Masters in Economics-uc3m, Microeconomics II
Midterm Exam (March 23, 2022)

Exercise 1. An exchange economy operates over two dates, today and tomorrow. There is a single perishable good, consumption. The state of nature tomorrow can either be sunny or cloudy. There are two consumers whose preferences over consumption today (x), tomorrow if sunny (y) and tomorrow if cloudy (z) are represented by the utility functions $u_1(x, y, z) = xy$, and $u_2(x, y, z) = xz$, respectively. Each consumer is endowed with 2 units of consumption today and tomorrow regardless of the state of nature. There are only two markets, both operating today: a credit market and a market for a security that pays returns tomorrow only when the state is cloudy. Consumption is the numeraire good: that is, a consumer that borrows $b \geq 0$ (respectively, lends $b \leq 0$) units of consumption today pays back (receives) $(1 + r)\, b$ units of consumption tomorrow regardless of the state of nature; likewise, a consumer that buys (sells) s of the security pays (receives) qs units of consumption today, and receives (pays) s unit of consumption tomorrow if cloudy.

(a) (45 points) Determine how much each consumer borrows or lends and how many units of the security she buys or sells, and verify that the competitive equilibrium interest rate and security price are $(q^*, r^*) = (1/2, 0)$. Also, calculate the competitive equilibrium allocation.

Solution: The consumers budget constraints are

$$\begin{aligned} x &= 2 + b - qs \\ y &= 2 - (1 + r)b \\ z &= 2 - (1 + r)b + s. \end{aligned}$$

Thus, Consumer 1 sets $z = 0$, *and hence* $s = (1+r)b-2$ *and* $x = 2+b-q\left((1 + r)b - 2\right)$, *where* b *solves the problem*

$$\max_b \ \left(2 + b - q\left((1 + r)b - 2\right)\right)\left(2 - (1 + r)b\right)$$

Therefore

$$b_1(q, r) = \frac{2q\,(1 + r) + r}{(1 + r)\,(q(1 + r) - 1)}.$$

and

$$s_1(q, r) = \frac{2q\,(1 + r) + r}{q(1 + r) - 1} - 2.$$

Consumer 2 set $y = 0$, *hence* $b = 2/\,(1 + r)$, $z = s$, *and* $x = 2 + 2/\,(1 + r) - qs$, *where* s *solves the problem*

$$\max_s \ \left(2 + 2/\,(1 + r) - qs\right) s$$

Therefore

$$s_2(q, r) = \frac{r + 2}{q(1 + r)},$$

and

$$b_2(q, r) = \frac{2}{1 + r}.$$

Hence $b_2(q^*, r^*) = -b_1(q^*, r^*) = 2$, *and* $s_2(q^*, r^*) = -s_1(q^*, r^*) = 4$. *The CE allocation is*

$$[(x_1^*, y_1^*, z_1^*), (x_2^*, y_2^*, z_2^*)] = [(2, 4, 0), (2, 0, 4)].$$

(b) (15 points) Calculate the Arrow-Debreu prices for all three contingent goods x, y and z, implicitly defined by the equilibrium interest rate and security price, (r^*, q^*) you found in part (a). (Hint. You can find these prices by consolidating a consumer's budget constraints into a single linear equation involving the consumptions and endowments of goods x, y, and z. The parameters in this linear equations, which will be functions of r and q, will give you the Arrow-Debreu CE prices when you replace r and q by their Radner CE values.)

Solution. Using the first two equations in system of budget constraints

$$\begin{aligned} x &= 2 + b - qs \\ y &= 2 - (1+r)b \\ z &= 2 - (1+r)b + s, \end{aligned}$$

to solve for b and s, we get

$$\begin{aligned} b &= \frac{2-y}{1+r} \\ s &= \frac{2+b-x}{q} = \frac{2+\frac{2-y}{1+r}-x}{q}. \end{aligned}$$

Substituting in the equation for z we have

$$z = 2 - (2-y) + \frac{2+\frac{2-y}{1+r}-x}{q}.$$

Rearranging we may write the consumer's budget constraint as

$$x + (\frac{1}{1+r} - q)y + qz = 2 + 2(\frac{1}{1+r} - q) + 2q.$$

Because there is a one to one correspondence between the Arrow-Debreu CE and the CE of the economy in part (a), the CE Arrow-Debreu prices are

$$p_x^* = 1,\ p_y^* = \frac{1}{1+r^*} - q^* = \frac{1}{2},\ p_z^* = q^* = \frac{1}{2}.$$

Exercise 2a. Each of the 10 inhabitants of a village must choose whether to own a cow to provide milk for his family. The more cows the village's common grazing land is required to support, the lower is each cow's yield of milk; specifically, a cow yield of milk is $(12 - x)$ quarts, where x is the total number of cows in the grazing land. Each inhabitant wants to obtain as much milk as she can.

