WORKSHEET 5: Integration

1. (*) Calculate the following integrals:

$$a)\ \int \frac{x^2+x+1}{x\sqrt{x}}dx \qquad b)\ \int xe^{-2x}dx \qquad c)\ \int sin^{14}x\ \cos x\ dx$$
$$d)\ \int (x+1)(2-x)^{1/3}dx \qquad e)\ \int \frac{x^4}{1+x^5}dx \qquad f)\ \int (1+\tfrac{1}{x})^3\tfrac{1}{x^2}dx$$
$$g)\ \int sin^3x\ dx \qquad h)\ \int xe^{ax^2}\ dx \qquad i)\ \int \frac{1}{3+x^2}dx$$
$$j)\ \int \frac{\sqrt{x-1}}{1+\sqrt[3]{x-1}}dx \qquad k)\ \int \frac{x}{\sqrt{16-x^2}}dx \qquad l)\ \int x^4\ln x\ dx$$
$$m)\ \int \frac{dx}{\sqrt[4]{x^3}-\sqrt{x}} \qquad n)\ \int (\ln x)^2dx \qquad \tilde{n})\ \int \frac{40x}{(x-1)^{40}}dx$$
$$o)\ \int \frac{4x+6}{(x^2+3x+7)^3}dx \qquad p)\ \int \frac{2x-6}{(x-2)^2}dx \qquad q)\ \int \frac{x^2+1}{x^3-4x^2+4x}dx$$
$$r)\ \int \frac{2x+1}{x^3+6x}dx \qquad s)\ \int \frac{1}{\frac{x^2}{2}-2x+4}dx \qquad t)\ \int \frac{x^4}{x^4-1}dx$$

2. How many different intersection points can two different primitives of the same function have?

3. (*) Let $f:[0,2]\longrightarrow\mathbb{R}$ be continuous, increasing in $(0,1)$, decreasing in $(1,2)$ and, also, satisfying that: $f(0)=3$, $f(1)=5$ and $f(2)=4$. Between which values can we guarantee that $\int_0^2 f(x)\,dx$ is located?

4. (*) Certain company has determined that its marginal cost is $\frac{dC}{dx}=4(1+12x)^{-1/3}$.
   Find the cost function if $C=100$ when $x=13$.

5. (*) Given that the marginal cost of producing x units is $x+5$ and the average cost has a minimum in $x=4$, find the fixed costs of the firm.

6. (*) Calculate $F'(x)$ in the following cases:

$$a)\ \int_x^{x^3} t\cos t\,dt \quad b)\ \int_1^{x^2}\sqrt{t^4+2t}\ dt \quad c)\ \int_1^{x^2}\left(t^2-2t+5\right)\ dt$$

7. Calculate $F'(x)$ in the following cases:

   (a) $\int_{-x}^{x^2}\tan^2 t\,dt$, supposing that $x^2<\frac{\pi}{2}$.

   (b) $\int_{x^2}^{2x} f^2(2t)\,dt$, supposing that f is continuous.

8. (*) What are the values of x where $F(x)=\int_{-3}^{x}\frac{t^2-4}{3t^2+1}dt$ has a local maximum or minimum?

9. Let $F(x)=\int_{x^2}^{2x} f(t^2)dt$ be such that $f(1)=1$, $f(2)=f(4)=4$ and f is continuous. Calculate $F'(1)$.

10. (*) Calculate observing the symmetry of the functions:

$$a)\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^{27}x\cos^{28}x\,dx \quad b)\ \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(\sqrt[3]{x^5\cos 3x}+\cos\tfrac{x}{3}+\tan^3 x\,)dx$$

11. Let f be a function with period T, such that $\int_0^T f = b$. Find $\int_a^{a+nT} f$.

12. (*) Find the area located between the following curves:

$$a)\ f(x)=x^2-4x+3,\ g(x)=-x^2+2x+3$$
$$b)\ f(x)=(x-1)^3,\ g(x)=x-1$$
$$c)\ f(x)=x^4-2x^2+1,\ g(x)=1-x^2$$

13. (*) Graph the functions $y=2e^{2x}$ and $y=2e^{-2x}$. Calculate the area located between those graphs and the lines $x=-1$ and $x=1$.

14. Let $f:[1,3]\longrightarrow[2,4]$ be increasing, continuous and bijective such that $\int_1^3 f\,dx=5$. Calculate $\int_2^4 f^{-1}(x)\,dx$

15. (a) Given $f:[0,4]\to\mathbb{R}$, convex and increasing with values $f\,(0)=0$, $f\,(2)=\alpha$, $f\,'(2)=\beta$, $f\,(4)=16$. Estimate as a function of α and β, the value of $\int_0^2 f(x)dx$.

    (b) Given $f:[0,4]\to\mathbb{R}$, concave and increasing with values $f\,(0)=0$, $f\,(2)=\alpha$, $f\,'(2)=\beta$, $f\,(4)=2$. Estimate as a function of α and β, the value of $\int_0^2 f(x)dx$.

16. The sales of a product are given by the formula $S(t)=10+5sin(\frac{\pi t}{6})$ where S is measured in thousands of units and time t in months. Calculate the average sales during the year $(0\le t\le 12)$.

17. Calculate:

$$a)\int_0^1\frac{1}{\sqrt{x}}dx \quad b)\int_0^3\frac{1}{x^3}dx \quad c)\int_1^\infty\frac{1}{x^2}dx$$
$$d)\int_1^\infty e^{-x}dx \quad e)\int_{-\infty}^\infty\frac{dx}{1+x^2} \quad f)\int_{-2}^4\frac{dx}{x^2}$$

18. Calculate $\displaystyle\int_0^\infty\frac{dx}{\sqrt{x}(1+x)}$