Dynamic Games

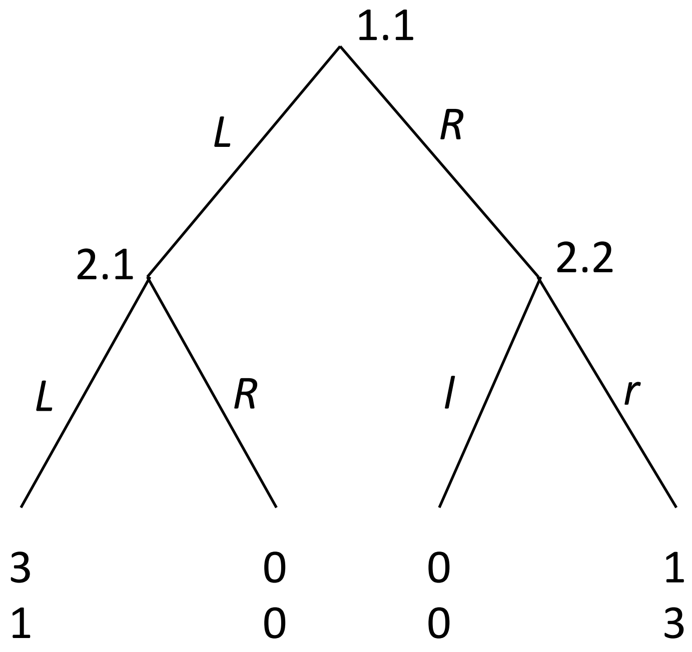
2: Imperfect information

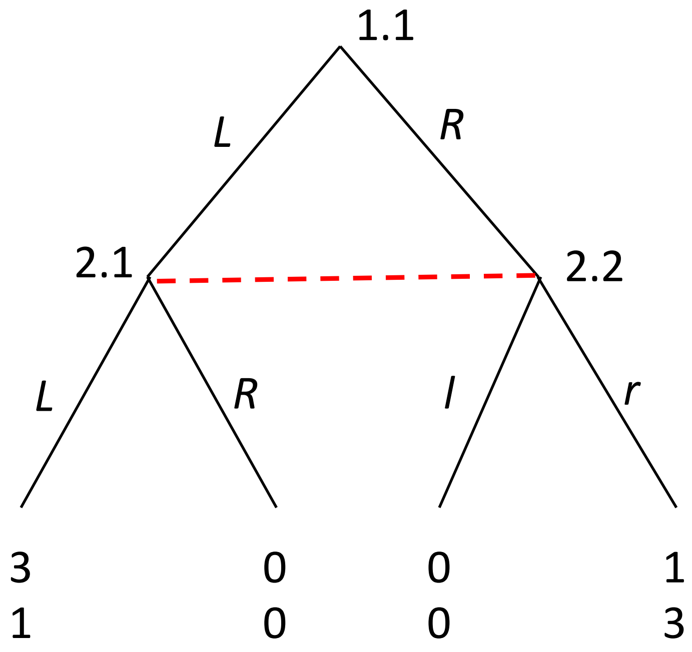
Universidad Carlos III de Madrid

Dynamic Games with Imperfect Information

- Games in which at least one of the following happens:
  - A player does not know which action some other player has taken.
  - Some players have different information over a result of a nature move.
- This translates into the fact that some players don't know with certainty in which one of their nodes actually are at some point in the game.
- The nodes a player cannot tell apart are nodes in which the player has the same information. Each set of nodes in which this occurs is called an information set.
- Trivially, when a player knows that she is a node, that node is an information set of one element.
- Graphically, we will join the nodes belonging in an information set with a dotted line or a "cloud".

