Dynamic games

5. Negotiation

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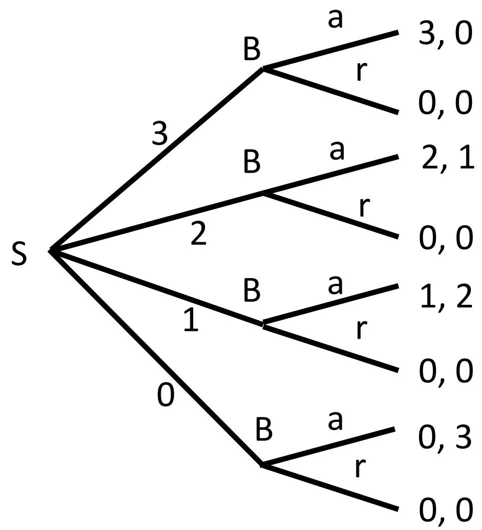
Negotiation / bargaining

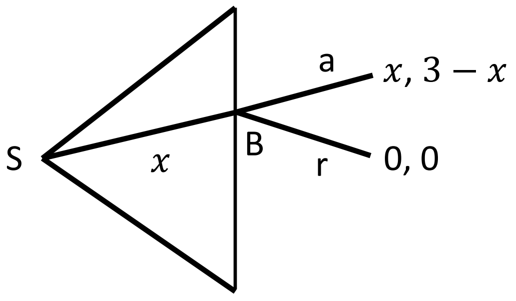
- We'll study negotiation processes as a kind of dynamic game. For instance:
  - Price negotiation between sellers and buyers.
  - Wage negotiation between unions and firms.
  - A treaty negotiation between countries.
- These processes usually have the following features:
  - Offers and counter-offers are made.
  - There is a limited time for negotiations.
  - The game ends if there is no agreement.
  - Players prefer an early agreement rather than a late one.
  - Everyone prefers an agreement to a disagreement.

The ultimatum game

- A seller and a buyer (S and B).
- The seller has a good that values at 0. The buyer values it at 3.
- The seller offers to sell for $x \in [0, 3]$.
- The buyer accepts or rejects.

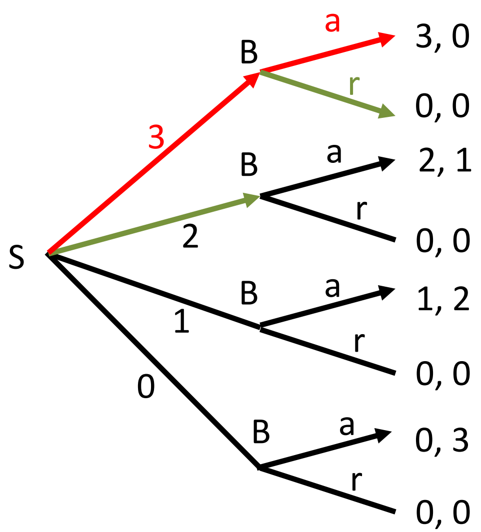


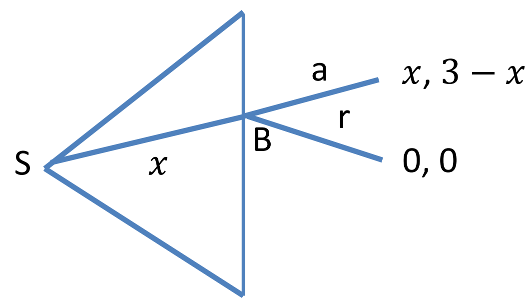
Game if variable x is discrete and takes only integer values



