Bayesian games

2. Economic applications

Universidad Carlos III de Madrid

Economic applications

- Auctions with asymmetric information over valuations:
 - First price auctions.
 - Second price auctions.
- Cournot duopoly with asymmetric information over:
 - Rival's costs.
 - Demand.
- Public goods with asymmetric information over the rival's valuation or costs.

Auctions

- Two individuals participate in a closed-envelope auction to buy an object.
- They must choose their bids, b₁ and b₂, simultaneously, and the object is sold to the one with the highest bid. In case of a tie, the winner is selected randomly.
- Valuations are v_1 and v_2 .
- Player *i*'s utility is:

$$u_i(b_1, b_2; v_1, v_2) = \begin{array}{cc} v_i - p & \text{if } b_i > b_j \\ \hline v_i - p \\ 2 & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{array}$$

where *p* depends on the auction type used to sell the item.

- The individual placing the highest bid gets the item. She pays her bid.
- Players: buyers 1 and 2.
- Possible actions: $b_i \in [0, \infty)$, i = 1, 2.
- Types: each buyer has a valuation v_i , that may be their private information.
- Player *i*'s utility:

$$v_i - b_i \quad \text{if } b_i > b_j$$

$$u_i(b_1, b_2; v_1, v_2) = \begin{array}{c} \frac{v_i - b_i}{2} & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{array}$$

Let us see a simplified version of a first price auction:

- Buyer 1 may be of two types, with equal probabilities, one type values the item at 60 and the other values it at 100.
- Buyer 2 has only one type, that values the item at 60.
- Only bids allowed are 40, 60 and 80.
- We will not consider equilibria in weakly dominated strategies.

 $v_1 = 60$, prob = 0.5

Player 2

		$b_2 = 40$	$b_2 = 60$	$b_2 = 80$
	$b_1 = 40$	10, 10	0, 0	0, -20
Player 1.60	$b_1 = 60$	0, 0	0, 0	0, 20
	<i>b</i> ₁ = 80	-20, 0	-20, 0	-10, -10

 $v_1 = 100, \text{ prob} = 0.5$ Player 2 $b_2 = 40$ $b_2 = 60$ $b_2 = 80$ $b_1 = 40$ 30, 10 0, 0 0, -20 Player 1.100 $b_1 = 60$ 40, 0 20, 0 0, -20

Player 1.100 $b_1 = 60$ 40, 020, 00, -20 $b_1 = 80$ 20, 020, 010, -10

Eliminate weakly dominated strategies for 1.60 and 1.100.

$v_1 = 60$, prob = 0,5			Player	2		l
		$b_2 = 40$	<i>b</i> ₂ =	60	b ₂ =	= 80
	<i>b</i> ₁ = 40	10, 10	0,	0	0,	-20
Player 1.60	$b_1 = 60$	0, 0	0,	0	0,	-20
	$b_1 = 80$	-20, 0	-20	,0	-10	, -10
$v_1 = 100$, prob = 0,5			Playe	r 2		
		$b_2 = 40$	$b_2 =$	60	b ₂ =	= 80
	$b_1 = 40$	30, 10	0,	0	0,	-20
Player 1.100	$b_1 = 60$	40, 0	20	, 0	0,	-20
	$b_1 = 80$	20, 0	20	, 0	10,	-10

Eliminate weakly dominated strategies for 2.

- Eliminate weakly dominated strategies for Type $v_1 = 60$: $b_1 = 60$ and $b_1 = 80$ are weakly dominated by $b_1 = 40$.
- Eliminate weakly dominated strategies for Type $v_1 = 100: b_1 = 40$ is weakly dominated by $b_1 = 60$.
- Eliminate weakly dominated strategies for Player 2: $b_2 = 60$ and $b_2 = 80$ are weakly dominated by $b_2 = 40$.

After the first round of elimination of weakly dominated strategies we get:

$$v_{1} = 60, \text{ prob} = 0.5$$
Player 2
$$b_{2} = 40$$
Player 1.60
$$b_{1} = 40$$
10, 10
Player 2
$$b_{2} = 40$$
Player 2
$$b_{2} = 40$$
Player 1.100
$$b_{1} = 60$$
40, 0
$$b_{1} = 80$$
20, 0

- Now $b_1 = 80$ is weakly dominated for 1.100.
- We are left with one action for each type, and that will be the Bayesian equilibrium: ((40, 60), 40).
- Equilibrium Payoffs: Player 1 obtains $0.5 \times 40 + 0.5 \times 10 = 25$, Player 2 obtains $0.5 \times 10 + 0.5 \times 0 = 5$.
- The auctioneer's expected revenues are $0.5 \times 40 + 0.5 \times 60 = 50$.

