Midterm (Micro II, Masters in Economics-UC3M, March 25, 2019)

Answer exercise 1, and either exercise 2 or exercise 3. Each exercise is worth 5 points.

Exercise 1. In an economy that extends over two dates, today and tomorrow, there is a single perishable good, consumption. There is uncertainty about whether the economy's pension system tomorrow will be sound (S) or weak (W). There are two consumers with identical preferences for consumption today (x), consumption tomorrow if S (y) and consumption tomorrow if W (z), represented by the utility function $u(x, y, z) = x + \ln y + \ln z$. Their endowments are $(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (2, 2, 2)$ and $(\bar{x}_2, \bar{y}_2, \bar{z}_2) = (2, 4, 1)$, respectively. The only markets in the economy are a credit market and a market for security that pays 3 units of consumption when the pension system is weak, and nothing when it is sound. Calculate the competitive equilibrium (CE). Verify that the CE allocation is Pareto optimal. (Hint. The consumers' problems are identical – they differ only on the endowments. Write down a consumer's budget constrains denoting by $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ her endowments, by b the units of consumption she borrows and by w the number of units of the security she buys. Calculate her utility as a function of these variables, $v(b, w)$. Maximizing v you obtain the demands of credit and security. Identify the CE security price q^* and interest rate r^* by solving the system of market clearing conditions. You may want try the value $r^* = 0$.)

Exercise 2. An economy is populated by 4 individuals who only care about their consumption. There is a technology freely available that allows to produce x units of consumption good for each hour of labor used as input. The value of x represents the *state of knowledge*; specifically, $x = z/2$, where z is the total time these 4 individuals spend improving the technology. Each individual is endowed with 8 hours that she can use to produce consumption good and/or to improve the technology. What would be the state of knowledge if each individual decides how much of her time she allocates to improving the technology and to production activities (i.e., under voluntary contribution)? (You may use, without proving it, that the equilibrium is symmetric.) Calculate the optimal state of knowledge x^* and the Lindahl equilibrium prices.

Exercise 3. A competitive market provides insurance to a population of individuals with preferences represented by the Bernuilli utility function $u(x) = \ln x$, where x is the individual's disposable income. Each individual has an initial wealth $W = 5$ euros, and faces the risk of a monetary lose $L = 4$ euros. For a fraction $\lambda \in (0, 1)$ of the individuals the probability of losing L is $p_H = 1/2$, whereas for the remaining fraction $1 - \lambda$ this probability is $p_L = 1/4$. This information is common knowledge to all market participants. At the time an individual is signing a policy, an insurance company does not know whether her probability of loosing L is p_H or p_L.Which policies will be offer in a competitive equilibrium (CE)? (Identify the policies offered, and determine for which value of λ such policies form a CE.) If the government passes a law that forbids insurance companies offering policies involving a positive deductible, i.e., only full insurance policies are allowed, which policies will companies offer for each value of λ? Which individuals will subscribe them?

Solutions

Exercise 1. Consumer $i \in \{A, B\}$ budget constraints are:

$$\begin{aligned} x &\leq \bar{x}_i + b - qw \\ y &\leq \bar{y}_i - (1+r)b \\ z &\leq \bar{y}_i + 3w - (1+r)b \end{aligned}$$

Hence she chooses b and w to solve the problem

$$\max_{(b,z)\in\mathbb{R}\times\mathbb{R}_+} v_i(b,w) = \left(\bar{x}^i + b - qw\right) + \ln\left(\bar{y}_i - (1+r)b\right) + \ln\left(\bar{z}_i + 3w - (1+r)b\right)$$

Taking derivatives

$$\begin{aligned} \frac{\partial v_i}{\partial b} &= 1 - \frac{1+r}{\bar{y}_i - (1+r)b} - \frac{1+r}{\bar{z}_i + 3w - (1+r)b} \\ \frac{\partial v_i}{\partial w} &= -q + \frac{3}{\bar{z}_i + 3w - (1+r)b}. \end{aligned}$$