Example: dynamic battle of the sexes

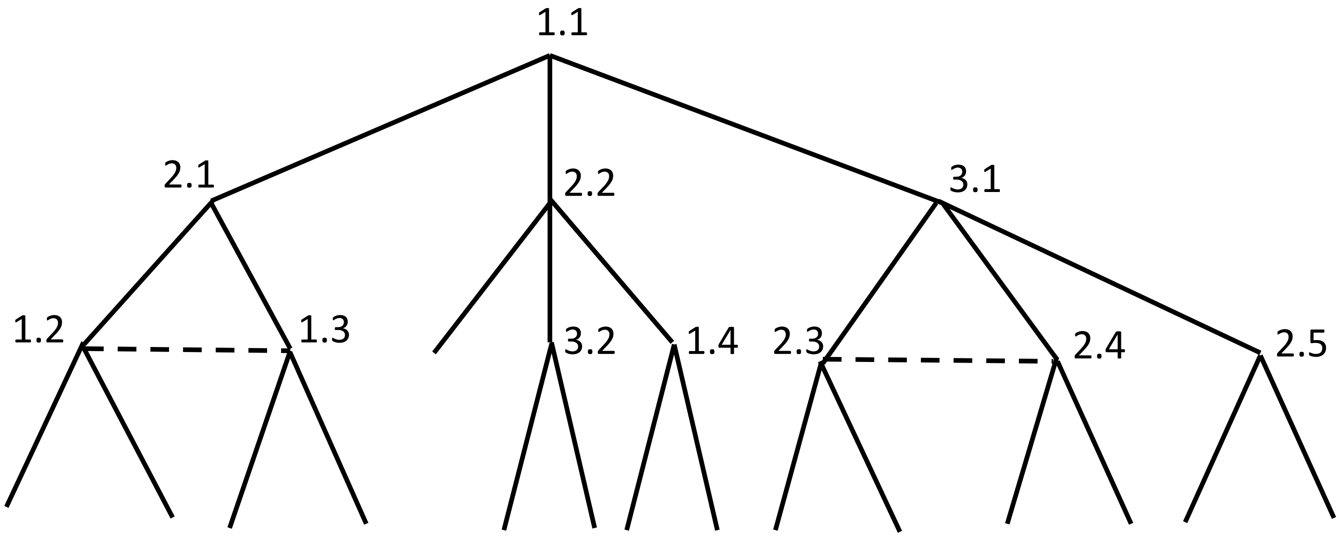




- Player 2 knows what player 1 did.

- Player 2 does not know what player 1 did.
- Nodes 2.1 y 2.2 belong in an information set.
- There is no backward induction equilibrium, but there are SPNE.

A complicated example

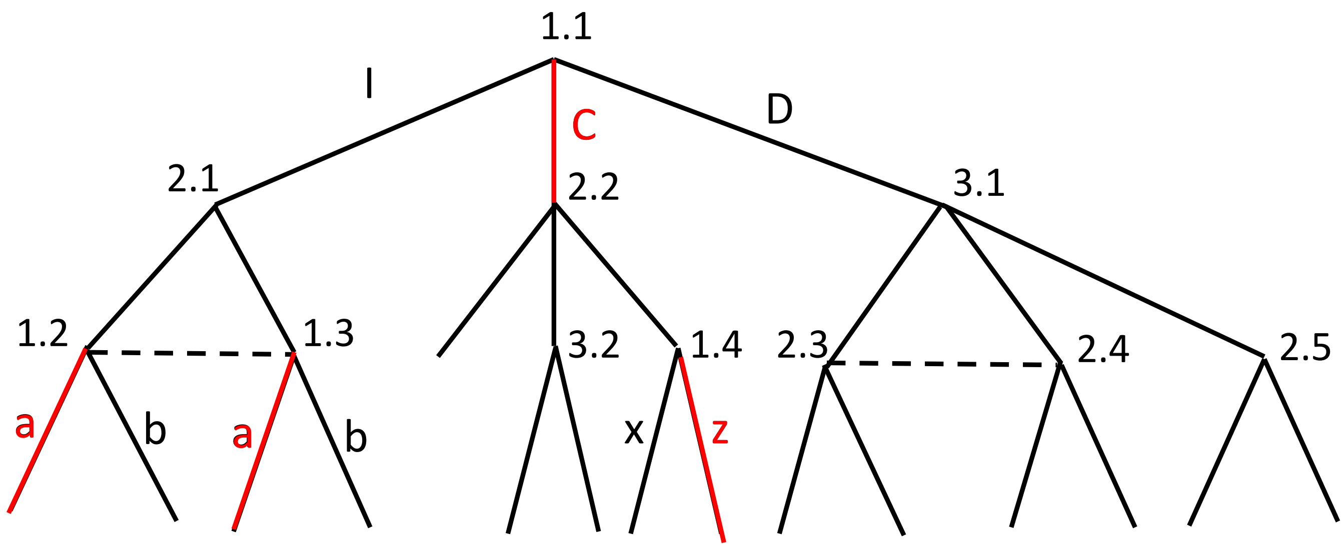


- Information sets:
  - Player 1: {1.1}, {1.2, 1.3} y {1.4} .
  - Player 2: {2.1}, {2.2}, {2.3, 2.4} y {2.5}.
  - Player 3: {3.1} y {3.2}.
- Static and dynamic subgames:
  - At 2.1 begins a static (sub)game.
  - At 2.2 begins a dynamic (sub)game.
  - At 3.1 begins a (sub)game with characteristics of both kinds of game.