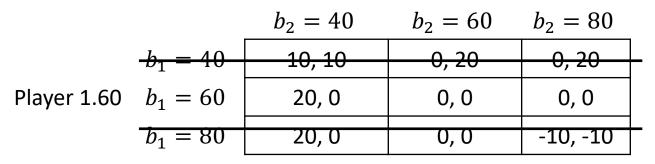
- The player placing the highest bid gets the item. She pays the second highest bid.
- Players: buyers 1 and 2.
- Possible actions : $b_i \in [0, \infty)$, i = 1, 2.
- Types: each player has a valuation v_i , that may be their private information.
- Player *i*'s utility:

$$v_i - b_j \quad \text{if } b_i > b_j$$

$$u_i(b_1, b_2; v_1, v_2) = \begin{array}{c} \frac{v_i - b_j}{2} & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{array}$$

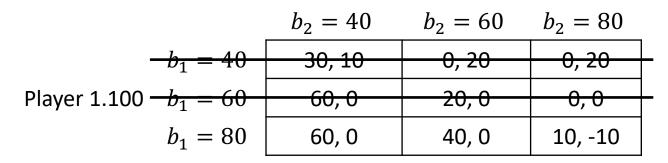
 $v_1 = 60$, prob = 0.5

Player 2

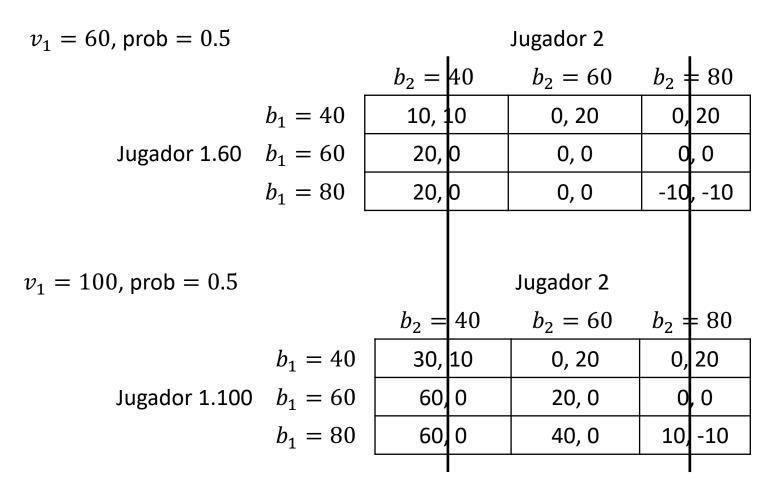


 $v_1 = 100$, prob = 0.5

Player 2



Eliminate weakly dominated strategies for 1.60 and 1.100.



Eliminate weakly dominated strategies for 2.

- Eliminate weakly dominated strategies for Type $v_1 = 60$: $b_1 = 40$ and $b_1 = 80$ are weakly dominated for $b_1 = 60$.
- Eliminate weakly dominated strategies for Type $v_1 = 100 : b_1 = 40$ and $b_1 = 60$ are weakly dominated for $b_1 = 80$.
- Eliminate weakly dominated strategies for Player 2: $b_2 = 40$ and $b_2 = 80$ are weakly dominated for $b_2 = 60$.
- After the elimination we are left with the Bayesian equilibrium: ((60, 80), 60).
- The equilibrium expected payoffs are:
 - $0.5 \times 0 + 0.5 \times 40 = 20$ for Player 1,
 - $0.5 \times 0 + 0.5 \times 0 = 0$ for Player 2.
- The auctioneer's expected revenue is 0.5×60 + 0.5×60 = 60 (recall that he gets the second price).

Cournot: private info. over costs

- Two firms compete on quantity in a market with demand p = A Q.
- Marginal costs for Firm 1 are either c_A or c_B with probabilities 1/3, 2/3 and with $c_A > c_B$.
- Marginal cost for Firm 2 are c_2 .
- Each firm knows its own costs.
- Firm 1 knows the costs for Firm 2.
- Firm 2 does not know the costs for Firm 1, but knows that they are either c_A or c_B with the above probabilities.

