1. The next figure shows the tree of a perfect information game $G$ between two players.

![Game Tree Diagram]

a) Identify the information sets of each player (use a Greek letter).

b) Which are the pure strategies of each player? Which are the actions in each information set?

c) What is the outcome after playing the strategy combination $(rll,LM)$, where $rll$ is the strategy of the first player and $LM$ the strategy of the second player?

d) Identify all possible combinations of strategies (one for each player) that result in the path $rRl$.

2. Consider the following extensive form game.

a) Indicate which are the feasible strategies for each player and find the subgame perfect Nash Equilibria.

b) Write the equivalent normal form of this game and find its Nash Equilibria.
3. Imagine that the market for vacuum cleaners was dominated by a firm called Rapilimpia and that a new firm, Neolimpia, was considering entry into this market. If Neolimpia enters, Rapilimpia has two choices: either to accommodate the entry of Neolimpia, accepting a decrease in its market share, or to fight entry starting a war price. Suppose that if Rapilimpia decides to accommodate the entry of Neolimpia, the latter would have a profit of 10 millions euros; but if Rapilimpia chooses a war price, Neolimpia would lose 20 millions euros. Obviously, if Neolimpia does not enter to the market, its profits are zero. Additionally, suppose that as a monopoly, Rapilimpia can obtain profits of 30 millions of euros, that to share the market with its competitor will reduce its profits to 10 million and that a war price will cost to the firm 10 millions.

a) Draw the extensive form game.

b) Now use the extensive form game to obtain the strategic form game and obtain all the Nash equilibria in pure strategy. Which of these Nash equilibria are subgame perfect?

4. Merche and Antonio have to decide where to go on vacation. They have three options: Alicante (A), Barcelona (B) or Córdoba (C), but they do not reach an agreement where to go. In order to take a decision they use the following mechanism. First, Merche vetoes one of the three places. Then, Antonio, after observing Merche’s veto, vetoes another place. They go to the place that has not been vetoed. Merche prefers A to B and B to C; Antonio prefers C to B and B to A. Assuming that each player assigns an utility of 3 to the favored place, an utility of 2 to the second best alternative and an utility of 1 to remaining city, and that both players want to go together on vacation, answer to the following questions:

a) Represent the game in extensive and normal form.

b) Find the Nash equilibrium/a in pure strategies.

c) Which of the Nash equilibria previously found are subgame perfect Nash equilibria? Explain your answer. Where do Merche and Antonio go on vacation?
5. Two Spanish firms share the dairy market of Getafe. One of them, called OBESA, only sells fat products. The other firm, called LISA, only sells non-fat products. It is well known that in Getafe people are not too worried about being thin and that if LISA does not launch an aggressive advertisement campaign about the risks of being overweight, LISA and OBESA profits would be 1 and 6 millions euros respectively. On the contrary, if LISA launch its campaign, OBESA has the choice of fighting back with a publication of a dossier warning consumers about the lack of vitamins in non-fat products of her rival. In this case, LISA can even do something else, by launching a public message about the lack of healthy and cleaning measures in the production facilities of OBESA. The marketing department of both firms forecast that if LISA launch its campaign against overweight and OBESA does not react with the dossier, profits would be of 4 millions of euros for LISA and of 3 for OBESA. On the contrary, if OBESA reacts, after LISA launches its campaign, by publishing the dossier and LISA does not react to this action, profits would be of 2 millions euros for LISA and 4 for OBESA. However, if LISA reacts to the publication of the dossier with the public message about the lack of healthy and cleaning measures in the production facilities of OBESA, profits would be of 3 million euros for LISA and of only 1 for OBESA.

a) Represent the game in normal and extensive form. Obtain all the Nash equilibria (in pure and mixed strategies).

b) Which of the Nash equilibrium are subgame perfect?

