Game Theory
Exam January 2014

Name: Group:

You have two and a half hours to complete the exam.

I. Short questions (20 points).

I.1 When a player chooses a mixed strategy in equilibrium, the pure strategies that are part of it may give different payoffs. True or false, explain.

No. For instance, if mixed strategy S plays pure strategy A with probability p and B strategy with probability q and if A gives higher payoffs than B, then another mixed strategy X that is as S except that it assigns p+q probability to A and zero probability to B will give higher payoffs.

I.2 By eliminating one or more of his strategies, a player may be able to induce a better outcome for him. Explain why or why not. Provide an example.

Yes, it may happen:

	A	B
A	5,5	0,4
B	6,0	1,1

If the row player eliminates her strategy B, the equilibrium will be (A,A) instead of (B,B) and her payoff 5 instead of 1.

I.3 In a finitely repeated game, for players to obtain payoffs above the ones in the Nash equilibria it is necessary that the game has more than one Nash equilibrium. True or false? Explain.

Yes. Say there are two NE and that one offers payoffs higher than the other. If there is a strategy profile that gives even higher payoffs, there is room for a SPNE that implies playing this strategy profile for non-last periods and the "good" NE in the last period and that punishes a deviation by playing the "bad" NE in all periods after the deviation.

I.4 If a player in a static Bayesian game has two types, each of which has to take one out of three actions, how many pure strategies does this player have?

The strategy of a player must indicate an action for all of his types. If there are two types and each has three actions, the number of strategies is 3x3=9.

II. Problems. You must answer four out if the next five problems. (20 points each).

II.1 Two firms, A and B are the only producers of milk in a small tropical island. Their milk is identical, in taste, color, nutrients, even in packaging. They compete in prices in the Bertrand sense: whoever puts the lowest price gets the entire market, while when they put the same price they share the market equally. The demand in this market is: $D = 100 - p$, and the marginal cost of Firm A is 0, while the marginal cost of Firm B is 40. Firm i can only bid prices that are integer numbers between its marginal cost and 100.

(a) Find the profit function of each firm as a function of the two prices. (5 points)
(b) Find the reaction function of each firm. (7 points)
(c) Find the Nash Equilibria and the profits each firm makes in each equilibrium. (8 points)

(a) If $p_A > p_B$: $\Pi_A = 0$, $\Pi_B = (100 - p_B)(p_B - 40)$.
If $p_A = p_B = p$, $\Pi_A = ((100 - p)/2)p$, $\Pi_B = ((100 - p)/2)(p - 40)$.
If $p_A < p_B$, $\Pi_A = (100 - p_A)p_A$, $\Pi_B = 0$.

(b) BR_A: If $p_B > 50$, $p_A = 50$ (A's monopoly price)
If $40 < p_B \leq 50$, $p_A = p_B - 1$

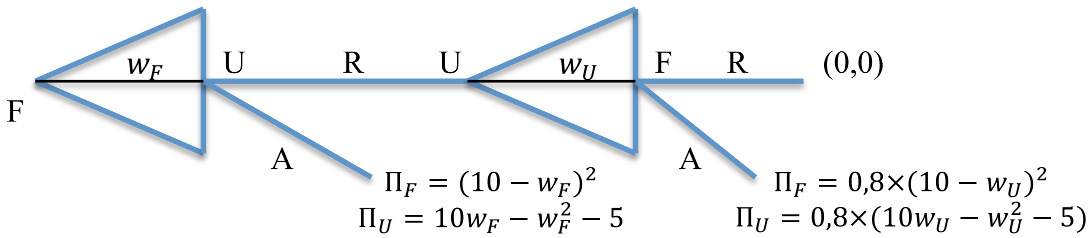
BR_B: If $p_A > 70$, $p_B = 70$ (B's monopoly price)
If $41 < p_A \leq 70$, $p_B = p_A - 1$
If $p_A = 41$, $p_B = 41$
If $p_A \leq 40$, $p_B \geq 40$.

(c) Nash equilibria = {(40, 41), (39, 40)}. $\Pi_A(40,41) = 60\times 40 = 2400$, $\Pi_B(40,41) = 0$, $\Pi_A(39,40) = 61\times 39 = 2379$, $\Pi_B(39,40) = 0$.

II.2 A firm negotiates with its labor union about the firm-specific wage. The firm's profit function is $\Pi_E = (10 - w)^2$, where w is the wage. The objective of the labor union is to maximize $\Pi_S = 10w - w^2 - 5$. The rules of wage negotiations are as follows. The firm makes a wage offer that the union can accept or reject. In case of acceptance the negotiation ends. In case of rejection, the union makes a counter-offer that the firm may accept or reject. If the counter-offer is rejected, there is no production and both parties get zero. If an offer is accepted then the parties get their respective profits according to the accepted offer. The discount factor for both players is 0.8.

(a) Provide the extensive form of the game. Indicate the strategies of the two players and their information sets. (5 points)
(b) What wage will the union offer if the game reaches the second stage? Which are the parties' profits in this case? (5 points)
(c) Find the unique subgame perfect Nash equilibrium of the negotiation game. What will be the agreed wage? Which are the profits? When will the agreement take place? (10 points)

(a)



(b) The union solves the problem:

$$\max \Pi_U = 10w - w^2 - 5$$
$$\text{s.a: } \Pi_F = (10 - w)^2 \geq 0$$

From there: $w = 5$. Union's profits = 20, Firm's profits = 25.

