Game Theory
Final Exam
January 17, 2012

Name:
Group:
INSTRUCTIONS:
You have 2 hours and 45 minutes to complete the exam. Write in the space after the problem. You may use the backside of each sheet.

1. (30 points) General A defends his territory against the attacks by General B. General A's territory can be accessed by two mountain passes. General A has 4 regiments and General B has 3. Each general allocates his regiments between the two passes (they have to use all of them). If both send the same number of regiments to pass 1, no one gets any advantage and get a payoff of zero. If one general sends more regiments than the other to pass 1 the payoffs are computed as follows: if x is the number of regiments of the general that sends the most, this general obtains a payoff of x and the other general a payoff of $-x$. Payoffs in pass 2 are calculated in the same manner. Total payoffs for each general are just the sum of the payoffs in the two mountain passes. Military reasons demand that a regiment cannot be divided. Both generals choose their strategy simultaneously.

(a) Which are the strategies of each general if General A must assign at least one regiment to each pass and General B is free to assign them as she pleases. (5 points)
(b) Represent the normal form of the game. (5 points)
(c) Which are the strategies that survive the iterative elimination of strictly dominated strategies? (5 points)
(d) Find all the Nash equilibria of the game. (15 points)

(a) Strategias of General A: {(1,3), (2,2), (3,1)}, B: {(0,3), (1,2), (2,1), (3,0)}.
(b)

A/B	**0,3**	**1,2**	**2,1**	**3,0**
1,3	1+0, -1+0	0+3, 0-3	-2+3, 2-3	-3+3, 3-3
2,2	2-3, -2+3	2+0, -2+0	0+2, 0-2	-3+2, 3-2
3,1	3-3, -3+3	3-2, -3+2	3+0, -3+0	0+1, 0-1

A/B	**0,3**	**1,2**	**2,1**	**3,0**
1,3	1, -1	3, -3	1, -1	0, 0
2,2	-1, 1	2, -2	2, -2	-1, 1
3,1	0, 0	1, -1	3, -3	1, -1

(c) For General B strategy (1,2) is strictly dominated by (0,3) and strategy (2,1) is strictly dominated by (3,0). Eliminating those strategies, (2,2) is strictly dominated by (1, 3) and (3, 1) for General A. We are left with

A/B	**0,3**	**3,0**
1,3	1, -1	0, 0
3,1	0, 0	1, -1

(d) There are no Nash equilibria in pure strategies (3 puntos). The only NEME is p =q = 1/2. (Write the problem: 6 points, solve: 6 points.)

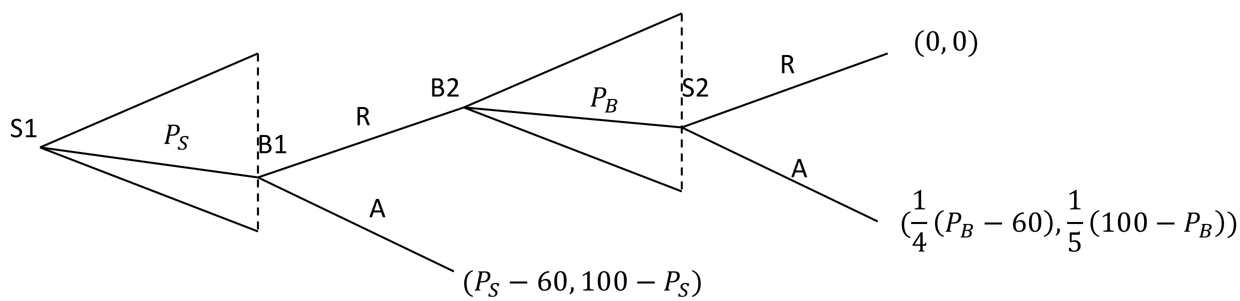
2. (30 points) A seller can produce an item at a cost of 60 euros. A buyer values this item in 100 euros. The buyer and the seller will negotiate the selling price in a two stage game. First, the seller offers a price that, if accepted by the buyer, means that the item is sold at that price and the game is over. If the offer is rejected, it is the buyer's turn to make an offer. If the seller accepts the offer, the item is sold at that price and the game ends. If the seller rejects it, the game ends with no trade. Let p the selling price, then the utility for the seller is $p - 60$ and for the buyer is $100 - p$. In case there is no agreement to sell, the utility is zero for both. The discount rates are $\delta_S = 1/4$ for the seller and $\delta_B = 1/5$ for the buyer. In case of indifference we will assume that players accept the offer.

