Game Theory
Exam, June 2015

Name **Group:**

You have two and a half hours to complete the exam.

I. Short questions (5 points each)

I.1 Show an example of a static game that has no Nash equilibrium in pure strategies, but that has one in mixed strategies.

I.2 Show an example of a dynamic game withe perfect information in which the subgame perfect equilibrium is not Pareto optimal.

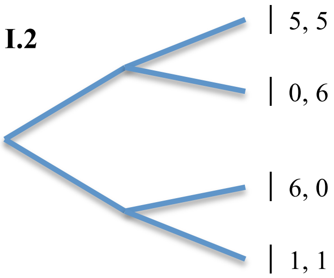
I.3 In a finitely repeated game, the only subgame perfect equilibria consist on repetition of the Nash equilibria of the stage game. True or false?

I.4 Define a Bayesian game with two players and two types of player 1 and one type of player 2. Draw the corresponding extensive form of that Bayesian game.

I.1:

1, -1	-1, 1
-1, 1	1, -1

I.2



I.3

False: If repeated twice, (6, 6) can be obtained in the first stage.

6, 6	0, 7	0, 0
7, 0	3, 3	0, 0
0, 0	0, 0	1, 1

I.4 See class notes.

II. Problems (20 points each)

II.1 Two firms are involved in developing a new technology that makes it possible for consumers to smell products over the Internet, which potentially will increase online sales in perfume and body care products. Given the risks and the relatively small expected size of this market, compatibility of the technologies is very important. Firm DigiScent is far advanced in developing its DigiFleur technology. WebOdor has been expanding into the Internet smell arena with its incompatible product, BreezeWeb. The two companies agree that if they both adopt the same technology, they each may gross €200M from the developing industry. If they adopt different technologies, no consumer will take the technology seriously, and neither product will be purchased, leading to a gross of €0. Switching costs to the competing (nonproprietary) technology for WebOdor amount to €100M and for DigiScent to €250M. Decisions are made simultaneously without consulting each other.

(a) Set up the above scenario as a normal form (simultaneous) game.
(b) What is the Nash equilibrium? How much are the firms making?

Both firms adopt the DigiFleur technology; that is, DigiScent keeps its technology, and WebOdor switches.

		WebOdor	
		DigiFleur	***BreezeWeb***
DigiScent	***DigiFleur***	200, 100	0, 0
	BreezeWeb	-250, -100	-50, 200

II.2 Consider the following situation. A monopoly needs to hire labor L to produce its goods and each unit of labor produces one unit of the good. Thus, $Q = L$. Market demand is represented by $p = 100 - Q$, where p is the market price and Q the produced quantity. Each unit of labor costs w, the wage, which is determined by a union. The monopoly maximizes its profits $\Pi = pQ - wQ$, the union maximizes the total wage bill $U = wL$. The order of moves is the following: First the union set wages, then the monopoly firm hires labor L to produce the good.

(a) Calculate and draw the monopolist's reaction function.
(b) Determine the subgame perfect equilibrium, that is the equilibrium wage and equilibrium quantity produced.

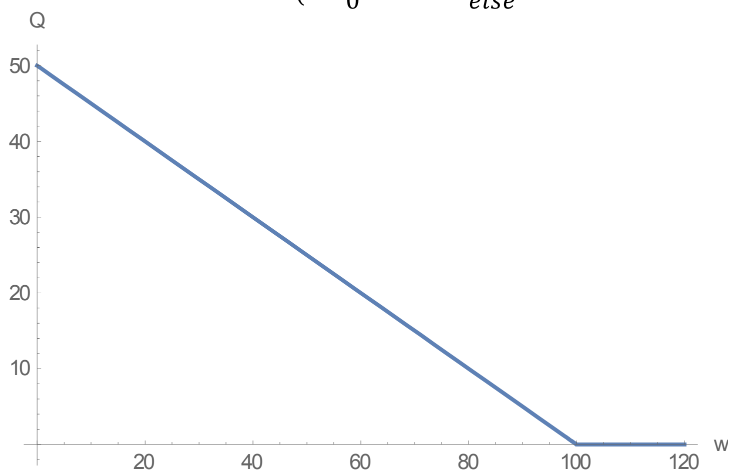
Assume now that instead of maximizing only the wage bill, the union puts merely a weight of 1/3 on the wage bill and assigns another weight of 2/3 to the number of employed workers L. The union's payoff is thus $U = \frac{1}{3}w\,L + \frac{2}{3}L$

(c) What is the new SPNE?

