Dynamic Games

4. Economic Applications

Universidad Carlos III de Madrid

Economic Applications

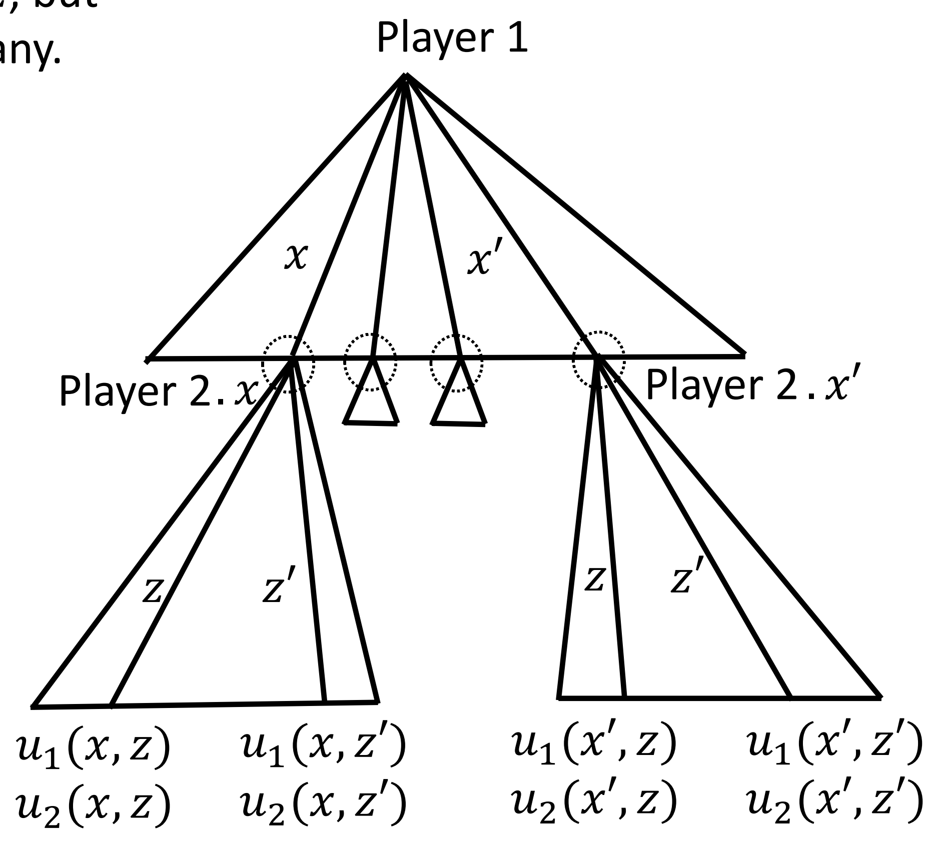
- Dynamic games of perfect information.
  - Sequential competition in quantities: Stackelberg.
  - Unions and firms: Negotiations.
  - Others: Contribution to a public good, price competition,…
- Dynamic games of imperfect information.
  - Voting Games:
    - Sincere or strategic voting.
    - Agenda manipulation.