6. The president of Real Madrid think that he can do without several of the club’s players. He has asked their coach, Fabio Capello, and sporting director, Pedja Mijatovic, to try to sell two players before the end of February. Capello and Mijatovic have three options: Ronaldo (R), Beckham (B), and Guti (G), but they cannot agree on which one of the three should remain in the club. To reach an agreement, they will resort to the following mechanism. Firstly, Capello (Player I) will decide if he wants to get rid of Ronaldo or Guti. Next, Mijatovic (Player II), after observing Capello’s decision, will choose who to eliminate from the set of remaining players. Below are the utility levels that each person obtains depending on the player that remains in the club:

<table>
<thead>
<tr>
<th></th>
<th>Capello’s Utility</th>
<th>Mijatovic’s Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ronaldo</td>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>Beckham</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Guti</td>
<td>b</td>
<td>1</td>
</tr>
</tbody>
</table>

where \(a\) and \(b\) are positive integers. Considering only pure strategies, answer the following questions:

a) Represent the game in both its extensive and normal forms.

b) Does there exist any strictly dominated strategy for some player?

c) Find the Nash equilibria of the normal form if \(a = b = 3\). What football player will be kept? Which of these equilibria is subgame perfect?
d) Find for which other combinations $a = b$ this game has the same Nash equilibria as those found in part c).

7. Consider the following extensive form game which we will call *Game A*:

![Game A Diagram]

a) Write the equivalent normal form of *Game A* and find all the Nash equilibria in pure and mixed strategies.

b) Consider *Game B* which is the same as *Game A* but now Player B observes Player A’s choice before making his own decision. Find the pure strategy Nash equilibria of *Game B*. Is there a subgame perfect Nash equilibrium in pure strategies?

c) Consider *Game C* which is as follows. Player A chooses between two actions $\alpha$ and $\beta$. The choice of $\alpha$ implies payoffs of 5 and of 25 for him and Player B, respectively. Choosing $\beta$ leads to *Game A*. Draw the extensive form game of *Game C*. How many subgames does this game have? How many information sets has each player? Calculate the subgame perfect equilibrium/a in pure and mixed strategies.

8. A torturer proposes both his prisoners a macabre game. Prisoner 1 can choose whether the game remains at stage A or moves on to stage B. If the game remains at stage A, both prisoners would be given a soft torture (which provides both a utility level of 2). If they move on to stage B, both prisoners have to choose simultaneously and independently a number (integer) between 1 and 100. If the sum of these numbers is even, Prisoner 1 will receive a strong torture (which provides him with a utility level of 1) and Prisoner 2 will not receive any torture (in which case he receives a utility level of 3). If the sum is odd, Prisoner 2 receives a strong torture (utility 0 in this case) and Prisoner 1 does not receive any (and receives a utility of 5).
a) Find all the Nash equilibria in pure and mixed strategies of this game. (Note that the set of strategies can be simplified into 2 strategies, choose an even number or an odd one., given that the sum of two even integers or two odd integers is an even one, and the sum of one even integer and one odd one is an odd number).

b) Show that for Prisoner 1, the strategy which consists in remaining at stage A is strictly dominated by the mixed strategy which consists in moving on to stage B, and then play a Nash equilibrium in mixed strategies at stage B.

c) Calculate the subgame perfect Nash equilibria.

9. Carlos and Natalia face the following situation. Natalia has to choose between two actions: S to stop playing with Carlos, or C to continue playing with him. In case she chooses S, she gets a payoff equal to y. In case she chooses C, they will have to play a simultaneous game where Natalia chooses between U and D while Carlos chooses between L and R. The payoff matrix for the simultaneous game is:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>3, 1</td>
<td>2, -1</td>
</tr>
<tr>
<td>D</td>
<td>1, 0</td>
<td>4, 5</td>
</tr>
</tbody>
</table>

a) Find all the Nash equilibria for the simultaneous game that starts after Natalia chooses C.

b) Find Natalia’s payoffs in each of the Nash equilibria from part a).

c) Find all possible values for y such that Natalia’s first action is always C in all and each of the subgame perfect Nash equilibria. List all the subgame perfect Nash equilibria for such values of y.

10. Two investors have put 10,000 € each in a bank. The bank has invested that money in a long run project. After the investment matures, it is going to generate a gross return of 25,000 €. However, in case the bank has to liquidate the investment before it, the returns will be only 15,000 €. There are two possible dates at which the investors can withdraw their money from the bank: the date 1 is before the investment maturation, and the date 2 is after that. In each of these dates, each investor decides whether he withdraws his money or not, without knowing the other investor’s decision. If both withdraw the money at the date 1 (before the investment maturation), each one gets 7,500 € and the game ends. If one withdraws at date 1 and the other doesn’t, the first one gets 10,000 € while the other gets only 5,000 € and the game ends. Finally, if no one withdraws at the date 1, the investment matures, and each investor has to decide whether or not he withdraws the money at the date 2. If both withdraw at date 2, each one gets 12,500 € and the game ends. If one withdraws and the other doesn’t, the first one gets 15,000 € while the other gets only 10,000 €, and the game ends. If no one withdraws, the bank gives 12,500 € back for each investor and the game ends.

a) Represent the game in the extensive form and indicate which the information sets are for each player and the subgames of such game. Indicate how many and which are the possible strategies for each player.

b) Find the subgame perfect Nash equilibria in pure strategies for this game.