Extensive form, normal form and subgames

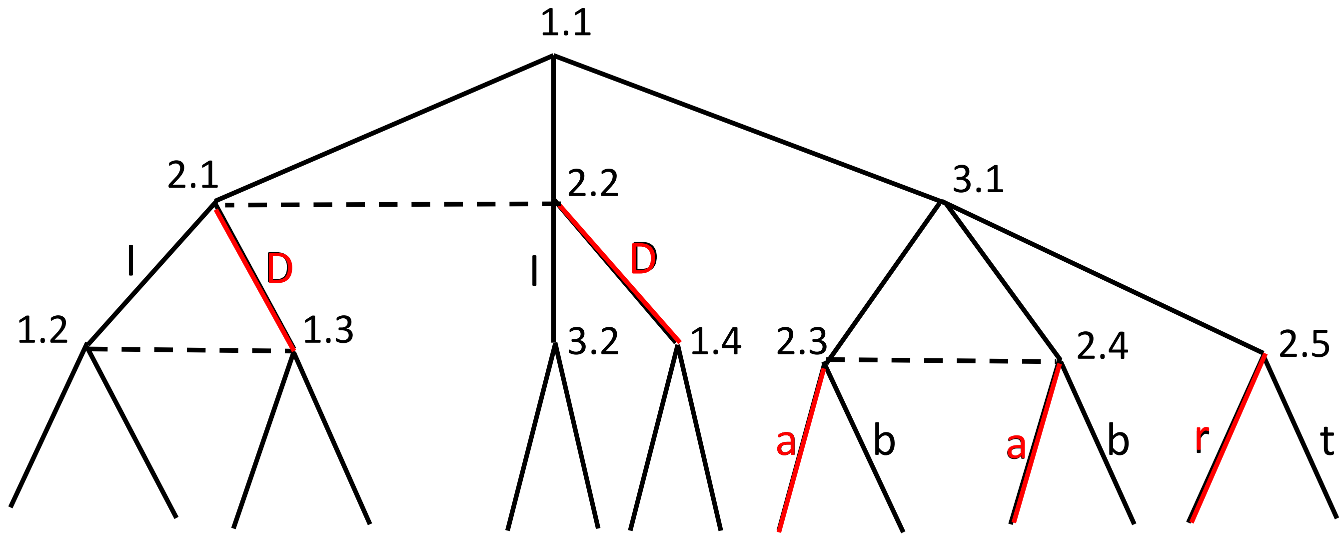
- We have to add or change the following in the extensive form definition for games of imperfect information:
  - Group the nodes of a player in information sets.
  - Define actions in each information set (not in each node): informally, an action implies choosing the same edge in each node of a given information set.
- To obtain the normal form, it is enough to define a player' strategy as a vector that defines an action in each information set (rather than in each node).
- Subgames are defined as before, but with a new rule "do not break information sets".

A complicated example



- Which subgames are there?
  - Those starting at 1.1, 2.1, 2.2, 3.1, 3.2, 1.4 and 2.5.
- Which is the set of strategies for Player 1?
  - {(I,a,x), (I,a,z), (I,b,x), (I,b,z), (C,a,x), (C,a,z), (C,b,x), (C,b,z), (D,a,x), (D,a,z), (D,b,x), (D,b,z)}.
  - Example: (C,a,z) in red.