Dynamic games of perfect information. and continuous variable

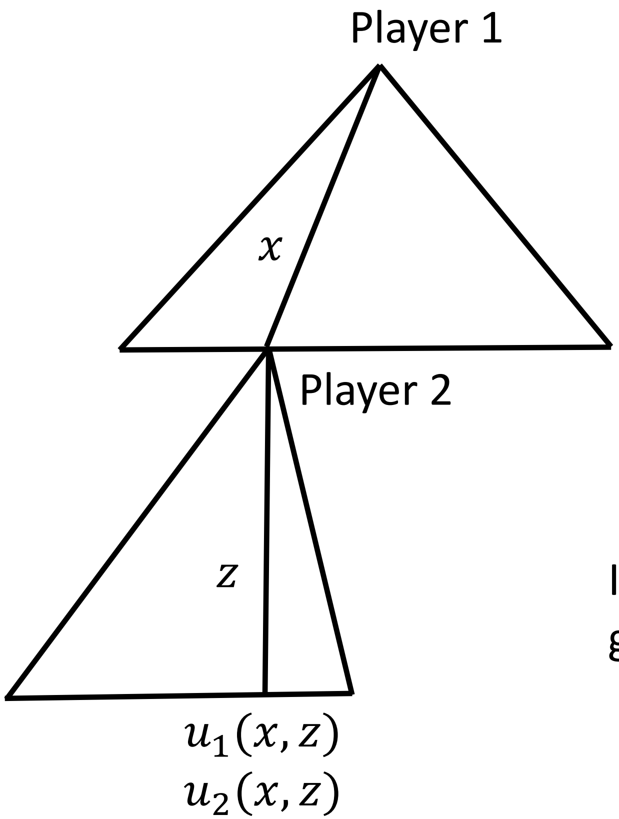
- Two players. Player 1 must choose x, $x \geq 0$, and Player 2 must choose z, $z \geq 0$.
- Payoffs are: $u_1(x, z)$ and $u_2(x, z)$.
- Player 1 chooses first, and her choice will be known by 2 before choosing z.
- Player 1 has one information set. Her strategy is a number x.
- Player 2 has infinitely many information sets (one for each possible value of x).
- A strategy for Player 2 must contain infinitely many values. Thus, it is a function: $z = f(x)$. The set of strategies is the set of possible functions.

Sketch of the extensive form with continuous variable

We only show two values for x and two values for z, but there are infinitely many.