Solving $\left(\frac{\partial v_i}{\partial b}, \frac{\partial v_i}{\partial w}\right) = (0,0)$ we get

$$\begin{aligned} b_i(q,r) &= \frac{\bar{y}_i}{1+r} + \frac{3}{q\,(1+r) - 3} \\ w_i(q,r) &= \frac{\bar{y}_i - \bar{z}_i}{3} + \frac{1}{q} + \frac{1+r}{q\,(1+r) - 3}. \end{aligned}$$

Market clearing requires

$$\begin{aligned} \frac{\bar{y}_1}{1+r} + \frac{3}{q\,(1+r)-3} + \frac{\bar{y}_2}{1+r} + \frac{3}{q\,(1+r)-3} &= 0 \\ \frac{\bar{y}_1 - \bar{z}_1}{3} + \frac{1}{q} + \frac{1+r}{q\,(1+r)-3} + \frac{\bar{y}_2 - \bar{z}_2}{3} + \frac{1}{q} + \frac{1+r}{q\,(1+r)-3} &= 0 \end{aligned}$$

Substituting the endowments and solving the system we get $(q^*, r^*) = (2, 0)$. Hence

$$\begin{aligned} b_2(q^*, r^*) &= \frac{4}{1+0} + \frac{3}{2\,(1+0) - 3} = 1 = -b_1(q^*, r^*) \\ w_2(q^*, r^*) &= \frac{4-1}{2} + \frac{1}{2\,(1+0) - 3} = \frac{1}{2} = -w_1(q^*, r^*). \end{aligned}$$

Using the budget constraints we calculate the resulting allocation as

$$(x_1^*, y_1^*, z_1^*) = (x_2^*, y_2^*, z_2^*) = (2, 3, 3/2).$$

Since

$$\begin{aligned} MRS^1_{x,y}(x_1^*, y_1^*, z_1^*) &= y_1^* = y_2^* = MRS^2_{x,y}(x_2^*, y_2^*, z_2^*), \\ MRS^1_{x,z}(x_1^*, y_1^*, z_1^*) &= z_1^* = z_2^* = MRS^2_{x,z}(x_2^*, y_2^*, z_2^*), \end{aligned}$$

this allocation is Pareto optimal.

Exercise 2. If x is the state of knowledge, and individual who spends y_i hours in production activities obtains $u(x, y_i) = xy_i$ units of consumption. Hence an individual's marginal rate of substitution for x and y is

$$MRS(x, y_i) = \frac{y_i}{x}.$$

Under voluntary contribution, each individual contributes to improve the technology up to the point where her $MRS(x, y_i) = 2$ (the marginal cost of an additional unit of knowledge); that is, in equilibrium $y_i = 2x$ holds for each i. Hence

$$\sum_{i=1}^{4} y_i = 8x.$$

Also, the level of knowledge as a function of the individuals' contribution is

$$2x = \sum_{i=1}^{4} z_i.$$

Using that equilibrium is symmetric, i.e., $y_i = y$ and $z_i = z$ we have the system

$$4y = 8x,\ 2x = 4z,\ y + z = 8,$$

which solution is

$$x^v = \frac{16}{5},\ y_i^v = \frac{32}{5},\ z_i^v = \frac{8}{5}.$$

Thus, each individual consumption is

$$\frac{16}{5}\left(\frac{32}{5}\right) = 20.48.$$

The optimal state of knowledge satisfies

$$\sum_{i=1}^{4} MRS(x, y_i) = \frac{\sum_{i=1}^{4} y_i}{x} = 2.$$

Also, for an allocation to be feasible we must havwe

$$\sum_{i=1}^{4} y_i + 2x = 32.$$

Hence $x^* = 8$ and $\sum_{i=1}^{4} y_i = 16$. The Lindahl equilibrium solves the system

$$MRS(x^*, y_i) = \frac{y_i}{8} = p_i,\ y_i = 8 - 8p_i,\ \sum_{i=1}^{4} p_i = 2.$$

Solving we get $p_i = 1/2$. The Lindahl allocation is $(x^*, y_i^*) = (8, 4)$ and each individual consumption is 32 units.