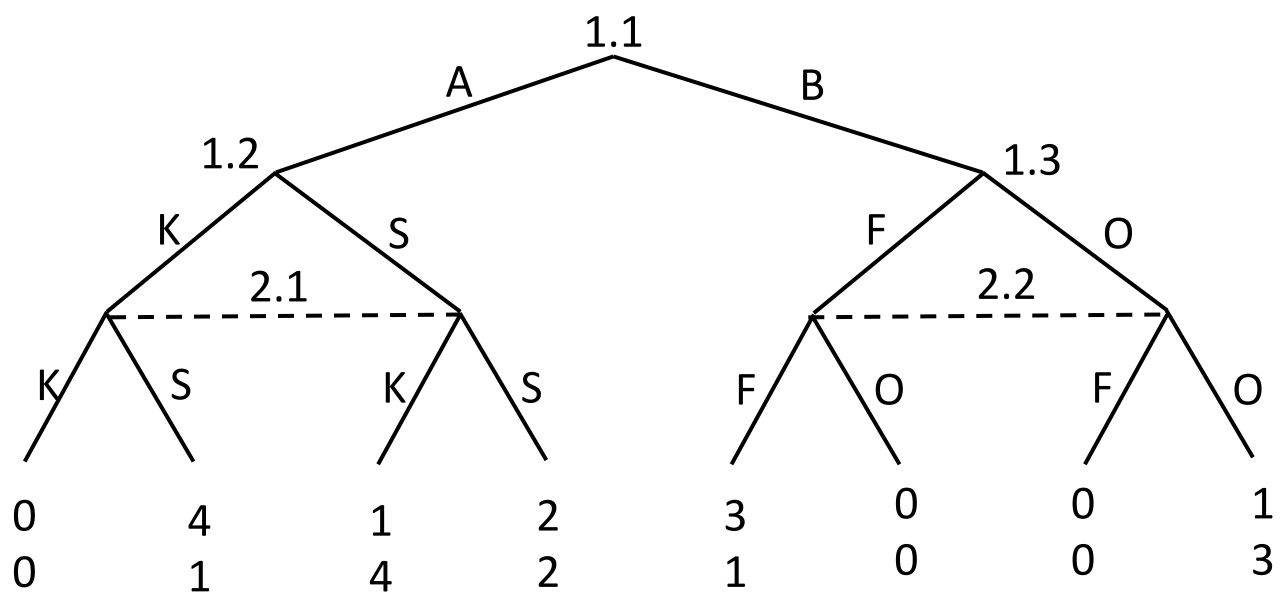
Complicated example 2



- Which subgames are there?
  - Those starting at 1.1, 3.1, 3.2, 1.4 y 2.5.
- Which is the set of strategies for Player 2?
  - {(I,a,r), (I,a,t), (I,b,r), (I,b,t), (D,a,r), (D,a,t), (D,b,r), (D,b,t)}.
  - Example: (D,a,r) in red.

Example to find SPNEa

- Player 1 chooses between A and B.
- If he chooses A, he and Player 2 play the chicken game.
- If he chooses B, they play the battle of the sexes.



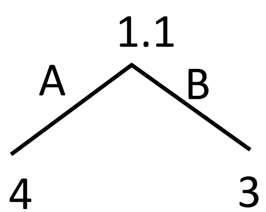
- We have numbered the information sets (rather than the nodes).
- Which subgames are there?

    Three, starting at 1.1, 1.2 and 1.3.
- Begin by solving 1.2 and 1.3.

Example to find SPNEa

- To simplify, we only consider pure strategies.
- The subgame starting at 1.2 is the chicken game with NE in pure strategies: (K, S) and (S, K).
- The subgame starting at 1.3 is the battle of the sexes with NE in pure strategies: (F, F) and (O, O).
- To find the equilibrium action at 1.1, we must consider four possibilities:

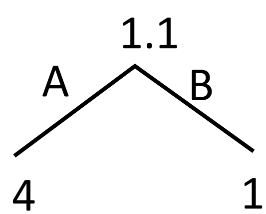
1.2: (K, S)
1.3: (F, F)



1.1 prefers A

SPNE:
((A,K,F), (S,F))

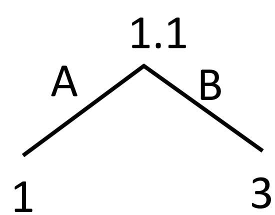
1.2: (K, S)
1.3: (O, O)



1.1 prefers A

SPNE:
((A,K,O), (S,O))

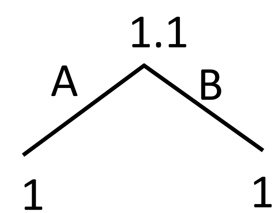
1.2: (S, K)
1.3: (F, F)



1.1 prefers B

SPNE:
((B,S,F), (K,F))

1.2: (S, K)
1.3: (O, O)



1.1 is indifferent

SPNEa:
((A,S,O), (K,O)) and
((B,S,O), (K,O))

Two ways to write the SPNEa

- The canonic way: sort by players.
- The convenient way: sort by subgames.
- In the example before, the equilibrium ((A,K,F), (S,F)) is written in the canonic way.
- The convenient way is: (A, (K,S), (F,F)).

