UC3M Niu: __________ Group: ____

Mathematical Optimization for Economics

Final Exam, 19 May 2023 **Name:** ____________________

Question:	1	2	3	4	Total
Points:	30	30	30	30	120
Score:					

1

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function $f(x,y) = xy$. Consider the order of Pareto defined on the set

$$A = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2, \quad x \geq 0\right\}.$$

(a) (10 points) Draw the set A. Justify whether the function f and the set A satisfy the hypotheses of the Theorem of Weiersstrass.

(b) (10 points) Calculate the maximal and minimal elements, and the maximum and the minimum of A, justifying your answers. If some of these elements do not exist, justify why.

(c) (10 points) Draw several level curves and several directions of fastest increase for the function f. Identify the global maximum and the global minimum of f on A on your drawing and calculate these points. Justify your answers.

2

Consider the function $f(x,y) = (1 - xy)^2$.

(a) (10 points) Find the critical points of f.

(b) (10 points) Classify the critical points found above.

(c) (10 points) Say whether the function f has global maximum and/or global minimum and find them when they exist.

3

The production function for a firm is $5x + xy + 3y$, where x is labor and y is capital. Each unit of labor costs 15 monetary units (m.u.), whereas each unit of capital costs 3 m.u. The total budget that the company spends is 3000 m.u.

(a) (15 points) By solving the Lagrange equations associated to the optimization problem, find the optimal level of production for the firm. (Note that the answer which is not based on the Lagrange equations will have 0 points).

(b) (15 points) How does respond the maximum production to changes in the total budget? How does it impact a budget increase of 1 m.u.? If we want our production to increase 1%, what increase of budget should we made approximately.

4

Consider the Kuhn–Tucker problem

$$\max \ -x^2 + 2y^2 - 4y + 10$$

$$\text{subject to: } x^2 + (y-2)^2 \leq 1.$$

(a) (10 points) Justify if the function f and the set A satisfy the conditions of the Theorem of Weierstrass. Is the problem a convex problem?

(b) (10 points) Find all the points that satisfy the necessary Kuhn–Tucker conditions (do not forget the value of the multiplier).

(c) (10 points) Find the solution(s) of the Kuhn–Tucker problem.