Sketch if variable x is continuous

The ultimatum game



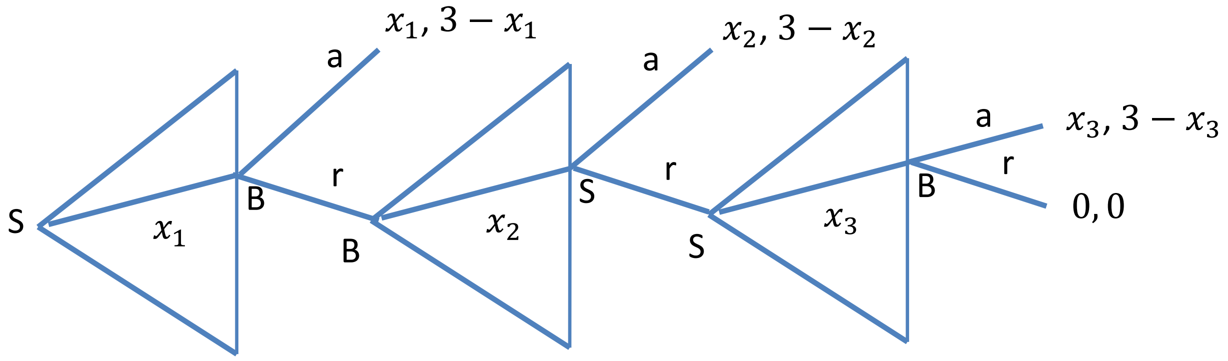


Sketch if variable x is continuous

- Only one NE in each subgame after offers smaller than 3: a.
- Two NE in subgame after offer 3: a (red line) and r (green line).
- SPNE:
  - S offers 3 and B accepts always: (3, (a,a,a,a)).
  - S offers 2 and B accepts offers smaller than 3 and rejects 3: (2, (r,a,a,a)).
- For simplicity, only equilibria of the first type will be considered, in both cases, the buyer gets a very small payoff: payoffs are (3, 0) and (2, 1), respectively. If the seller can make offers up to the centesimal, the payoffs will be (3, 0) and (2.99, 0.01), respectively,
- In the game with continuous variable there is only one SPNE: (3, accept any offer).

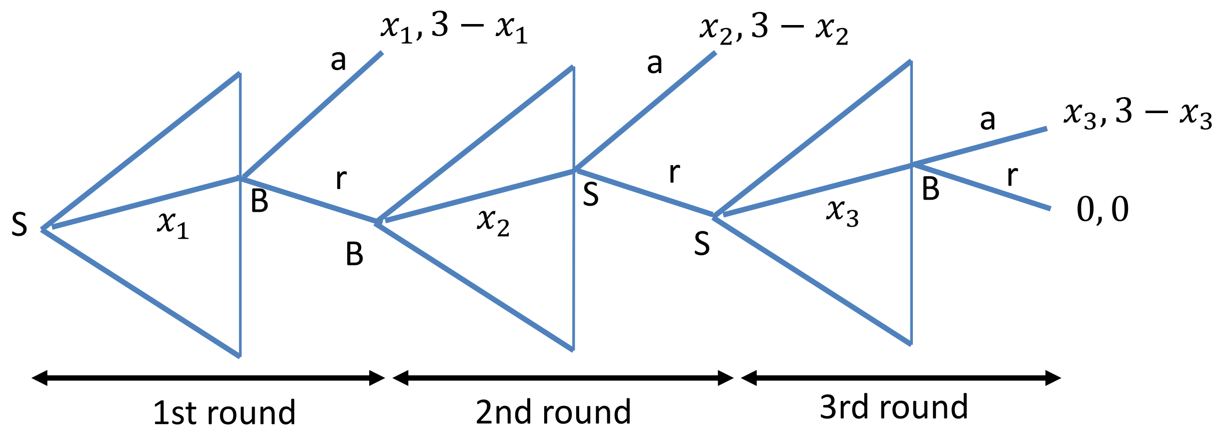
Negotiation model of offers and counteroffers

- In the ultimatum game there is a clear disadvantage to be the player that can only accept or reject.
- Let us see what happens if we add counteroffers.
- x_t is the payoff for S at period t.



The player making the offer in the last period can always reject all offers and wait until the end to keep the advantage.

Negotiation model of offers and counteroffers



SPNE: 3rd round:

B accepts any x_3 such that $3 - x_3 \geq 0$.

S offers $x_3 = 3$.

2nd round:

S accepts any $x_2 \geq 3$ (3 is what he makes if he rejects the offer).

B offers any x_2 (if S accepts or rejects, he gets 0).

1st round:

B accepts any x_1 such that $3 - x_1 \geq 0$.