(a) Represent the game in the extensive form indicating the information sets of each player and the possible actions for each player. (10 points)

(b) Find the Subgame Perfect Nash Equilibrium of this game. (15 points)

(c) What will be the agreement reached and the associated payoffs in equilibrium and in which period will the agreement be reached? (5 points)

(a) Extensiva Form:



(Tree representation with actions: 4 points)
(Payoffs in final nodes: 3 points)
(Description of information sets: 3 points)

(b) S2 accepts (A) if $p_S - 60 \geq 0$, rejects otherwise. (5 points)
B2 offers $p_B = 60$. (2 points)
B1 accepts (A) if $100 - P_S \geq \frac{1}{5}(100 - 60)$, i.e., if $p_S \leq 92$. (6 points)
S1 offers $p_S = 92$. (2 points)

(c) An aggreement is reached in the first period (1 point). The seller offers $p_S = 92$ and the buyer accepts (1 point). Payoffs are $(92 - 60, 100 - 92) = (32, 8)$ (3 points).

3. (20 points) The Roman and the Parthian Empires play the following game each period, where *P* means being peaceful and *A* means to raid across the border with the other empire. Emperors on both sides assume the empires will last forever. Compute the discount rate for a "pacific" subgame perfect equilibrium to exist. In a pacific equilibrium both empires play peacefully as the equilibrium result.

		Parthian Empire	
		P	A
Roman Empire	P	1,1	-1,3
	A	3,-1	0,0

Use the trigger strategy where each empire (Roman ro Parthian) plays as follows:

At t=1 play P, at t>1 play P if at all previous periods (P,P) was played, play A otherwise. (5 points)

The best deviation is top lay A at t=1 and, then, always play A, as there is nothing to gain in subgames after not having played (P,P) because the trigger strategy dictates a NE in those cases. (5 points)

We have to check that it is a EN in the whole game (and in subgames after a history of (P,P) play):

If both empires follow the trigger strategy, then $u_I = 1 + \delta + \delta^2 + \delta^3 + \cdots = \frac{1}{1-\delta}$.

The best deviation gives $u_I = 3 + 0 + 0 + \cdots = 3$.

(5 points)

Fort the deviation not to be profitable we must have $\frac{1}{1-\delta} \geq 3$, i.e. $\delta \geq \frac{2}{3}$. (5 points)

4. (20 points) Consider a Cournot duopoly in a market with inverse demand function given by $p(q) = A - q$, where $q = q_1 + q_2$ is the aggregate market quantity. For simplicity, assume that firms have zero production costs. Parameter A in the demand may take values 10 (low demand) or 20 (high demand) with equal probabilities. Before taking the decision about how much to produce, Firm 1 has truthful information over the value of A. Firm 2, however, does not have this information. Firm 1 knows that Firm 2 does not have the information, and Firm 2 knows that Firm 1 does have the information. All this is common knowledge.

(a) Represent this situation as a Bayesian game. Namely, indicate the set of players, the set of types for each player, beliefs about other player's types and types' strategies. Indicate also the utility (profits) functions of the firms as a function of the strategies. (5 points).

(b) Compute the Bayesian Nash equilibrium of the game. Compute also the equilibrium profits. (15 points)

(a) Players: {1,2}
Types for 1: {1A, 1B}, types for 2: {2} (Players and types: 1 point)
1A's beliefs: $p_{1.A}(2) = 1$,
1B's beliefs: $p_{1.B}(2) = 1$,
2's beliefs: $p_2(1.A) = \frac{1}{2}$, $p_2(1.B) = \frac{1}{2}$. (beliefs: 1 point)
Strategies of type *t*: $q_t \in [0, \infty)$. (strategies: 1 point)
Profits: $\Pi_{1.A} = (20 - q_{1.A} - q_2)q_{1.A}$, $\Pi_{1.B} = (10 - q_{1.B} - q_2)q_{1.B}$,
$\Pi_2 = \frac{1}{2}(20 - q_{1.A} - q_2)q_2 + \frac{1}{2}(10 - q_{1.B} - q_2)q_2$. (2 points)

(b) 1.A solves $max_{q_{1.A}}\,(20 - q_{1.A} - q_2)q_{1.A}$, first order condition (FOC) is $q_{1.A} = \frac{20-q_2}{2}$. (3 points)

1.B solves $max_{q_{1.B}}\,(10 - q_{1.B} - q_2)q_{1.B}$, FOC is
$q_{1.B} = \frac{10-q_2}{2}$. (3 points)

2 solves $max_{q_2} \frac{1}{2}(20 - q_{1.A} - q_2)q_2 + \frac{1}{2}(10 - q_{1.B} - q_2)q_2$, FOC is
$q_2 = \frac{30-q_{1.A}-q_{1.B}}{4}$. (4 points)

Solving the three equations one obtains the NE: $(q_{1.A}, q_{1.B}, q_2) = (7{,}5,\ 2{,}5,\ 5)$. (3 points)

NE profits are $(\Pi_{1.A}, \Pi_{1.B}, \Pi_2) = (56{,}25,\ 6{,}25,\ 25)$. (2 points)