11. Consider the following game in the extensive form:
a) How many information sets has Player 1? What about Player 2? Represent the game in the normal form. How many rationalizable strategy profiles are there? Find the pure strategy Nash equilibria for the game. Which of them is subgame perfect?

b) Consider now that in the second time Player 1 has to choose an action, he doesn’t know the action taken by Player 2. How many information sets has Player 1? What about Player 2? Represent the game in the extensive form.

c) Find all the Nash equilibria in the subgames different than the whole game. Find all the subgame perfect equilibrium for the whole game.

12. Go back to Problem 18 from the list of static games, but suppose now that the game is sequential and that neighbor 1 moves first (that is, neighbor 1 chooses first $c_1$ and after this knowing $c_1$ neighbor 2 chooses $c_2$).

a) Draw a schema of the extensive form of this game, indicate the subgames and the information sets of each neighbor, and calculate the subgame perfect Nash equilibria.

b) Determine the time each neighbor dedicates to cleaning the street if both follow the strategy prescribed by the subgame perfect Nash equilibrium.

c) Calculate the utility of each neighbor in each equilibrium both in part a) in Problem 18 and in part b) of the present problem. Who improves from switching from a static game in Problem 18 to the present dynamic game and why?

13. Suppose that Extra and Ultra are the only car-producers that compete in the Spanish car market. The demand for cars in Spain is given by $p(Q) = 10 - Q$, where $Q$ is the total quantity produced by the two car-producers, $Q = q_E + q_U$. The total costs faced by Extra and Ultra are given respectively by $C_E(q_E) = 3q_E$ and $C_U(q_U) = 2q_U$.

a) Assume that Extra and Ultra choose simultaneously the quantities that they will produce, $q_E$ and $q_U$ respectively. Determine the reaction function of each car-producer and the Nash Equilibrium of this game. Compare the equilibrium profits of the two car-producers.

b) Assume now that the game changes. In the new game, Extra chooses its quantity $q_E$ first. Ultra chooses its own quantity $q_U$ after observing the decision of Extra. Represent this game in extensive form. Find the Subgame Perfect Nash Equilibrium (SPNE) of this game.
c) What is the amount of money that Ultra would have to pay to Extra in order to make Extra agree to choose its quantity simultaneously with Ultra? Is Ultra willing to pay this amount of money?

14. In the market for video game consoles, two firms offer the Box-X and Station-Y products respectively. The two products are similar, but consumers can tell some differences and demand them according to the following functions:

\[ q_X = 168 - 2p_X + p_Y \]
\[ q_Y = 168 - 2p_Y + p_X \]

Assume zero production costs.

a) Compute and draw the best reply functions for both firms if they simultaneously choose prices.
b) Find the Nash equilibrium in this static game. Find the quantities sold by each firm and their profits.
c) Find the subgame perfect equilibrium if Box-X chooses first its price and Station-Y chooses after observing its rival’s price. Who wins and who losses in the change from the static game to this dynamic game.

15. The firms Andesa and Bertola (A and B) are going to be the only electricity generators in Fierro Island from June 2020 on. They must decide simultaneously which of the two available technologies in the market they will use. The nuclear one has zero marginal costs but a fixed cost of 2500; the other technology (combined cycle) fixed cost is 1000 and would allow them to produce electricity at a cost of 30 per unit. The decisions taken by each of the firms will be announced the first of May. The market demand function for power \( P(Q) = 180 - Q \), where \( Q = q_A + q_B \).

a) Find all subgame perfect Nash equilibria in pure strategies of this game.
b) Suppose that both firms have adopted the combined cycle. The 1st of June of 2010 the aim of Firm A will continue to be maximizing profits, but Firm B will no longer be interested in profits but in the difference between its production and the production of Firm A. More concretely, the new utility function of B will be given by

\[ U_B(q_A, q_B) = (q_B - q_A)^2. \]

Find the SPNE of the new game given that firm B is now the market leader, so that Firm A observes how much Firm B chooses to produce before having to decide its own production. How much does each firm produce in SPNE?