(a) (20 points) If the decisions whether to own a cow are made independently, how may inhabitants you predict will own a cow? How much milk will consume each inhabitant?

Solution. Since at most 10 people may own a cow, an individual owning a cow gets at least

$$(12 - 10)1 = 2$$

quarts of milk. Since the cost of owning a cow is negligible, every individual will own a cow.

Using our standard notation, in equilibrium $z_i^* = 1$*, and hence the number of cows in the grazing land will be*

$$z^* = \sum_{i=1}^{10} z_i^* = 10,$$

and the per capita consumption of milk will be

$$m^* = (12 - 10)1 = 2.$$

(b) (10 points) Determine the number of cows that maximizes the total yield of milk, z^{PO}.

Solution. The number of cows grazing the in common land z^{PO} *solves the problem*

$$\max_{z\in\{0,1,\ldots,10\}} M(z) = (12 - z)\, z.$$

We have

$$M'(z) = 12 - z = 0 \Leftrightarrow z = 6,$$

that is the number of cows that maximize total milk production is $z^{PO} = 6$.

(c) (10 points) Assume that the city council imposes a Lindahl fee of L quarts of milk to those who chose to own a cow, and distributes the milk revenues evenly among the inhabitants who choose not to own a cow. For which values of L would z^{PO} inhabitants chose to own a cow. (Hint. Assume that z^{PO} inhabitants chose to own a cow. The fee L must be such that each inhabitant who owns a cow does not prefer to not own a cow, and each inhabitant who do not own a cow does not prefer to own a cow.) Is there a Lindahl fee L^* that assures a per capita consumption of milk equal to $\left(12 - z^{PO}\right) z^{PO}/10$?

Solution. If a Lindahl fee L is to induce exactly 6 *inhabitants to own a cow, then anyone of the* 6 *cow owners must prefer to own a cow than collecting its share of the revenue raised by the fee, that is*

$$12 - 6 - L \geq \frac{5L}{5} \Leftrightarrow L \leq 3.$$

Likewise,the 4 *inhabitants who own no cow must prefer collecting its share of the revenue raised by the fee than owning a cow, that is*

$$\frac{6L}{4} \geq 12 - 7 - L \Leftrightarrow L \geq 2.$$

Thus, an optimal Lindahl satisfies $L \in [2, 3]$.

If we want the fee to assure an equal consumption of milk to all inhabitants, then L must satisfy

$$12 - 6 - L = \frac{6L}{4} \Leftrightarrow L = \frac{12}{5}.$$

Exercise 2b. NYC used to be a pickpocket's playground. In a typical day, a fraction $p_L = 1/4$ of *alert* tourists reported that his wallet was stolen, while this fraction was $p_H = 1/2$ for *inattentive* tourists. Each tourist typically carried $W = 150$ euros in his wallet for the daily expenses, and the typical loss was $L = 100$. Tourists' preferences are described by the Bernoulli utility function $u(x) = \ln x$.

(a) (10 points) Assume that there is a competitive insurance market where tourist may subscribe a policy covering this risk. Determine the policies that will be offered assuming that insurance companies can tell whether a tourist is of the alert or the inattentive type.

Solution. Since the market is competitive, under complete information companies will offer the fair premium full insurance policy to each type; that is, they will offer the policy

$$(I_H, 0) = (100p_H, 0) = (50, 0)$$

to the inattentive tourists, and the policy

$$(I_L, 0) = (100p_L, 0) = (25, 0)$$

to the alert tourists.

(b) (20 points) Assume now that insurance companies cannot tell whether a tourist subscribing a policy is of the alert or the inattentive type, and that there are twice as many inattentive tourists than alert tourists. Which insurance policies will be offered? (To solve an equation you will encounter, these formulae will be useful: $a \ln x + b \ln y = \ln(x^a y^b)$; also, the solution to the equation $ax^2 + bx + c = 0$ is $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.)

Solution. As established in class, in a competitive equilibrium, when it exists, insurance companies offer separating fair policies $(I_H, 0) = (50, 0)$ and $(\hat{I}_L, \hat{D}_L)$, where

$$\hat{I}_L = (100 - \hat{D}_L)p_L,$$

and $\hat{D}_L$ is such that the inattentive tourists are indifferent between the two policies, that is

$$\frac{1}{2}\ln\left(150 - (100 - \hat{D}_L)p_L - \hat{D}_L\right) + \frac{1}{2}\ln\left(150 - (100 - \hat{D}_L)p_L\right) = \ln(150 - 50).$$

This equation may be written as

$$\left(150 - (100 - \hat{D}_L)/4 - \hat{D}_L\right)\left(150 - (100 - \hat{D}_L)/4\right) = (100)^2.$$