(c) Knowing the equilibrium in the second stage in the first stage the firm will look for the minimum w that leaves the union with the same discounted payoffs as in the second stage. Thus, it will offer w such that:

$$10w - w^2 - 5 = 0{,}8\times 20$$

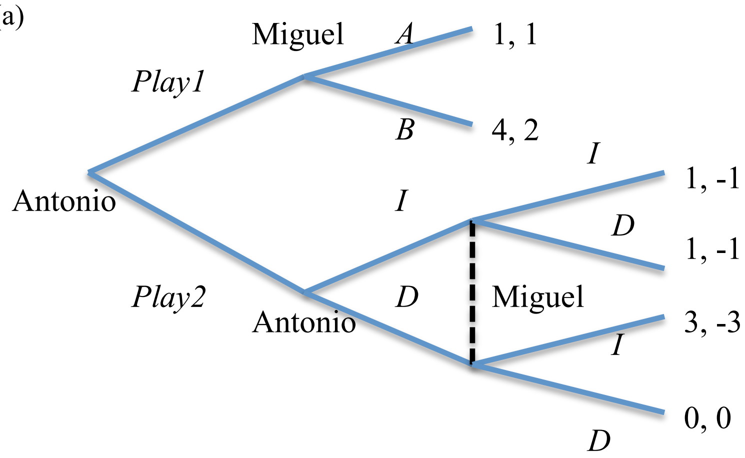
with $w = 3$. The union accepts. In case of rejection, follow as in (b).
Union's profits = 16, Firm's profits = 49.

II.3 Antonio and Miguel face the following game. Antonio must choose between two actions: *Play1* and *Play2*. If he chooses *Play1*, then Miguel must choose between *A* and *B*. If he chooses *A* both players get a payoff of one, whereas if he chooses *B*, Antonio gets 4 and Miguel gets 2. On the other hand, if Antonio chooses *Play2*, both players then play a simultaneous game where Antonio (row player) choose between *I* and *D*, the same options that Miguel (column player) has. The payoffs of this simultaneous game are:

	I	*D*
I	1, -1	1, -1
D	3, -3	0, 0

(a) Represent the extensive form of the game. (3 points)
(b) Find all the Nash equilibria of the simultaneous game that starts if Antonio chooses *Play2*. (10 points)
(c) Find the subgame perfect Nash equilibria of the whole game. (7 points)

(a)



(b) NE = {(I, $q[I] + (1-q)[D]$) with $q \leq \frac{1}{3}$}

(c) SPNE = {(*Play1*, *I*), (B, $q[I] + (1-q)[D]$), with $q \leq \frac{1}{3}$}

II.4 Consider the infinitely repeated version of following the prisoners' dilemma with $x \geq 5$. Each player discounts future payoffs by $\delta_1 = \delta_2 = \frac{1}{2}$.

	C	D
C	4, 4	0, x
D	x, 0	1, 1

(a) Find the values for x such that the trigger strategy supports the outcome (C,C) in each period in a subgame perfect equilibrium. (10 points)

(b) Let $x \geq 6.5$ and consider the following strategy profile: Player plays C as long as nobody deviates from playing C. If somebody does, they play D for 2 periods, then they return to playing C and plays until somebody deviates again and the 2 period punishment is implemented again, and so on. Is this strategy a Subgame perfect Nash equilibrium? (10 points)

(a) $u_i(\text{trigger strategy}) = 4\frac{1}{1-\delta} = \frac{4}{1-\frac{1}{2}} = 8.$

$u_i(\text{deviation to } D) = x + \frac{\frac{1}{2}}{1-\frac{1}{2}} = x + 1.$

$8 \geq x + 1 \rightarrow x \leq 7.$

(b) $u_i(\text{strategy with two periods of punishment}) = 4\frac{1}{1-\delta} = \frac{4}{1-\frac{1}{2}} = 8.$

$u_i(\text{deviation to } D) = x + \frac{1}{2}1 + (\frac{1}{2})^2 1 + \left(\frac{1}{2}\right)^3 \frac{4}{1-\frac{1}{2}} = x + 1{,}75 \geq 8{,}25.$ It is not a NE.

II.5 Consider the following Bayesian game. Two players, *A* (who plays rows) and *B* (columns), play the Chicken game depicted next. Both of them can decide to continue driving towards the cliff (*C*) or stop the car (*S*). Player A can be of two types: either Player *A* is normal, in which case the payoffs are the standard ones of the Chicken game

	C	*S*
C	-5, -5	1, -1
S	-1, 1	0, 0

or Player *A* is a wimp, in which case the payoffs are as follows

	C	*S*
C	-5, -5	1, -1
S	0, 1	2, 0

Player *A* knows whether he is a wimp or not. Player *B* thinks that with probability ½, Player *A* is a wimp and with probability ½ Player *A* is normal.

(a) Describe all the elements of this Bayesian game. (5 points)
(b) Find its Bayesian Nash equilibria in pure strategies. (15 points)

(a) Players = $\{A, B\}$
Types of A = {Normal, Wimp}
Types of B = $\{B\}$

$(p(A = \text{normal}/B) = \frac{1}{2}, p(A = \text{wimp}/B) = \frac{1}{2})$,
$p(B = B/A = \text{normal}) = 1$,
$p(B = B/A = \text{wimp}) = 1$.

Actions of A = $\{C, S\}$, actions of B = $\{C, S\}$

Payoffs after actions and types are as shown in the tables.

(b) BNE in pure strategies = $\{\big((S,S),C\big),\big((C,S),S\big)\}$.