16. Consider again problem 15. a) to discuss what a government could do if it were interested in promoting the technological diversity in the energy sector.
17. In the following bargaining game, a firm \((F)\) and a syndicate \((S)\) have to share the benefits generated by their economic activity. Assume that the benefits are equal to 2 million euros. The game has three stages. Offers are alternating between F, S and F. In each stage, the player who has not offered how to share the benefits has the choice of accepting or rejecting the proposal made by the other player. If she accepts the proposal, the game ends and if she rejects, in the next stage, she will become the proposer. If the players do not reach any agreement, after the third proposal, both of them get a zero payoff.

a) What will be agreement reached in equilibrium and in which time period will the agreement be reach if the discount factor of both players is \(\delta = 1/4\)?

b) What will be agreement reached in equilibrium and in which time period will the agreement be reach if \(F\) has a discount factor \(\delta_F = 1/4\) and \(S\) has \(\delta_S = 1/2\)?

c) Compare the two agreements and try to provide an intuitive argument to support the results you have found.

18. Consider a bargaining (negotiation) game of 2 periods. In the first one, Player \(A\) offers Player \(B\) to share 1 million Euros \((x, 1 - x)\), where \(x\) is the quantity that \(A\) would receive. Player \(B\) can then choose to accept or reject \(A\)’s proposition. If he accepts, the game is over. If he rejects, they move on to period 2 where both have to make simultaneously an offer of share. If \(A\) proposes \((x, 1 - x)\) and \(B\) proposes \((1 - y, y)\), payoffs are \((x, y)\) if \(x + y \leq 1\) and \((0, 0)\) otherwise. Payoffs are discounted with the discount factor \(\delta = 1/4\).

a) Solve the subgame that starts when \(B\) rejects the offer. Find best reply functions of \(A\) and \(B\), and find the Nash equilibria of the subgame. Find the expected payoff of the equilibrium in this subgame.

b) Find all the subgame perfect Nash equilibria.

Note: If \(B\) is indifferent between accepting and rejecting, we assume that he always accepts.

19. The firms Ford Motor and General Motors (GM) are bargaining over the selling price of Ford’s luxury cars division, which GM is willing to buy. For Ford, the division has a value of 2 billions of euros, while it worth 4 billion euros for GM. In the first meeting, the two firms agreed on the following bargaining procedure: in the first negotiation round one of them offers a price, and then the other decides either to accept, in which case the negotiation is over, or to reject, and they move to the next negotiation round. In the second round, the firm that reject the initial offer makes a new offer that must be accepted or rejected by the other firm. But now, in case that the offer is rejected, the negotiation ends, and both gets zero (while Ford keeps the division). We assume that in any case where a firm is indifferent between accepting and rejecting an offer, it accepts. Finally, the two firms give the same value for future or present payments.

a) For the case where Ford is the first to offer a price, draw the extensive form of the game, clearly showing the information sets, the strategies and the payoffs. Find the subgame perfect Nash equilibrium.

b) If GM starts offering the price in the first round, what price it will offer in a subgame perfect Nash equilibrium?
Extra Exercises

20. Three neighbors (Ana, Bea, Cruz) have to choose one among three projects ($a$, $b$, $c$). Preferences are represented in the following table. Each column represents the order of preferences of the corresponding neighbor, the preferred project being located above in each column.

<table>
<thead>
<tr>
<th>Ana</th>
<th>Bea</th>
<th>Cruz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
</tr>
<tr>
<td>$b$</td>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>$c$</td>
<td>$c$</td>
<td>$b$</td>
</tr>
</tbody>
</table>

The choice is realized using a simple majority rule in a two step vote. In the first step, the neighbors choose between $a$ and $b$, and the winner of this step competes against $c$. From this second step is selected the project that will be implemented.

a) What would be the result if, in each step, preferences are truly revealed? (i.e., they vote for the project they prefer).

We analyze now this election mechanism as a game (the neighbors can vote strategically)

b) Assume that $a$ has been chosen in the first step. Explain why the fact that all neighbors vote for $c$ in the second stage is a Nash equilibrium.

c) Why isn’t this equilibrium very plausible? Which refinement (or selection criterion) would eliminate this equilibrium?

d) Which subgame perfect Nash equilibrium would satisfy this refinement in both steps?