By players:
((1.1, 1,2, 1.3), (2.1, 2.2))
((A , K , F), (S , F))

By subgames:
(1.1, (1,2, 2.1), (1.3, 2.2))
(A , (K , S), (F , F))

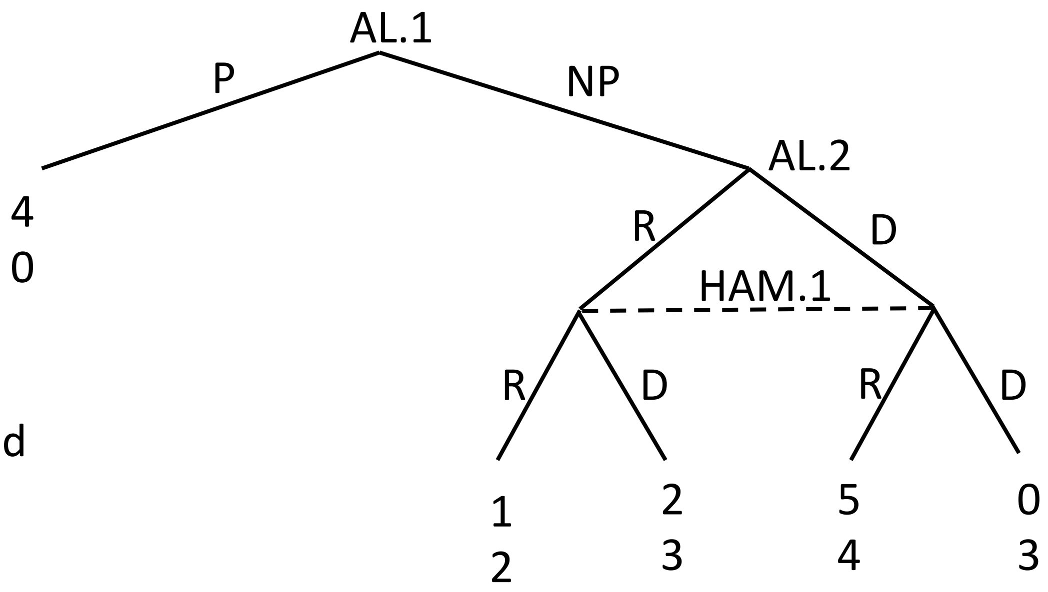
Example 2 on how to find ENPS

Formula 1 Game

- Before deciding what type of tires to use, Al can make a strategic maneuver that would prevent Ham from participating in the race.
- Thus, in a first stage, Al must decide whether to prevent or not Ham’s participation in the race (decisions P and NP).
- If Al prevents the participation of Ham, Al will have 4 points at the end of the race, and Ham will have none.
- If Al does not prevent Ham’s participation, both pilots must choose simultaneously the type of tires (rain or dry), with the results shown next.

Example 2 on how to find ENPS

Formula 1 Game

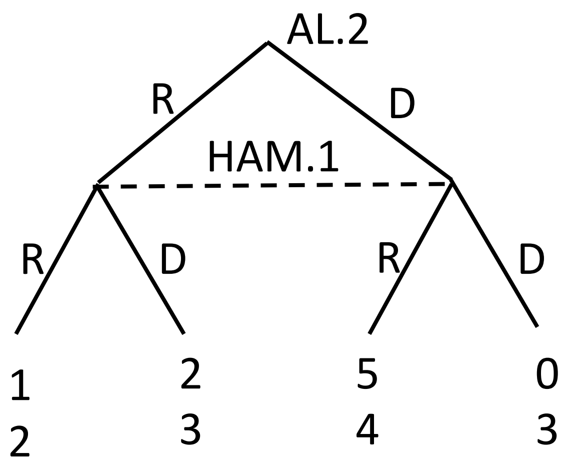


Subgames:
starting at AL.1 and
starting at AL.2

Example 2 on how to find ENPS

Formula 1 Game

- Start by solving subgame at AL.2 after NP:



The normal form is:

		HAM.1	
		R	D
AL.2	R	1, 2	2, 3
	D	5, 4	0, 3

- NE = {(D, R), (R, D), (1/2[R]+1/2[D], 1/3[R]+2/3[D])}
- Payoffs in NE of subgame after NP for AL: 5, 2 and 5/3, respectively.
- If AL.1 plays P he will get 4. Thus, if at the subgame after NP the NE is (D, R), he will choose NP. For any other NE he will choose P.
- Hence: SPNE : {((NP,D), R), (P,R), D), ((P,1/2), 1/3)}.