Exercise 3. *As seen in class a competitive equilibrium, when it exists, is separating. In a separating equilibrium high risk agents get full insurance, that is* $(I_H, D_H) = (p_H L, 0) = (2, 0)$. *Hence the expected utility for a high risk individual is*

$$u(W - p_H L) = \ln(W - p_H L) = \ln\left(5 - \left(\frac{1}{2}\right)4\right) = \ln 3$$

Low risk individuals get partial insurance $(I_L, D_L) = (p_L(L - D), D)$, *where* D *must leave the high risk individuals indifferent between the policy* (I_H, D_H) *and the policy* (I_L, D_L), *i.e.,*

$$p_H u(W - p_L(L - D) - D) + (1 - p_H)\, u(W - p_L(L - D)) = u(W - p_H L)$$

Substituting, this equation becomes

$$\frac{1}{2}\ln\left(5 - \frac{1}{4}(4 - D) - D\right) + \frac{1}{2}\ln(5 - \frac{3}{4} + \frac{D}{2}) = \ln 3.$$

which may be written as

$$\left(5 - \frac{1}{4}(4 - D) - D\right)\left(5 - \frac{3}{4} + \frac{D}{2}\right) = 3^2.$$

Solving this equation we get

$$D_L = -\frac{4}{3}\left(\frac{19}{16} - \sqrt{\left(\frac{19}{16}\right)^2 + 12}\right) \simeq \frac{10}{3}.$$

The expected utility of the low risk type with the policy $(p_L(L - D_L), D_L)$ *is*

$$\begin{aligned} u_L^* &= p_L u(W - p_L(L - D_L) - D_L) + (1 - p_L)\, u(W - p_L(L - D_L)) \\ &= \frac{1}{4}\ln\left(5 - \frac{1}{4}(4 - \frac{10}{3}) - \frac{10}{3}\right) + (1 - \frac{1}{4})\ln\left(5 - \frac{1}{4}(4 - \frac{10}{3})\right) \simeq 1.28. \end{aligned}$$

For these policies to be a separating equilibrium, the full insurance pooling policy $(\bar{p}L, 0)$, *where*

$$\bar{p} = \lambda p_H + (1 - \lambda)\, p_L = \frac{\lambda}{2} + \frac{1 - \lambda}{4} = \frac{1}{4}(1 + \lambda),$$

must not be preferred by a low risk individual, i.e.,

$$u(W - \bar{p}L) = \ln\left(5 - \frac{1}{4}(1 + \lambda)4\right) = \ln(4 - \lambda) \leq u_L^*,$$

which may be written as $\lambda \geq 4 - e^{u_L^*} := \underline{\lambda} \simeq 0.39$.

If low risk individuals are willing to subscribe the pooling policy $(\bar{p}L, 0)$, *that is,* $\lambda > \underline{\lambda}$, *then in a CE all insurance companies will offer this policy: If only the policy* $(p_H L, 0) = (2, 0)$ *were too to be offered, then an insurance company offering the policy* $(\bar{I}, 0) = (\bar{p}L + \varepsilon, 0)$ *will be able to attract all individuals for* $\varepsilon > 0$ *sufficiently small) and make profits. If* $\lambda < \underline{\lambda}$, *then the insurance companies will offer only the policy* $(p^H L, 0) = (2, 0)$, *and only high risk type individuals we get insurance. If* $\lambda = \underline{\lambda}$, *then both offering only the policy* $(p^H L, 0) = (2, 0)$, *and offering only the policy* $(\bar{I}, 0) = (\bar{p}L, 0)$ *are possible CE.*