Cournot: private info. over costs

- This is the problem for Firm 1:
 - If costs are c_A :

$$\max_{q_A} (A - q_A - q_2)q_A - c_A q_A,$$
$$q_A = \frac{A - q_2 - c_A}{2}.$$

• If costs are c_B :

$$\max_{q_B} (A - q_B - q_2)q_B - c_B q_B,$$
$$q_B = \frac{A - q_2 - c_B}{2}.$$

• Firm 2:

$$\begin{split} \max_{q_2} \frac{1}{3} (A - q_A - q_2 - c_2) q_2 + \frac{2}{3} (A - q_B - q_2 - c_2) q_2, \\ q_2 &= \frac{1}{3} \frac{A - q_A - c_2}{2} + \frac{2}{3} \frac{A - q_B - c_2}{2}. \end{split}$$

• Using the three reaction functions we will find the equilibrium.

Cournot: private info. over the demand

- Two firms compete on quantity in a market with demand p = A - Q where A can take values A_A or A_B, with probabilities ½, ½, and A_A > A_B.
- Marginal costs for Firm i are c_i .
- Firm 1 knows with certainty the value for the demand.
- Firm 2 does not know the demand, but knows that it will be either A_A or A_B with the above probabilities.

Cournot: private info. over the demand

- This is the problem for Firm 1:
 - If demand is A_A :

$$\max_{q_A} (A_A - q_A - q_2)q_A - c_1 q_A,$$
$$q_A = \frac{A_A - q_2 - c_1}{2}.$$

• If demand is A_B :

$$\max_{q_B} (A_B - q_B - q_2)q_B - c_1 q_B,$$
$$q_B = \frac{A_B - q_2 - c_1}{2}.$$

• Firm 2:

$$\max_{q_2} \frac{1}{2} (A_A - q_A - q_2 - c_2)q_2 + \frac{1}{2} (A_B - q_B - q_2 - c_2)q_2,$$
$$q_2 = \frac{1}{2} \frac{A_A - q_A - c_2}{2} + \frac{1}{2} \frac{A_B - q_B - c_2}{2}.$$

• Using the three reaction functions we will find the equilibrium.

- Two players must decide simultaneously whether to contribute towards the provision of a public good.
- Their actions are "contribute" and "do not contribute".
- Each player gets a utility of 3 if they both contribute, of 1 only one decides to contribute and 0 no one contributes.
- For Player 1, the cost of contributing is $c_1 \in (1,3)$.
- For Player 2, the cost of contributing is either $c_A \in (0,1)$ or $c_B \in (1,3)$, with probabilities 2/5, 3/5 respectively.
- Each player knows his own costs.
- Player 2 knows the costs for Payer 1.
- Player 1 does not know the costs for Player 2, but knows that they
 must be either c_A or c_B with the above probabilities.

• Payoffs are:

 $t_2 = c_A$, Prob 2/5

Player 2 Contribute Do not contribute

Player 1	Contribute	$3 - c_1, 3 - c_A$	$1 - c_1, 1$
	Do not contribute	1, 1 − <i>c</i> _A	0, 0

 $t_2 = c_{\rm B}$, Prob 3/5

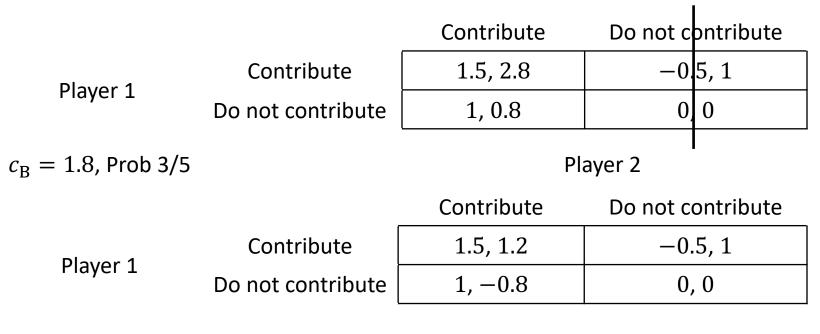
Player 2

		Contribute	Do not contribute
Player 1	Contribute	$3 - c_1, 3 - c_B$	$1 - c_1, 1$
	Do not contribute	1, $1 - c_B$	0, 0

• For example, let $c_1 = 1.5$, $c_A = 0.2$ and $c_B = 1.8$:

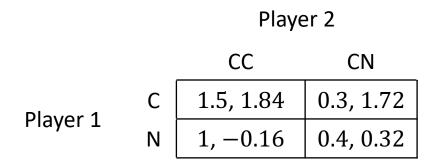
 $c_A = 0.2$, Prob 2/5

Player 2



Strategies after eliminating "Do not contribute" for 2.c_A:
1: (C, N),
2: (CC, CN).

Let us build the normal form.



- There are two EN in pure strategies : (C, CC) and (N, CN), that will also be BNE in pure strategies
- There will also be a third equilibrium in mixed strategies.