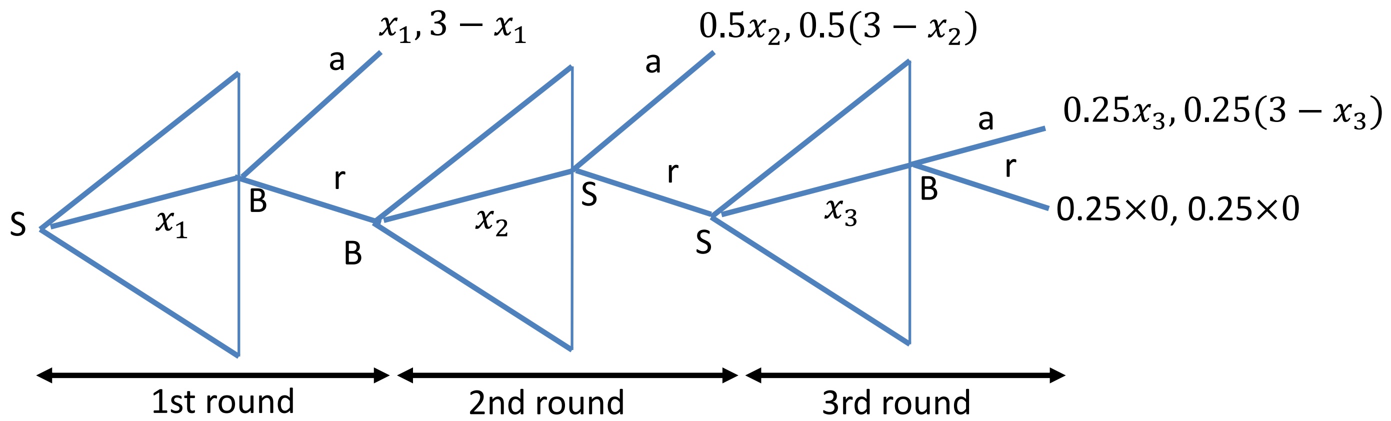
S offers $x_1 = 3$.

Equilibrium path: S offers $x_1 = 3$, B accepts: (3, a).

Equilibrium payoffs: (3, 0).

Impatience

Introduce a discount rate $\delta = 0.5$.



SPNE: 3rd round: B accepts any x_3 such that $0.25(3 - x_3) \geq 0.25\times 0$: $x_3 \leq 3$.
S offers $x_3 = 3$.

2nd round : S accepts any x_2 such that $0.5x_2 \geq 0.25\times 3$: $x_2 \geq 1.5$.
B offers $x_2 = 1.5$.

1st round : B accepts any x_1 such that $3 - x_1 \geq 0.5(3 - 1.5)$: $x_1 \leq 2.25$.
S offers $x_1 = 2.25$.

Equilibrium path: S offers $x_1 = 2.25$, B accepts: (2.25, a).

Equilibrium payoffs: (2.25, 0.75).

Impatience

- 20 periods, players negotiate over one unit, $\delta = 0.8$.
- At the time to accept or reject, a player accepts if she is offered what she expects to win in the next round multiplied times δ.
- Anticipating this, at the time to offer, a player offers the minimum to be accepted by the other player.

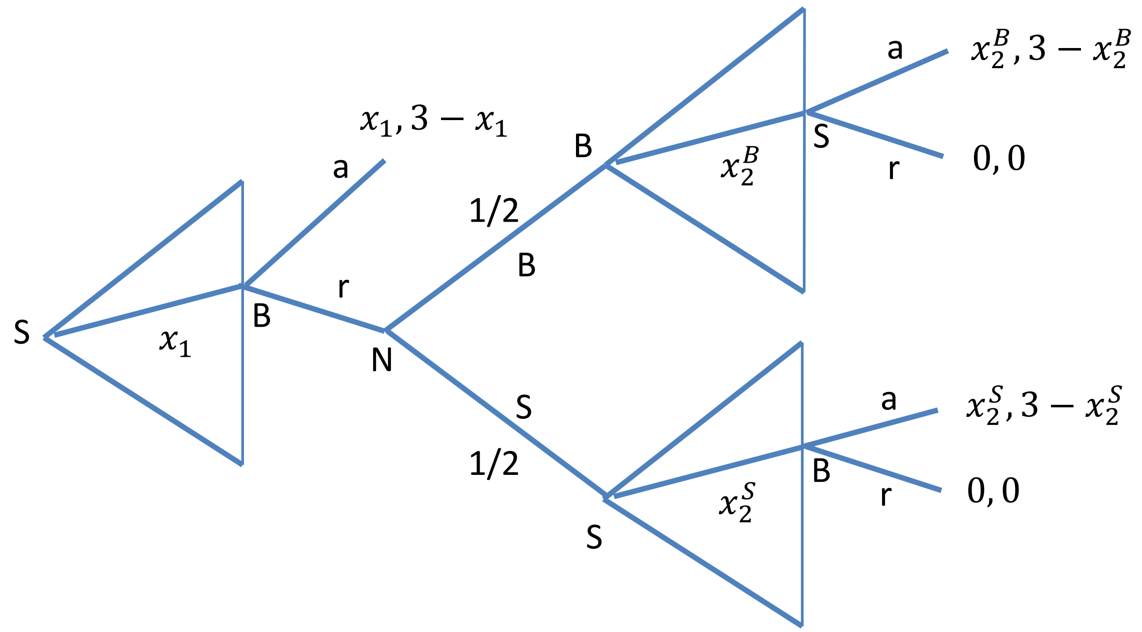
Round	Player making the offer	Payoff for Player 1	Payoff for Player 2
20	Player 2	0	1
19	Player 1	0.2	0.8 $(= 1\times0.8)$
18	Player 2	0.16 $(= 0.2\times0.8)$	0.84
17	Player 1	0.328	0.672 $(= 0.84\times0.8)$
16	Player 2	0.262 $(= 0.328\times0.8)$	0.738
15	Player 1	0.41	0.59 $(= 0.738\times0.8)$
14	Player 2	0.328 $(= 0.41\times0.8)$	0.672
13	Player 1	0.462	0.538 $(= 0.672\times0.8)$
12	Player 2	0.37 $(= 0.462\times0.8)$	0.63
11	Player 1	0.496	0.504 $(= 0.63\times0.8)$
10	Player 2	0.397 $(= 0.496\times0.8)$	0.603
9	Player 1	0.517	0.483 $(= 0.603\times0.8)$
8	Player 2	0.414	0.586
7	Player 1	0.531	0.469
6	Player 2	0.425	0.575
5	Player 1	0.54	0.46
4	Player 2	0.432	0.568
3	Player 1	0.545	0.455
2	Player 2	0.436	0.564
1	Player 1	0.549	0.451

