General Equilibrium with Market Power: An Exercise.

(Toy example based on Azar, J. and X. Vives: General Equilibrium Oligopoly and Ownership Structure, *Econometrica* 89 (2021): 999-1048.

On the effect of market power: Eeckhout, J.: *The Profit Paradox. How Thriving Firms Threaten the Future of Work*, Princeton University Press, 2021.)

In an economy there are two goods, labor (l) and consumption (c), and there are two identical firms producing consumption using labor l according to the production function $f(l) = 2l$. The economy is populated by a *worker,* and the *owners* of firms 1 and 2 (each firm is entirely own by a different agent.) The worker has no resources other than her labor income, and her preferences over labor (the counterpart of leisure) and consumption are represented by a utility function $u_i(l, c) = -l + 2\sqrt{c}$. Owners only care about consumption and have no resources other than the firms profits of the firm each owns. Firms are aware of their market power both, in the market for consumption, where they form a duopoly, and in the labor market, where they form a duopsony. Their objective is to maximize the real returns (the profits in units of consumption). Calculate the economy's *Cournot-Walras* equilibrium prices and allocation. What if there are $n > 2$ with the given technology? Discuss the properties of the this allocation compare to those of the competitive equilibrium allocation.

Let us denote by w the wage and by p the price of consumption. Worker's labor supply and consumption demand.

$$\max_{(c,l)\in\mathbb{R}^2_+} -l + 2\sqrt{c}$$

$$\text{subject to: } pc \leq wl.$$

Calculating

$$MRS(l, c) = \sqrt{c}$$

and solving the system

$$\begin{aligned} \sqrt{c} &= \frac{w}{p} \\ pc &= wl \end{aligned}$$

we get the worker's demand for consumption and supply of labor. Denoting $\omega = w/p$ for the real wage, we get

$$c(\omega) = \omega^2, \; l(\omega) = \omega,$$

Owners are passive since they have no decision to make, and simply use their portfolio returns to buy consumption.

If firms 1 and 2 use l_1 and l_2 units of labor, the equilibrium the real wage satisfies

$$l_1 + l_2 = l(\omega) \Leftrightarrow \omega = l_1 + l_2.$$

Hence the real profits of firm i are

$$\frac{\pi_i(l_1, l_2)}{p} = \frac{p\,(2l_i) - wl_i}{p} = 2l_i - \frac{w}{p}l_i = 2l_i - \omega(l_1, l_2)l_i = (2 - l_1 - l_2)\,l_i.$$

In the *Cournot-Walras equilibrium* of the economy each firm maximizes it real profit given its rival's labor decision; i.e., l_i solves the problem

$$\max_{l_i \in \mathbb{R}_+} (2 - l_1 - l_2)\,l_i.$$

Hence the *reaction function* of firm i solves the equation

$$2 - l_j - 2l_i = 0 \Leftrightarrow l_i(l_j) = 1 - \frac{l_j}{2}.$$

Since equilibrium is symmetric, i.e., $l_1 = l_2 = l$, each firm equilibrium labor is

$$l^* = \frac{2}{3}.$$

Hence the Cournot-Walras equilibrium real wage is

$$\omega^* = \frac{4}{3}.$$

The consumptions of the worker and each owner are

$$c_w^* = (\omega^*)^2 = \frac{16}{9},\ c_o^* = (2 - \omega^*)\,l^* = \frac{4}{3}\left(\frac{2}{4}\right) = \frac{8}{9},$$

The case $n > 2$ firms

In the *Cournot-Walras equilibrium* of the economy each firm maximizes its real return given its rival's labor decision; i.e., l_i solves the problem

$$\max_{l_i \in \mathbb{R}_+} \left(2 - l_i - \sum_{j \neq i} l_j\right) l_i.$$

Hence, the reaction function of firm i solves the equation

$$2 - l_i - \sum_{j \neq i} l_j - l_i = 0 \Leftrightarrow l_i(l_{-i}) = 1 - \frac{1}{2} \sum_{j \neq i} l_j.$$

Since equilibrium is symmetric, i.e., $l_i = l$ for all i, each firms equilibrium labor is

$$l^*(n) = \frac{2}{n+1}.$$

Hence the equilibrium real wage is

$$nl^*(n) = \frac{2n}{n+1} = \omega^*(n),$$

and the workers consumption is

$$c_w^*(n) = \left(\frac{2n}{n+1}\right)^2 = 4\left(\frac{n}{n+1}\right)^2,$$

and each owner consumption is

$$c_o^*(n) = (2 - \omega^*(n))\, l^*(n) = \left(2 - \frac{2n}{n+1}\right) \frac{2}{n+1} = \frac{2}{(n+1)^2}$$

Hence

$$\lim_{n\to\infty} \omega^*(n) = 2, \quad \lim_{n\to\infty} c_w^*(n) = 4, \quad \lim_{n\to\infty} c_o^*(n) = 0.$$

Comparison to the Competitive Equilibrium

In the competitive equilibrium constant returns to scale imply that firms profits are zero, and hence the owners consumption is zero. Since a unit of labor allows to produce 2 units of consumption, in equilibrium the real wage is

$$\omega = 2 > \frac{4}{3}$$

Further, since the market for consumption must clear, the total output, which entirely consumed by the worker, is

$$\sqrt{c} = 2 \Leftrightarrow c_w = 4 > \frac{16}{9}.$$

Thus, the implication of market power is a smaller real wage, a smaller production (leading to a loss of total surplus), and a shift of the surplus distribution in favor of the firms' owners. However, as n grows large, and the real wage and worker's utility in the Cournot-Walras equilibrium approaches that she has at the CE.