UC3M
Mathematical Optimization for Economics
Final Exam, 20 June, 2023 Name: ______________________ Niu: ______________ Group: ____

Question:	1	2	3	4	Total
Points:	30	20	25	25	100
Score:					

1

Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \frac{1}{2 - (x + y)}.$$

Consider the order of Pareto defined on the set

$$A = \left\{(x, y) \in \mathbb{R}^2 : -1 \le x + y \le 1, \quad -(1 - x)^2 \le y \le (1 + x)^2\right\}.$$

(a) (10 points) Draw the set A, reasoning your answer. Calculate, if they exist, the maximal and minimal elements, and the maximum and the minimum of A, justifying your answers. If some of these elements do not exist, give reasons.

(b) (10 points) Justify whether the function f and the set A satisfy the hypotheses of the Theorem of Weiersstrass.

(c) (10 points) Draw several level curves of f, superimposing them on the set A. Identify the points of A, if they exist, where f attains global maximum and/or minimum, and calculate the value of f on that points.

2

Let the function $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x, y) = y(2x^2 + y^2 + 2xy + 2y + 2).$$

(a) (10 points) Find the critical points of f.

(b) (10 points) Classify the critical points found in part (a) above.

3

Let the Lagrange optimization problem given by

$$\begin{aligned} &\text{optimize} \quad f(x, y) := 15x + 3y \\ &\text{subject to:} \quad g(x, y) := 5x + xy + 3y = 30. \end{aligned}$$

(a) (5 points) Check that the regularity condition holds. Write the Lagrangian and the Lagrange equations.

(b) (10 points) Solve the Lagrange equations, finding all critical points (x^*, y^*, λ^*) of the Lagrangian.

(c) (10 points) Classify the critical points (x^*, y^*, λ^*) obtained in part (b) above (local maximum, local minimum, or saddle points).

4

Let us consider the Kuhn–Tucker optimization problem given by

$$\max \ f(x,y) := x^3 + y^2$$

$$\text{subject to: } g(x,y) := x^2 + y^2 \leq 1.$$

(a) (5 points) Draw the feasible set S. Justify whether S and the function f satisfy the conditions in Weierstrass Theorem. Check that all points of the feasible set are regular points.

(b) (10 points) Write the necessary Kuhn-Tucker conditions for this problem (equalities and inequalities). Find all points which satisfy the Kuhn–Tucker conditions, multiplier included in each case.

(c) (10 points) Find the solution(s) to the Kuhn-Tucker problem.