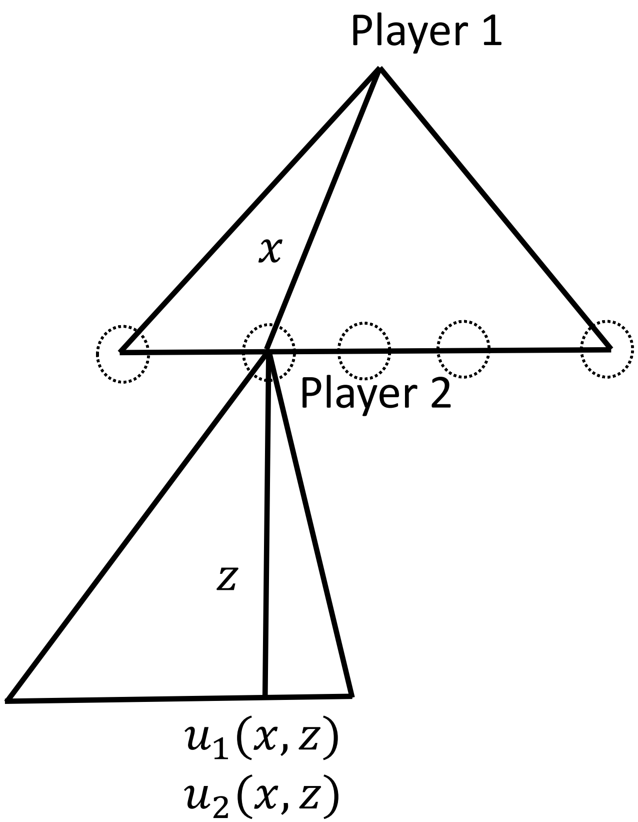


Sketch of the extensive form with continuous variable

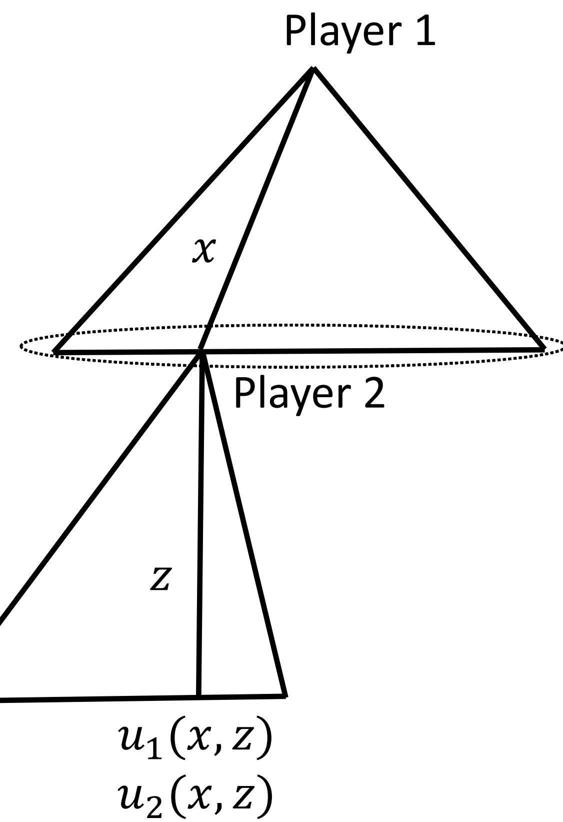


In fact, we usually just show generic actions x and z.

Sketch of the extensive form with continuous variable



Perfect information



Simultaneous

SPNE: solution

- By backward induction, start with Player 2. He will choose z to maximize his profits given the value of x:

$$\max_{z \geq 0} u_2(x, z)$$

- From the first order conditions we obtain the reaction function (or best reply function): $z = f(x)$.
- Substitute this reaction function by 2 in the expression for 1's payoff.
- Player 1 will choose the value x that maximizes her profits:

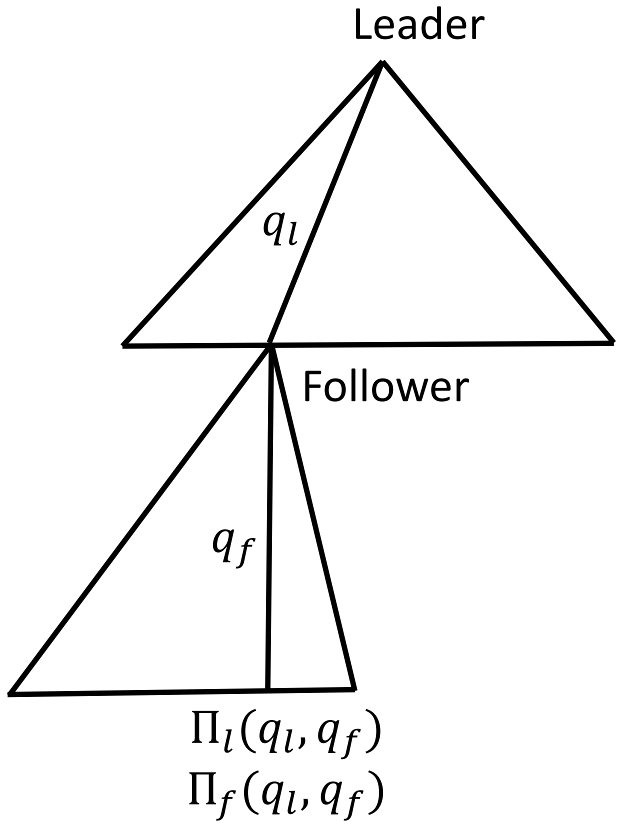
$$\max_{x \geq 0} u_1\big(x, f(x)\big)$$

- Solving the above problem we find x^* as a solution.
- The SPNE is $\big(x^*, f(x)\big)$.

Stackelberg model

- 2 firms produce a homogeneous good.
- Linear demand: $p = a - Q$.
- Constant marginal cost is equal for both firms: $c < a$.
- Decision sequence:
  - Firm 1 is Leader and decides its production q_l.
  - Firm 2 is Follower: observes the leader's output and then decides its own q_f.
  - $Q = q_l + q_f$.
- The difference with Cournot's model is that decisions are made sequentially and not simultaneously.

Stackelberg model



Stackelberg model

- Information sets:
  - The leader has one information set.
  - The follower has infinitely many, one in each node after observing q_l.

- Subgames: Besides the whole game, there are infinitely many subgames, one for each possible value of q_l.