Impatience

- Being impatient reduces the bargaining power, as one tends to accept worse offers.
- With a few rounds, the player playing last has an advantage.
- The advantage is reduced with larger number of periods.
- In the number of periods is large enough, the advantage goes to the player playing first.
- In the limit, when the number of rounds goes to infinite, the equilibrium payoffs are: $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$. In the previous example with infinitely many rounds this is $\left(\frac{1}{1+0.8}, \frac{0.8}{1+0.8}\right) = (0.555, 0.444)$.
- We will not prove this last result, but observe that, in the example, with 20 rounds we get close to that limit: (0.549, 0.451).
- Rejecting small offers in the last period does not affect the result if there are many periods (it is like, for instance, round 16 were the last, with the offer as shown in the table).

Risk aversion

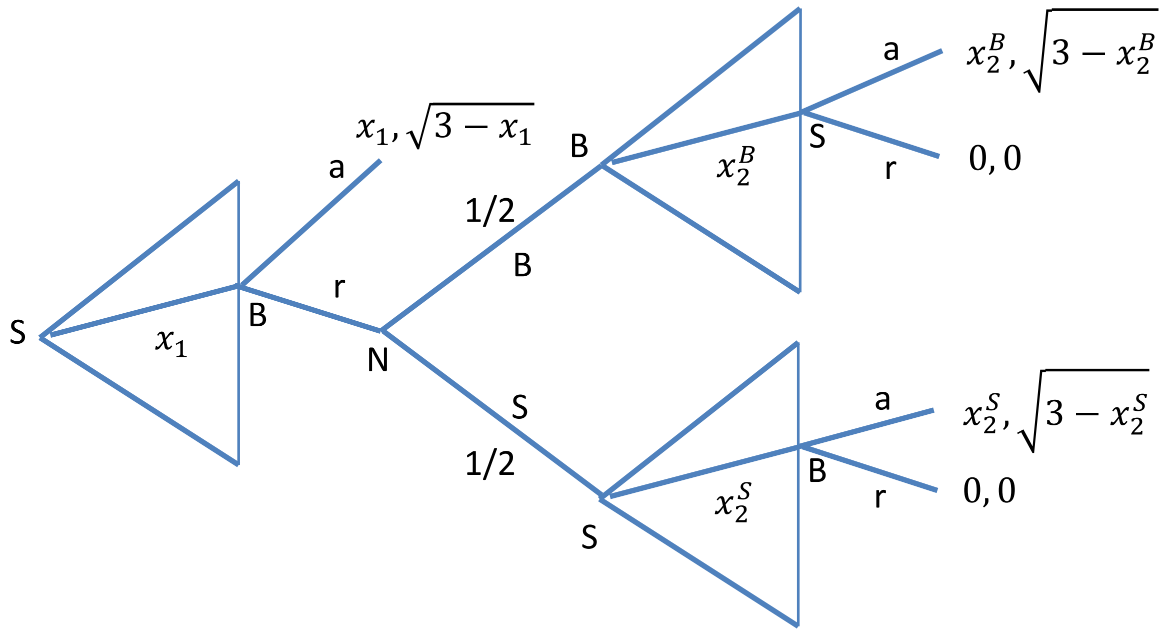
- In the last round, the player making an offer is chosen randomly (no discount).
- Players are risk neutral.