SOLUTION:

a)

$$BR(w) = \begin{cases} \dfrac{100 - w}{2} & if\ w \leq 100 \\ 0 & else \end{cases}$$



b) SPNE = {(50,25)}
c) SPNE = {(49,51/4)}. The wage decreases to increase employment.

II.3 Ana and Bruno play the following game n times.

		Bruno A	Bruno B
Ana	A	3,3	1,4
	B	4,1	0,0

(a) Is it possible to obtain the payoff (3,3) at some stage in a subgame perfect equilibrium if $n = 2$?
(b) If n is infinite?

(a) There are three NE in the game: (A,B), (B,A) and (½[A]+1/2[B], (½[A]+1/2[B]), with respective payoffs (1,4), (4,1) and (2,2). The following strategy is a SPE:
Period 1: Play (A,A).
Period 2:
If (A,A) was played in period 1, play (½[A]+1/2[B], (½[A]+1/2[B]).
If (A,B) was played in period 1, play (B,A).
If (B,A) was played in period 1, play (A,B).
If (B,B) was played in period 1, play any of the three NE.

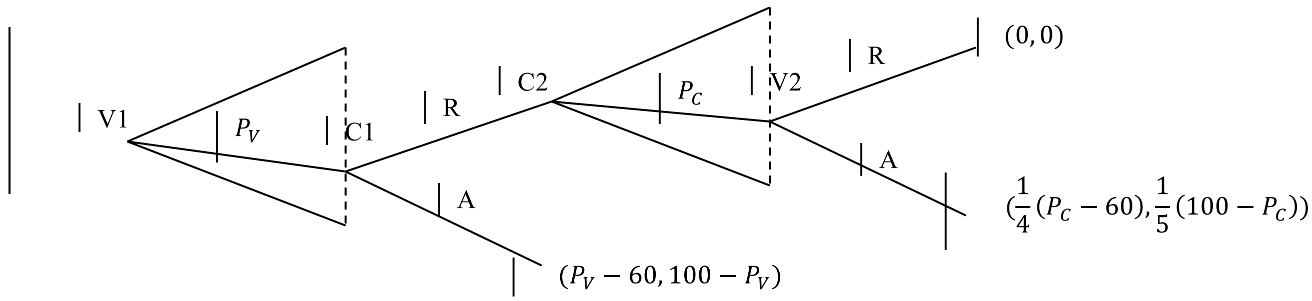
If no deviation, each player gets 3+2=5. If player i deviates, she gets 4+1=5.

(b) Define the trigger strategy:
Period 1: Play (A,A)
Period t: Play (A,A) if (A,A) was played for all T<t. Play (½[A]+1/2[B], (½[A]+1/2[B]) if (A,A) was not played for some T<t.
Check it is a NE: $U(\text{no dev.}) = \frac{3}{1-\delta} \geq U(\text{dev.}) = 4 + \frac{2\delta}{1-\delta}$, which implies $\delta \geq \frac{1}{2}$.
Check it is a NE in subgames after no deviation: the game is as the original. The previous argument applies.
Check it is a NE in subgames after deviation: it implies the play of a NE in each period forever on. It is a NE.

II.4 A seller can produce an item at a cost of 60 euros. A buyer values this item in 100 euros. The buyer and the seller will negotiate the selling price in a two-stage game. First, the seller offers a price that, if accepted by the buyer, means that the item is sold at that price and the game is over. If the offer is rejected, it is the buyer's turn to make an offer. If the seller accepts the offer, the item is sold at that price and the game ends. If the seller rejects it, the game ends with no trade. Let p the selling price, then the utility for the seller is p-60 and for the buyer is 100-p. In case there is no agreement to sell, the utility is zero for both. The discount rates are $\delta_S = 1/4$ for the seller and $\delta_B = 1/5$ for the buyer. In case of indifference we will assume that players accept the offer.

(a) Represent the game in the extensive form indicating the information sets of each player and the possible actions for each player.
(b) Find the Subgame Perfect Nash Equilibrium of this game.

(a) Extensive form:



(b) V2 accepts (A) if $p_c - 60 \geq 0$, rejects otherwise.
C2 offers $p_C = 60$.
C1 accepts (A) if $100 - P_V \geq \frac{1}{5}(100 - 60)$, i.e., if $p_V \leq 92$.
V1 offers $p_V = 92$.