I	II.1	II.2	II.3	II.4	Total

Game Theory
Exam June 2021

Name: **Group:**

You have two and a half hours to complete the exam. No calculators or electronic devices are permitted.

I Short questions (5 points each)

I.1 When a player chooses a mixed strategy in an equilibrium, the pure strategies that are part of it may give her different payoffs. True or false?

False. If one of the pure strategies gives a higher payoff than some other one, this last strategy would not be used in the equilibrium.

I.2 Give an example of a game that shows a first-mover advantage.

Stackelberg (see class notes).

I.3 Give an example of a game that shows a last-mover advantage.

Dynamic Bertrand with differentiated goods (see class notes).

I.4 In a finitely repeated game, the only subgame perfect equilibria consist on the repetition of the Nash equilibria of the stage game. True or false?

False. No NE outcomes may be sustained in the first periods (see class notes).

II. Problems (20 points each)

II.1 There are three firms operating in a market, where they compete a la Cournot. All firms have the same technology given by the cost function $c_i(q_i) = 10q_i$ ($i \in \{1,2,3\}$). The market demand is $Q = 130 - p$.

(a) (7 points) Find the equilibrium. Find also the market price and the firms' profits.

Suppose now that firms 1 and 2 are considering a merge, forming the firm 12, that will operate with the same cost function, $c_{12}(q_{12}) = 10q_{12}$. As a reputed game theorist, they hire you as a consultant.

(b) (7 points) Will you advise in favor or against the merge?

(c) (6 points) Will you change your advice if you happen to know that Firm 3 has a capacity limit and cannot produce more than 34 units?

(a) $\underset{q_1}{max}(130 - q_1 - q_2 - q_3)q_1 - 10q_1$

F.O.C. gives $q_1 = \frac{120-q_2-q_3}{2}$.

Similarly for Firms 2 and 3. Solving the equations formed by F.O.C.: $q_1 = q_2 = q_3 = \frac{120}{4} = 30$.

Market price is $p = 130 - 90 = 40$.

Individual profits are $\Pi_i = 30 \times (40 - 10) = 900$.

(b) $\underset{q_{12}}{max}(130 - q_{12} - q_3)q_{12} - 10q_{12}$

F.O.C. gives $q_{12} = \frac{120-q_3}{2}$.

Similarly for Firm 3. Solving the equations formed by F.O.C.: $q_{12} = q_3 = \frac{120}{3} = 40$.

Market price is $p = 130 - 80 = 50$.

Individual profits are $\Pi_i = 40 \times (50 - 10) = 1600$.

Firm 12's profits will be 1600, while joint profits in (a) where $\Pi_1 + \Pi_2 = 1800$.

The merge is not advisable.

(c) If Firm 3 has a capacity of 34, it will not be able to produce 40 as required by the equilibrium in (b), it will produce at its capacity limit of 34 (corner solution). Given this, Firm 12 will produce according to the best reply function: $q_{12} = \frac{120-34}{2} = 43$.

Market price is $p = 130 - 43 - 34 = 53$.

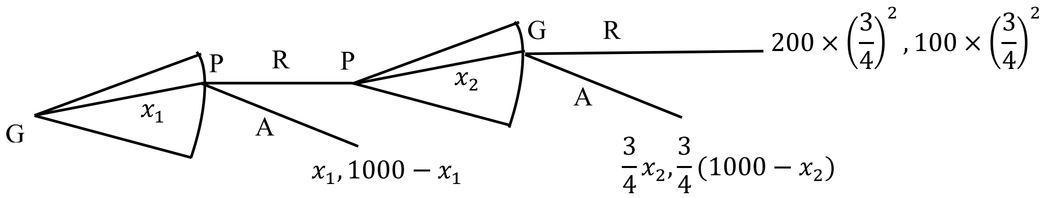
Firm 12 profits are $\Pi_{12} = 43 \times (53 - 10) = 1849 > 1800$.

Now the merger gives higher profits and is advisable.

II.2 Guillermo and Pedro are owners of a family business and have to negotiate on how to split the earnings of the month, €1000. Guillermo, who is the founder of the firm, first makes an offer to split the earnings with Pedro. If Pedro accepts, the division is made according to Guillermo's offer, but if Pedro rejects the offer, he can make a counter offer the next day. Guillermo has the opportunity of accepting or rejecting the offer in the same day. If he accepts, they split the money accordingly, but if Guillermo rejects he obtains €200 euros the next day, while Pedro obtains €100, also next day, and the rest of the money is reinvested. Both owners are impatient to obtain the money and have a discount factor of $\delta = \frac{3}{4}$ per day. To simplify matters, assume that, in their impatience, they do not care about the future earnings after the reinvestment.

(a) (6 points) Draw the dynamic game.
(b) (14 points) Find the subgame perfect Nash equilibrium.

(a)



(b) Second stage: G accepts any x_2 such that $\frac{3}{4}x_2 \geq 200 \times \left(\frac{3}{4}\right)^2$. This gives any $x_2 \geq 150$

Anticipating this, P offers $x_2 = 150$.

First stage: P accepts any x_1 such that $1000 - x_1 \geq \frac{3}{4}(1000 - 150)$. This gives any $x_1 \leq 362.5$.

Anticipating this, G offers $x_1 = 362.5$.

II.3 Consider the dynamic game that consists on playing the following game n times.