- In the last round, B earns 3 in the subgame where she makes the offer and 0 in the subgame where S makes the offer. The expected payoff is 1.5.
- Thus, in the first round, S offers to give her 1.5 and to keep 1.5.

Risk aversion

- In the last round, the player making an offer is chosen randomly (no discount).
- The buyer is risk averse, with $u_B = \sqrt{x}$.



- In the last round, B earns 3 in the subgame where she makes the offer and 0 in the subgame where S makes the offer. The expected utility is $Eu_B = 0.5\sqrt{3} + 0.5\sqrt{0} = 0.5\sqrt{3}$.
- Thus, in the first round, S offers to give her the quantity $3 - x_1$ such that $\sqrt{3 - x_1} = 0.5\sqrt{3}$, i.e.: $3 - x_1 = 0.75$, so he offers $x_1 = 2.25$.
- Being risk averse reduces payoffs when uncertainty is present.

Application: Coase theorem

- A physician is disturbed by the machines used by a baker working in a neighbouring store.
- The physician is currently earning 30, but she could make 70 if the baker goes somewhere else.
- She could also leave, but in this case she would earn 50 (including the costs of moving).
- The baker is currently earning 50 and would make 40 if he should leave (including the costs of moving).

		Baker: Stays	Baker: Leaves
Physician	Stays	30, <u>50</u>	<u>70</u>, 40
	Leaves	<u>50</u>, <u>50</u>	50, 40

- In the absence of an agreement, in the equilibrium, the physician stays and the baker leaves.
- However, total earnings are maximized if the baker leaves and the physician stays.

Application: Coase theorem

		Baker: Stays	Baker: Leaves
Physician	Stays	30, 50	70, 40
	Leaves	50, 50	50, 40

- Case 1: After suing, a judge rules that the baker has the right to stay in the building.
- The baker can negotiate a payment x to agree to leave.
  - For the baker to agree, it must be $40 + x \geq 50$.
  - For the physician to accept, it must be $70 - x \geq 50$.
  - Thus, there exists room for an agreement : $x \in [10, 20]$.
- In a negotiation with one round where the baker makes the offer, in the SPNE, the baker offers $x = 20$ and the physician accepts all $x \leq 20$ and rejects any $x > 20$. The baker leaves. Payoffs are (50, 60).
- In negotiations with more rounds and with discount, the equilibrium offer will be a quantity $x \in (10, 20)$.
- In any case, the baker leaves.

Application: Coase theorem

		Baker: Stays	Baker: Leaves
Physician	Stays	30, 50	70, 40
	Leaves	50, 50	50, 40

- Case 2: After suing, a judge rules that the physician has the right to ask the baker to leave.
- The physician can negotiate a payment x for not exercising her right.
  - For the physician to agree, it must be $50 + x \geq 70$.
  - For the baker to accept, it must be $50 - x \geq 40$.
  - Now there is no room for agreement: $x \geq 20, x \leq 10$.
- The bake will leave. Payoffs are (70, 40).
- Giving the right to the physician or to the baker does not alter the efficiency, but does alter how the profits are shared: (50, 60) in Case 1 vs (70, 40) in Case 2.
- This result, in its more general version, is known as Coase theorem.
- The theorem is not satisfied if negotiation costs are too big (e.g., when there are many people involved).

The hijacking of the Alakrana

Diez años del 'Alakrana': un rescate polémico y más seguridad

iez-anos-alakrana-20191002214550-ntrc.html#:~:text=Han pasado diez años desde,costa del sur de Somalia.

La Audiencia Nacional dictaminó que el Gobierno pagó los 2,5 millones de la liberación del barco vasco, en el que había 16 marineros españoles

El 'Alakrana', seguido de los piratas, momentos antes del secuestro. / REUTERS

ÁLVARO SOTO
Madrid
Miércoles, 2 octubre 2019, 06:33

The hijacking of the Alakrana

- On Oct 2nd, 2009 some Somalian pirates hijack the Basque tuna fishing boat Alakrana and demand a millionaire ransom.
- The Spanish government negotiates under pressure by families and press.
- The government sends the frigate Canarias, but cannot free the boat.
- The capture two pirates when they were about to reach land.
- Questions: how do the following circumstances affect the outcome?
  - Pressure by families and press.
  - Sending the frigate.
  - Capturing the two pirates.