21. Two partners $A$ and $B$ are trying to finish a project. Each of them will receive 25 million euros when the project is completed, but nothing before its completion. The project needs 7 millions in order to be complete. None of the partners can commit in a credible way to pay such a sum in the future, so they decide the following: In a first stage, Partner $A$ chooses to contribute with $c_A$. If this quantity is enough to complete the project, the game is over and each partner receives the 25 millions. In case it is not enough ($c_A$ is less than 7 millions), Partner $B$ chooses its contribution $c_B$. If the sum of both contributions allows completing the project, each of them receives the 25 millions, if not, they do not receive anything. The only way to get the money and contribute to the project is to remove it from the other activities of both partners. We assume that from these activities, each partner can obtain $c_i^2$, ($i = A,B$).

a) Find the subgame perfect Nash equilibrium.

b) Assume now that the sum that is missing in order to complete the project is 12 millions. Find the new subgame perfect Nash equilibrium.

22. Two firms, Peurot and Fol, compete in the market for cars, where demand is $P(Q) = 2 - Q$ and the state of technology is such that marginal cost is $c = 1$. In this market, Peurot is the leader (chooses first) and Fol is the follower (chooses its quantity knowing the leader’s chosen
quantity). Fol cares not only about its own profit but also the volume of sales, since it must capture some market share. Specifically, the utility function for Fol is given by

\[ U_F(q_F, q_{G}) = \alpha \Pi_F(q_F, q_{G}) + (1 - \alpha) q_F, \]

whereas the leading firm only cares about own profit, i.e.,

\[ U_F(q_F, q_{G}) = \Pi_F(q_F, q_{G}). \]

a) If \( \alpha > 1/2 \), calculate the subgame perfect Nash equilibrium of this game. Determine for what values of \( \alpha \) the leader produces more than the follower in the subgame perfect Nash equilibrium.

b) Now suppose that \( \alpha = 0 \). Calculate the subgame perfect Nash equilibrium.

23. A gazelle is threatened by \( N \) lions in a line. Each lion prefers eating the gazelle or eating the lion who ate the gazelle (if it is in second in line) or eating a lion who ate the lion … who ate the gazelle rather than not eating. To eat a lion that did not eat, directly or through a chain of lions, the gazelle is the same as not eating at all. Finally, each lion prefers not eating rather than to being eaten. The first lion has to decide whether to eat or not the gazelle, the second one has to decide whether to eat or not the first lion, and the \( n \)-th lion whether to eat or not the \( (n-1) \)-th lion. The gazelle decides nothing. What are the subgame perfect equilibrium, for each possible value \( N \)? (hint: start with \( N = 1 \), then \( N = 2 \), and then successively, until you figure out the general solution.)

24. Two firms produce differentiated goods, for example, Dell and Acer. Each of them chooses its price in order to maximize its own profits. Let \( p_1 \) be the price of Firm 1 and \( p_2 \) be the price of Firm 2. Given prices \( p_1 \) and \( p_2 \), Firm 1 will be able to sell \( q_1 = 100 - p_1 + 0.5p_2 \) and Firm 2 will be able to sell \( q_2 = 100 - p_2 + 0.5p_1 \). Both firms have marginal costs of 50.

a) Suppose the two firms move simultaneously. Find their best response functions. Find out the Nash equilibrium of this game. Also find each firm’s profits at the equilibrium prices.

b) Suppose now Firm 1 moves first, and Firm 2 observes Firm 1’s choice of \( p_1 \) before choosing \( p_2 \). Find the prices, quantities and profits in the subgame perfect Nash equilibrium.

c) Is there an advantage of moving first in this game?

25. Ester and Fernando play a game where each of them has to choose a number from the interval \([0, 1]\). First Ester writes a number \( x, x \in [0, 1] \). Then, after observing \( x \), Fernando chooses a number \( y, y \in [0, 1] \). Ester’s and Fernando’s utility functions are \( U_E(x, y) = \min (x, y) \) and \( U_F(x, y) = (2x - y)^2 \) respectively.

a) Represent this game in the extensive form, indicating if it is a perfect or imperfect information game, how many information sets each player has, and how many subgames the game has.

b) What are Fernando’s best responses for each of the following Ester’s choices: \( x = 0, x = 1/4, x = 1/2 \) and \( x = 1 \)?
c) Find the subgame perfect Nash equilibrium for the game. Suppose that, in case of being indifferent between two numbers, Fernando always chooses the greater one.

d) Find the utilities obtained by Ester and Fernando in the subgame perfect Nash equilibrium.