Solving this equation we get

$$\hat{D}_L = 100(2\sqrt{13} - 5)/3 \simeq 73.703.$$

Hence

$$\hat{I}_L = \frac{1}{4}\left(100 - \hat{D}_L\right) = \frac{1}{4}\left(100 - 100(2\sqrt{13} - 5)/3\right) \simeq 6.574\,1.$$

For these policies to form a competitive equilibrium the alert tourist must prefer the policy $(\hat{I}_L, \hat{D}_L)$ to the pooling policy $(100\bar{p}, 0)$, where

$$\bar{p} = \frac{2}{3}p_H + \frac{1}{3}p_L = \frac{5}{12}.$$

The expected utility of an alert tourist with the policy $(\hat{I}_L, \hat{D}_L)$ *is*

$$\frac{1}{4}\ln\left(150 - \left(200(4-\sqrt{13})/9\right) - \left(100(2\sqrt{13}-5)/3\right)\right) + \frac{3}{4}\ln\left(150 - \left(200(4-\sqrt{13})/9\right)\right) \simeq 4.766,$$

and his expected utility with the pooling policy is

$$\ln\left(150 - (100)\frac{5}{12}\right) \simeq 4.685.$$

Hence the policies $\{(I_H, 0), (\hat{I}_L, \hat{D}_L)\}$ *form a competitive equilibrium in this market.*

(c) (10 points) Assume that the market is monopolized by a single company, which by law must offer a single insurance policy to all tourists. (That is, the firm cannot "screen" tourists with a menu of policies.) Which policy will this company offer? (Hint. Should the firm offer full insurance? Should it offer a policy intended for both types of tourists or a policy that attracts only inattentive tourist?) Determine which tourists win and lose in this situation relative to that of part (b).

Solution. The company must decide whether to offer a policy that only inattentive tourist subscribe or one which both types of types of tourists subscribe. Obviously, in either case the company will offer full insurance since it can extract more surplus from the risk averse tourists.

If the firm offers a policy that both types subscribe, it has to offer it at the maximum premium the alert tourists are willing to pay, that is,

$$\ln(150 - x) = \frac{1}{4}\ln(150 - 100) + \frac{3}{4}\ln(150),$$

Solving this equation we get $\bar{I} = 150 - \sqrt[4]{50\,(150)^3} \simeq 36$. *The monopoly's expected profit per tourist is*

$$\bar{I} - \bar{p}L = 36 - \frac{5}{12}(100) = -\frac{17}{3}.$$

(The probability that the average tourist suffers the loss is $\bar{p}$*.) Hence the monopoly will not offer this policy.*

If the firm offers a policy that only inattentive tourists subscribe, then it charges the premium that solves the equation

$$\begin{aligned}
\ln(150 - I_H) &= \frac{1}{2}\ln(150-100) + \frac{1}{2}\ln(150) \\
&\Leftrightarrow \\
(150 - I_H)^2 &= 3\,(50)^2 \\
&\Leftrightarrow \\
I_H &= 150 - 50\sqrt{3} \simeq 63.397.
\end{aligned}$$

Since only $2/3$ *of the tourist are inattentive, the monopoly's expected profit*

$$\frac{2}{3}\left(\bar{I}_H - p_H L\right) = \frac{2}{3}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right) \simeq 8.93.$$

Hence the monopoly will offer the policy $\left(\bar{I}_H, 0\right)$.

(d) (Question added after the exam.) Assume that the fraction of inattentive tourists is $\lambda \in (0,1)$. For which values of λ would the monopoly of part (c) offer a pooling policy?

Solution. The aggregate probability is now

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{1-\lambda}{4} = \frac{1+\lambda}{4}.$$

If the firm offers a policy that both types subscribe, $(\bar{I}, 0)$*, where* $\bar{I} = 150 - \sqrt[4]{50(150)^3} \simeq 36.025$*, the monopoly's expected profit per tourist is*

$$\bar{I} - \bar{p}(\lambda) L = 150 - \sqrt[4]{50(150)^3} - \frac{1+\lambda}{4}(100).$$

If the firm offers a policy that only inattentive tourists subscribe, $(\bar{I}_H, 0)$*, where* $I_H = 150 - 50\sqrt{3}$*, then the monopoly's expected profit*

$$\frac{\lambda}{2}\left(\bar{I}_H - p_H L\right) = \frac{\lambda}{2}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right).$$

Solving

$$\frac{\lambda}{2}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right) = 150 - \sqrt[4]{50(150)^3} - \frac{1+\lambda}{4}(100)$$

we get.

$$\bar{\lambda} = \frac{\sqrt[4]{168750000} - 125}{25\sqrt{3} - 75} \simeq 0.34779$$

Thus, for $\lambda < \bar{\lambda}$ *the monopoly offers the policy* $(\bar{I}_H, 0)$*, which all tourists subscribe. Note that alert tourist are worse off, while inattentive tourist are better off, than in the competitive equilibrium of the market identified in part (b).*