- Strategies:
  - For the leader, one strategy is an action: a value for q_l.
  - For the follower, a strategy must have infinitely many elements, one for each one of its information sets. Its strategy is thus a function: $q_f = f(q_l)$.

Stackelberg model

- To compute the set of SPNE we solve backwards, starting with the last subgame where the follower produces the quantity that maximizes its profits given the quantity previously chosen by the leader :

$$\max_{q_f \geq 0} \Pi_f = (p(Q) - c)q_f = \left(a - q_l - q_f - c\right)q_f$$

- After computing the first order condition we find the best reply function:

$$\frac{\partial \Pi_f}{\partial q_f} = a - q_l - 2q_f - c = 0$$

$$q_f = BR_f(q_l) = \max\left\{0, \frac{a - c - q_l}{2}\right\}$$

- Notice that its best reply function is identical to the one we obtain in the static Cournot game.

Stackelberg model

- Given the follower's best reply, the leader maximizes its profits (it anticipates that best reply and takes it into account when solving its own maximization problem). In other words, we are moving backwards to the first stage of the game:

$$\max_{q_l \geq 0} \Pi_l = (p(Q) - c)q_l = (a - q_l - q_f - c)q_l,$$

subject to: $q_f = \frac{a-c-q_l}{2}$.

- Substitute the restriction in the objective function

$$\max_{q_l \geq 0} \Pi_l = (p(Q) - c)q_l = \left(a - q_l - \frac{a-c-q_l}{2} - c\right) q_l = \frac{a-c-q_l}{2} q_l.$$

- Calculate the optimal quantity for the Leader using the first order condition:

$$\frac{\partial \Pi_l}{\partial q_l} = \frac{a-c-2q_l}{2} = 0,$$

$$q_l = \frac{a-c}{2}.$$

- The only SPNE is: $\left(\frac{a-c}{2}, \max\left\{0, \frac{a-c-q_l}{2}\right\}\right)$.
- For better visualization write: $\left(q_l = \frac{a-c}{2}, q_f = \max\left\{0, \frac{a-c-q_l}{2}\right\}\right)$.

Stackelberg model

- SPNE: $\left(q_l = \frac{a-c}{2}, q_f = \max\left\{0, \frac{a-c-q_l}{2}\right\}\right)$.
- Equilibrium path: $\left(q_l = \frac{a-c}{2}, q_f = \frac{a-c-\frac{a-c}{2}}{2}\right) = \left(q_l = \frac{a-c}{2}, q_f = \frac{a-c}{4}\right)$.
- Equilibrium payoffs:

$$\Pi_l = \left(a - \frac{a-c}{2} - \frac{a-c}{4} - c\right)\frac{a-c}{2} = \frac{1}{8}(a-c)^2,$$

$$\Pi_f = \left(a - \frac{a-c}{2} - \frac{a-c}{4} - c\right)\frac{a-c}{4} = \frac{1}{16}(a-c)^2.$$

- Even though the firms have the same technology, the Leader makes more profits than the Follower.

- Compare Stackelberg (St) with Cournot (Co):

$$q_l = \frac{a-c}{2} > q_i^{Co} = \frac{a-c}{3}, \qquad q_f = \frac{a-c}{4} < q_i^{Co} = \frac{a-c}{3}.$$

$$Q^{St} = \frac{3}{4}(a-c) > Q^{Co} = \frac{2}{3}(a-c),$$

$$p^{St} = \frac{1}{4}(a-c) + c < p^{Co} = \frac{1}{3}(a-c) + c.$$

$$\Pi_l = \frac{1}{8}(a-c)^2 > \Pi_i^{Co} = \frac{1}{9}(a-c)^2,\ \ \Pi_f = \frac{1}{16}(a-c)^2 < \Pi_i^{Co} = \frac{1}{9}(a-c)^2.$$

The advantage of moving first … or second

- The advantage of moving first:
  - In the Stackelberg example the reaction function of the Follower is $q_f = \frac{a-c-q_l}{2}$, where the quantity depends negatively on the quantity chosen by the Leader.
  - In general this occurs when decision variables are strategic substitutes.
- The advantage of moving second:
  - In general this occurs when decision variables are strategic complements.
  - For instance, price competition with heterogenous goods as seen in static games, where one firm's price depends positively on the rival's: $p_i = BR_i(p_j) = \frac{a+c+bp_j}{2}$.

