UC3M
Mathematical Optimization for Economics
Final Exam July 2, 2024

Niu: ________________ Group: ______

Nombre: ________________________________

Ejercicio	1	2	3	4	Total
Puntos	40	20	35	25	120
Nota					

Instructions:

- **DURATION OF THE EXAM: 100'**.
- Calculators are **NOT** allowed.
- **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 4 questions, for a total of 120 points.
- Justify all your answers.

$\boxed{1}$

Consider the plane set
$$A = \left\{ (x, y) \in \mathbb{R}^2 : x > 1,\ y \geq \frac{3}{4}(x-1),\ y \leq \frac{x}{x-1} \right\}.$$

(a) (10 puntos) Represent A graphically.

(b) (10 puntos) Considering the order of Pareto defined on A, find the maximals, minimals, maximum and minimum of A, justifying your answers. If you figure out that some of those elements do not exist, justify your answer.

(c) (10 puntos) Study graphically the existence of global extrema of $f(x, y) = y - 2x$ on A, and find them when they exist. Use the concepts of level curve and fastest increase/decrease directions, drawing some of these elements on the graph of A.

(d) (10 puntos) Study graphically the existence of global extrema of $f(x, y) = y - 0.5x$ on A, and find them when they exist. Use the concepts of level curve and fastest increase/decrease directions, drawing some of these elements on the graph of A.

$\boxed{2}$

Consider the function $f(x, y) = x^3 + y^2 - 2xy - x + 6$.

(a) (10 puntos) Determine the critical points of f.

(b) (10 puntos) Classify the critical points obtained above. Clarify whether they are global extrema.

$\boxed{3}$

Consider the following problem of Lagrange:

$$\begin{aligned} &\text{optimize} && f(x,y,z) = 2x - y \\ &\text{subject to:} && g(x,y,z) = (x-y)^2 + 3y^2 + z^2 = 39 \end{aligned}$$

(a) (5 puntos) Show without solving that the problem admits global solutions.

(b) (10 puntos) Construct the Lagrangian and find its critical points.

(c) (10 puntos) Determine the global maximum and minimum of f subject to the constraint and the optimal value of f in each case.

(d) (10 puntos) Assuming that the constraint becomes $g(x,y,z) = (x-y)^2 + 3y^2 + z^2 = 39 + h$, where h is a negligible quantity added to the independent term, give approximations to the optimal (maximum and minimum) values of f subject to the new constraint.

$\boxed{4}$

Consider the Kuhn–Tucker problem

$$\begin{array}{ll} \text{maximize} & x+y \\ \text{subject to:} & x^2+y^2 \leq 4, \\ & y \leq 1. \end{array}$$

(a) (10 puntos) Discuss whether the optimization problem is a convex one, and if the necessary Kuhn–Tucker conditions are also sufficient.

(b) (15 puntos) Solve the Kuhn–Tucker necessary conditions and solve the optimization problem.