UC3M
Mathematical Optimization for Economics
Final Exam, 3 June 2024

Niu: ______________ Group: _____

Name: ____

Question:	1	2	3	4	Total
Points:	30	25	35	30	120
Score:					

$\boxed{1}$

Let the set
$$A = \left\{(x, y) \in \mathbb{R}^2 : \quad g(x) \leq y \leq h(x), \quad 2 \leq x \leq 4\right\}$$
with
$$g(x) = \ln(x - 1), \quad h(x) = \frac{1}{x} + 2$$
and let the function
$$f(x, y) = \ln\left(y + \frac{x}{2} - 1\right).$$

(a) (15 points) Draw the set A, justifying your drawing. Consider defined the order of Pareto on the set A; find maximals, minimals, maximum and minimum of A, justifying your answers. If you figure out that some of these elements do not exist, justify why.

(b) (5 points) Study if the function f and the set A fulfil the hypotheses of the Theorem of Weierstrass, and explain what does it mean.

(c) (10 points) Draw several level curves and several directions of fastest increase for the function f. Identify, the global maximum and the global minimum of f on A on your drawing and calculate these points if they exist. Justify your answers.

2

Consider the function $f(x, y) = 3xy - x^2y - xy^2$.

(a) (10 points) Find the critical points of f.

(b) (10 points) Classify the critical points found above (local maximum, local minimum or saddle point).

(c) (5 points) Determine whether the local extrema of f found above are global extrema.

$\boxed{3}$

Consider the following problem:

$$\max \qquad f(x, y) = x^3 + y^3$$

$$\text{subject to:} \quad g(x, y) = x + 4y = 63$$

(a) (10 points) Construct the Lagrangian of the problem and find its critical points.

(b) (10 points) Classify the critical points found above (local maximum, local minimum, or saddle point).

(c) (5 points) Determine whether the local extrema found above are global extrema of f subject to the constraint.

(d) (10 points) Now suppose that the constraint becomes $g(x, y) = 63$ and we add the nonnegative conditions $x \geq 0$, $y \geq 0$. Is it possible that any of the local extrema of f found above are global extrema of f subject to the new constraints?

$\boxed{4}$

Consider the program

$$\begin{array}{ll} \text{minimize} & x^2 + 2y^2 + 2x - 4y \\ \text{s.t.} & y \leq x \\ & x \geq -y \end{array}$$

(a) (20 points) Obtain the solutions of the Kuhn-Tucker equations for the program.

(b) (10 points) Justify that the program has a global solution.