Collective negotiation

- Let's have an economy with a Union and a Firm. The Union is the only provider of labor, and has exclusive power over the salary w. The Firm, on the other hand, decides how much labor L to hire.
- The Union's objective is to maximize total wage income wL.
- The Firm uses only labor as an input. It chooses L to maximize profits:

$$\max_{L} F(L) - wL$$

- Say the value of production is $F(L) = 8L - \frac{L^2}{2}$.
- The game has the following timing:
  1. The Union chooses salary w.
  2. The Firm observes w and then chooses L.

Collective negotiation

- Solve by backwards induction. Start with the Firm:

$$\max_L 8L - \frac{L^2}{2} - wL$$

- From the first order conditions we find $8 - L - w = 0$,
- and then $L(w) = 8 - w$.
- Substitute the Firm's reaction function in the Union's objective function:

$$\max_w wL = w(8 - w)\,.$$

- First order conditions : $8 - 2w = 0$, and then: $w = 4$.
- SPNE: $(4, 8 - w)$.
- Equilibrium path: $(w = 4, L = 4)$.
- Equilibrium payoffs: $U = 16$, $\Pi = 8$.

Voting

- Suppose that in a parliamentary committee there are three proposals: *A*, *B* and *C.*
- The committee has three members: 1, 2 and 3.
- The voting rules are as follows:
  - In a first round, the committee chooses between *A* and *B* in a simultaneous vote.
  - Then, the vote is between the winner in the previous round and *C*.
  - The overall winner is the option with most votes in this second round.
- The preferences of the committee members are:
  - Member 1: $A \succ C \succ B$
  - Member 2: $B \succ A \succ C$
  - Member 3: $C \succ A \succ B$
- To simplify the problem, assume that the best option gives a utility of 3, the second best, 2, and the last, 1.

Voting

- We are dealing with a dynamic game with 9 subgames: the 8 subgames that start after the first stage (one after each possible combination of votes: *AAA*, *AAB*, *ABA*, *ABB*, *BAA*, *BAB*, *BBA*, *BBB*) and the whole game.
- A strategy for each player must have 9 elements: it must show what to do in stage 1 and what to do in each subgame.
- Find first the *NE* in the 8 subgames of the second stage. Then, replace the subgames with the payoffs in the *NE* and compute the *NE* of the resulting game.