26. Consider the following game, where Jorge chooses between the actions $A$ and $B$, while Alicia chooses between $C$ and $D$:

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>$B$</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

a) Find all the Nash equilibria in pure and mixed strategies for this game. Indicate the utility each of the players gets in each of the equilibria.

b) Suppose now that Jorge and Alicia choose their actions sequentially, with Jorge choosing first, and Alicia being able to observe Jorge’s action before choosing her own action. Find ALL the subgame perfect Nash equilibria for this new game. How much gets each of the players in these equilibria?

27. Consider a firm ($F$) that selects the number of workers $L \geq 0$ and a union ($U$) that fixes wages, $w \geq 0$. Firm’s profits are given by $\Pi_F(w, L) = 100L - 0.1L^2 - wL$ whereas union’s payoff is given by total wage, $\Pi_U(w, L) = wL$. Suppose that the union chooses first the wage $w$, and the firm observes $w$ and then chooses labor input $L$.

a) Draw the extensive form of the game and find the subgame perfect equilibrium.

b) Suppose that the union is worried about reaching an employment level of $X$ at least, so that its payoff function is now $\Pi_U(w, L) = (L - E)w$.

Draw the extensive form of the game and find the subgame perfect equilibrium, as a function of $E$.

28. Consider the following effort-negotiation game between two partners in a joint project $X$. In a first stage partners 1 and 2 must choose simultaneously the effort level, $e_i \in [0, \infty)$ for $i = 1, 2$, to exert in the joint project $X$. Gains from project $X$ are:

$$\Pi(e_1, e_2) = e_1 + e_2 + e_1e_2.$$ 

The cost of exerting effort is given by

$$C(e_i) = \frac{1}{2}e_i^2, \text{ for } i = 1, 2.$$ 

In a second stage, once $e_1$ and $e_2$ have been chosen, the two partners agree to share those gains as follows. They flip a coin and if the result is heads Partner 1 proposes a division of the gains between him and Partner 2. The latter must decide whether to accept or reject that division. If
rejected, the game ends and both partners earn zero. If tails, the allocation procedure is the same but Partner 2 will be the one proposing the division of gains. Find all the subgame perfect equilibria of the game. Specify your results in terms of the strategies of each of the players.

29. Consider the following Extensive form game among three players

![Game Diagram]

Game problem 29

a) If the game is one of perfect information and all players can observe previous actions of the other players, determine all subgame perfect Nash equilibria in pure strategies, the equilibrium path and the equilibrium payoffs.

b) Assume now that Player $B$ cannot observe the actions of $A$. In this case:
   (i) Represent the new game in extensive form.
   (ii) Are there pure strategy subgame perfect Nash equilibria?

c) Suppose now that $A$’s actions are observable by $B$ and $C$, but that $C$ cannot observe the actions of $B$. In this case:
   (i) Represent the new game in extensive form.
   (ii) Are there any pure strategy subgame perfect Nash equilibria?

Note: If a player does not participate in a given subgame, his/her payoffs are not relevant for the calculation of the NE in that subgame.

30. Two companies competing in a market are considering the possibility of a mail sale campaign. The cost of this campaign is 200 euros. If only one company makes the campaign, it will ensure for itself the sale of 18 units at a price of 30 euros, while if the two companies do it, everyone would sell 12 units at a price of 15 euros. Both companies can produce any quantity at zero cost. Suppose that firms, after deciding whether or not to undertake the mail sale campaign,
compete in quantities in a market whose demand is given by the function $P = (99 - V) - q$, where $P$ represents price, $V$ is the total volume of sales and $q = q_1 + q_2$ is the total amount sold in this market (net of mail sales). Consider the sequential game in which the two companies must first decide simultaneously whether to make the mail sales campaign or not to do it, and, after knowing the decisions about the campaign, there is Cournot competition.

a) Draw the extensive form game
b) Show the information sets for each company. Show also the subsets of this game.
c) Compute the Nash equilibria of all subgames.
d) Find all the SPNE of the game.