		Player B		
		C	N	P
Player A	C	8, 8	1, 11	0, 0
	N	11, 1	3, 3	0, 0
	P	0, 0	0, 0	1, 1

(a) (7 points) How many periods can (C, C) be played in a subgame perfect Nash equilibrium using pure strategies if $n = 2$?
(b) (6 points) Repeat (a) if $n = 3$.
(c) (7 points) Repeat (a) if $n = 4$.

(a) Try the trigger strategy to sustain the play of (C, C) once, in the first stage.
First stage: Play (C, C).
Second stage: Play (N, N) if (C, C) was played in the first stage.
Play (P, P) if (C, C) was not played in the first stage.

Is it a SPNE? If players follow the strategy, they get $8 + 3 = 11$ each. If one deviates in the first stage (to N), the deviator gets $11 + 1 = 12$. The strategy is not a SPNE. No other strategy can do better to sustain (C, C) in the first stage. (C, C) cannot be played in any stage of a SPNE if $n = 2$.

The play of (C, C) cannot be sustained in the second (and last) stage, as it is not a NE of the stage game.

(b) Try the trigger strategy to sustain the play of (C, C) once, in the first stage.
First stage: Play (C, C).
Second and third stages: Play (N, N) if (C, C) was played in the first stage.
Play (P, P) if (C, C) was not played in the first stage.

Is it a SPNE? If players follow the strategy, they get $8 + 3 + 3 = 14$ each. If one deviates in the first stage (to N), the deviator gets $11 + 1 + 1 = 13$. There is no better deviation (in stages 2 and 3 the strategy demands to play NEa. The strategy is a SPNE. (C, C) can be played in one stage (the first) of a SPNE if $n = 3$.

The play of (C, C) cannot be sustained in the second stage, as the game after the first stage is exactly the same game as in (a), where (C, C) cannot be sustained in any stage. This means that it cannot be sustained in the second and third stages of the game repeated three times.

(c) Try the trigger strategy to sustain the play of (C, C) twice, in the first and second stages.
First stage: Play (C, C).
Second stage: Play (N, N) if (C, C) was played in the first stage.
Play (P, P) if (C, C) was not played in the first stage.
Third and fourth stages: Play (N, N) if (C, C) was played in the first and second stages.
Play (P, P) if (C, C) was not played in the first or second stage.

Is it a SPNE? If players follow the strategy, they get $8 + 8 + 3 + 3 = 22$ each. If one deviates in the first stage (to N), the deviator gets $11 + 1 + 1 + 1 = 14$. There is a better deviation: deviate in the second stage (to N), then the deviator gets $8 + 11 + 1 + 1 = 21$. There is no better deviation (in stages 2 and 3 the strategy demands to play NEa. The strategy is a SPNE. (C, C) can be played in two stages (the first and the second) of a SPNE if $n = 4$.

The play of (C, C) cannot be sustained in the third or four stages, as the game after the second stage is exactly the same game as in (a), where (C, C) cannot be sustained in any stage. This means that it cannot be sustained in the third and fourth stages of the game repeated four times.

II.4 In the pharmaceutical market, COLMEN Inc. can choose between expanding the plant, E, and not expanding it, NE. Profits after the choice depend on whether the competitor GETAFE Corp. innovates its product, I, or not, NI. COLMEN Inc. knows its own costs, low or high, in case it decides to expand, but GETAFE Corp. does not know those costs, but knows that the probability that they are low is p, $0 \leq p \leq 1$. Finally, firms must make their decisions simultaneously and payoffs are the following:

Low costs

		GETAFE Corp. I	GETAFE Corp. NI
COLMEN Inc.	E	2, 10	20, 5
	NE	0, 20	10, 10

High costs

		GETAFE Corp. I	GETAFE Corp. NI
COLMEN Inc.	E	2, 10	1, 5
	NE	10, 2	20, 10

(a) (6 points) Describe the elements of the Bayesian game.
(b) (14 points) Calculate the Bayesian equilibria of the game as a function of p.

(a) Players: {COLMEN Inc., GETAFE Corp.}
Types of COLMEN Inc.: $\{L, H\}$,
types of GETAFE Corp.: $\{G\}$.
Beliefs: $p(L|G) = p$, $p(H|G) = 1 - p$; $p(G|L) = 1$; $p(G|H) = 1$.
Actions by types: $A_L = A_H = \{E, NE\}$, $A_G = \{I, NI\}$.
Strategies by players: $S_C = A_L \times A_H = \{(E,E), (E,NE), (NE,E), (NE,NE)\}$; $S_G = \{I, NI\}$.
Payoffs: as shown in the tables.

(c) Type L of COLMEN Inc. has E as dominating, while type H has NE as dominating. This means that COLMEN Inc. will play (E, NE).
GETAFE Corp.'s utilities of I and NI are:

$$u_G((E,NE), I) = 10p + 2(1 - p) \text{ and}$$
$$u_G((E,NE), NI) = 5p + 10(1 - p).$$

Then, $u_G((E,NE), I) \geq u_G((E,NE), NI)$ if $p \geq \frac{8}{13}$.
The Bayesian equilibria are:
If $p > \frac{8}{13}$: ((E, NE), I),
if $p = \frac{8}{13}$: (E, NE), $q[I] + (1 - q)[NI]$) with $q \in [0, 1]$,
if $p < \frac{8}{13}$: ((E, NE), NI).