f on M.

(b) (6 points) Show that there is a unique constrained global minimum and find the minimum value of f on M.

(c) (6 points) Let $N = \{(x, y, z) : x+y+z = 0.5\}$. Without solving the new problem, give an estimate of the minimum value of f on N.

4

Consider the function $f(x, y) = 10xy$ defined on the set

$$A = \{(x, y) : x^2 + y^2 \leq 1\}.$$

(a) (6 points) Find the Kuhn–Tucker necessary optimality conditions to the problem

$$\max f(x, y) \quad \text{subject to } (x, y) \in A.$$

(b) (6 points) Find all the solutions of the Kuhn–Tucker conditions established in part (a).

(c) (6 points) Find the maximum of f on A. Is $(0, 0)$ the minimizer of f on A?
