UC3M
Mathematical Optimization for Economics
Final Exam, 20 May 2022

Niu: ________________ Group: ______

Name: ____

Question:	1	2	3	4	Total
Points:	15	15	15	15	60
Score:					

Instructions:

- **DURATION OF THE EXAM: 90'**.
- Calculators are **NOT** allowed.
- **Turn off** your smart phone.
- **DO NOT UNSTAPLE** the exam.
- Please show a valid ID to the professor if required.
- Read the exam carefully. The exam has 4 questions, for a total of 60 points.
- Justify all your answers.

1

Let $F : \mathbb{R}^2 \to \mathbb{R}$ be the function $F(x, y) = ax + y$, where $a \neq 0$. Consider the order of Pareto defined on the set

$$A = \left\{(x, y) \in \mathbb{R}^2 : y \geq x^2 - 2x;\ y \leq x + 4;\ x \geq 0\right\}.$$

(a) (5 points) Draw the set A. Calculate, if they exist, the maximal and minimal elements, the maximum and the minimum of A. Justify your answers.

(b) (5 points) Suppose $a = 1$. Draw the curves of level $c = 0, 1, 3$ of F. Represent the increasing direction of the function. Calculate, if they exist the global maximum and global minimum of F on A.

(c) (5 points) Find the range of values of a such that the global maximum of F on A is attained at the point $(0, 4)$.

2

A monopolistic firm produces two goods A and B, of which it sells x and y units per day, respectively. The cost function is given by

$$C(x, y) = x^2 + 4y^2 + 2xy - 20x + 30.$$

The unitary prices of the goods A and B are

$$p_A(x, y) = 60 - x - ay,$$
$$p_B(x, y) = 80 - 4y - ax,$$

respectively, where a is an unknown parameter.

(a) (5 points) Find the range of values of a for which the profit function of the firm is a concave function.

(b) (5 points) Let $a = 1$. Find the values of x and y which maximize the firm's profits.

(c) (5 points) Let $a = 1$. A new regulation requires to sell the products in packages formed by 1 unit of good B and 2 units of good A. Find the values of x e y which maximize the firm's profits.

3

Consider the problem of Lagrange:

$$\text{Optimize } f(x, y) = xy - 3x - 6y$$
$$\text{subject to: } g(x, y) = 2x + 4y = 40.$$

(a) (5 points) Find all critical points of the problem.

(b) (5 points) Find all local extrema of $f(x, y)$ subject to the constraint. Justify whether the local extrema are global extrema.

(c) (5 points) Suppose that $f(x, y)$ is the profit function of a firm and that $2x + 4y = 40$ is the budget constraint, both in thousands of euros.

Approximately, what would be the added benefit of increasing the company's funds by 1,000 euros?

4

Consider the Kuhn–Tucker problem

$$\begin{aligned} \max \quad & x + 2y \\ \text{s.t.} \quad & x^4 + 2y^4 \leq 3. \end{aligned}$$

(a) (10 points) Find all points that satisfy the Kuhn–Tucker conditions.

(b) (5 points) Justify that the problem admits global solutions and find them.