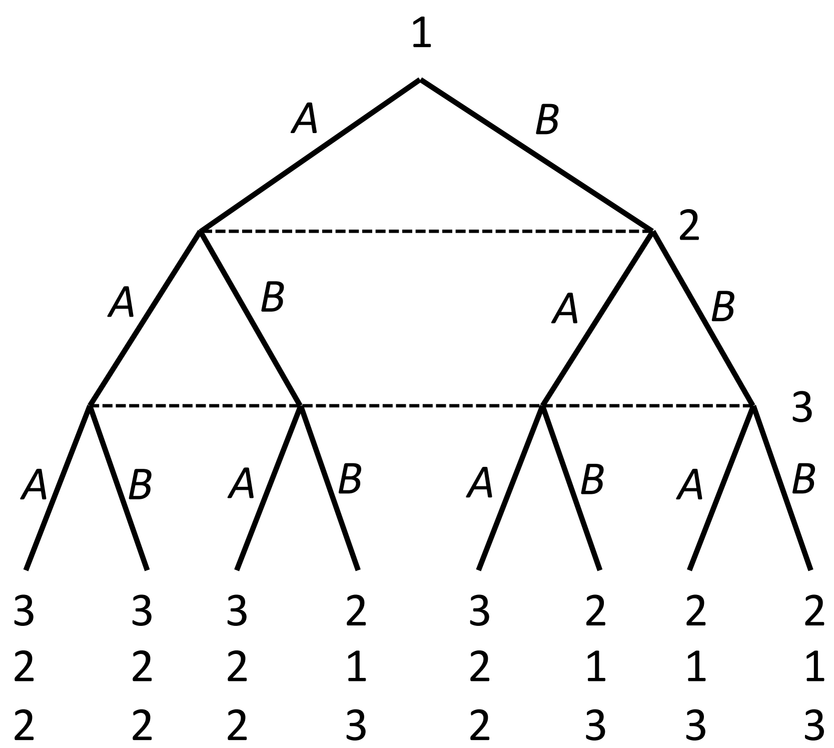
Voting

- Subgames in the second stage.
  - There are two types: subgames after *A* won and subgames after *B* won:
  - *A* wins after *AAA, AAB, ABA, BAA* in the first round.
  - *B* wins after *ABB, BAB, BBA, BBB* in the first round.

- NE in the subgames:
  - In subgames after *A* won.
    - Voters have two actions each: vote *A* or *C*.
    - Assume players don't use weakly dominated strategies, or, equivalently, that they vote for their preferred option, then the *NE* is: (*A,A,C*).
    - Utilities are (3, 2, 2).
    - Note that (*A,A,A*) is also an *NE*, but that Player 3 is using a weakly dominated strategy.
  - In subgames after *B* won.
    - Voters have to actions: vote for *B* or *C*.
    - Assume again that in case of indifference, they vote their most preferred option, the *NE* is: (*C,B,C*).
    - Utilities are (2, 1, 3).

Voting

Substitute subgames with their *NE* payoffs.



NE in undominated strategies: (*A*, *A*, *B*).

Voting

- The other NE are (*A, A, A*) and (*B*, *B*, *B*), but notice that in both of them there is at least one player that is using a weakly dominated strategy.
- Strategic voting in the NE (*A*, *A*, *B*).
  - Note that Player 2 votes *A* in the first stage even thought he prefers *B* before *A*:
    - This is because he knows that, in the final round, if the vote is between *B* and *C*, the winner will be *C*, that is worse than *A*.
    - In other words: a vote for *B* in the first round is to all effects a vote for *C*.
  - In a similar vein, Player 3 is voting for *B* in the first round even though she prefers *A*.
- The next strategies constitute a SPNE:
  - s_1 = (*A*, *A* if *A* wins, *C* if *B* wins),
  - s_2 = (*A*, *A* if *A* wins, *B* if *B* wins),
  - S_3 = (*B*, *C* if *A* wins, *C* if *B* wins).
- Equilibrium path: ((*A*, *A*, *B*), (*A*, *A*, *C*)).
- Equilibrium payoffs: (3, 2, 2), which are the utilities when A wins.

Agenda manipulation

- Consider the same voting problem with two rounds, but now with preferences:
  - Member 1: $A \succ B \succ C$
  - Member 2: $B \succ C \succ A$
  - Member 3: $C \succ A \succ B$
- If the vote is, as before, between *A* and *B* in the first round, which one wins?
  - Observe that in the second round, between *A* and *C*, *C* wins, and between *B* and *C*, *B* wins. A vote for *A* is a vote for *C*.
  - In the first round, members who prefer *C* before *B* will vote for *A* (member 3), and members who prefer *B* before *C* will vote for *B* (members 1 and 2).
  - Thus, *B* wins in the first round and also in the second round.
- If the president of the committee belongs to the same party of member 1 (and then has the same preferences), she can change the voting order and make *A* vs *C* be the vote in the first round.
  - If *A* wins in the first round, it will also win in the second against *B*. If *C* wins, *B* is chosen in the second round.
  - A vote for *C* is a vote for *B*.
  - In the first round, members who prefer *A* before *B* will vote for A (members 1 and 3), and members who prefer *B* before *A* will vote for *C* (member 2).
  - *A* wins in the first and second rounds.