Exercise List 4: Introduction to the Economics of Information

I. Risk Sharing and Moral Hazard

1. Consider the contract design problem of a risk neutral principal whose revenue depends on the effort exerted by an agent, $e \in \{0, 1\}$, as described in the following table:

X	0	10	25
$p(0)$	$\frac{1}{2}$	$\frac{1}{2}$	0
$p(1)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

The reservation utility of this agent is $\underline{u} = 1$, his cost of effort is $c(e) = e$ and his von Neumann-Morgenstern utility function is $u(w) = \sqrt{w}$. Determine the optimal contract when effort level is observed and when it is not. What is the impact of moral hazard on the welfare of the agent and on the social surplus?

2. A salesman's probability of selling an encyclopedia is $p \in (1/2, 1)$ if he exerts effort, and $1/2$ otherwise. His cost of exerting effort is 20, and his reservation utility is $\underline{u} = 50$. A sale generates a revenue of 200 euros to the company. Both the salesman and his company are risk-neutral.

(a) The company cannot monitor the salesman, but can pay him wages contingent on success. Write down the company's contract design problem. Calculate the optimal contract for each $p \in (1/2, 1)$. Is the optimal contract socially optimal? Explain this result.

(b) Now suppose the salesman is risk averse, and his von Neumann-Morgenstern utility function is $u(w) = 20 \ln w$. Determine the optimal contract for $p = 4/5$.

3. An individual's preferences are represented by the von Neumann-Morgenstern utility function $u(w) = \sqrt{w}$, and his wealth is $W = 100$ euros. The individual faces the risk of suffering a loss of $L = 36$ euros. He may take a precautionary action, whose cost is $c = 4/10$, that would reduce the probability of suffering this loss from $p_0 = 3/4$ to $p_1 = 1/2$. In addition, the individual may subscribe an insurance contract (I, D), where I is the premium, in euros, and $D \in [0, 36]$ is the deductible (that is, the part of the loss no covered by the policy). Determine the insurance policy that a monopolistic risk neutral insurance company will offer when the precautionary action is contractible, and when it is not.

4. Exercises 2 and 3 in chapter 3 in Macho and Perez-Castrillo's textbook.

II. Screening

5. Consider a perfectly competitive labor market in which there are three types of workers $i \in \{1, 2, 3\}$ present in equal fractions. The workers' reservation wages are $(w_1, w_2, w_3) = (1/2, 1, 22/10)$. The revenue of a firm that hires a worker of type $i \in \{1, 2, 3\}$ is $r_i = i$. Identify the pooling, separating and partially separating PBNE of this market.

6. A monopoly produces a good with cost $c(q) = q$ that sells to two consumers whose demands are $D_L(p) = \max\{8 - p, 0\}$ and $D_H(p) = \max\{10 - p, 0\}$.

(a) Calculate the monopoly equilibrium in the absence of price discrimination.

(b) Assume that the monopoly sells the good using a two-part tariff (T, p), where T is a fee a consumer must pay to access the market, and p is the unit price at which the good is sold. Calculate the two-part tariff the monopoly will offer to each consumer.

(c) Now assume that the monopoly cannot discriminate consumers because it does not observe their types. Calculate the menu of two-part tariffs the monopoly will offer in this case.

7. Exercises 2, 3, 4 and 6 in Chapter 4 in Macho and Perez-Castrillo's textbook.

III. Signaling

8. In a competitive labor market there workers of two types, high skilled (H) and low skilled (L) present in equal fractions. A firm (principal) that hires a high (low) skilled worker obtains an expected revenue of $\bar{x}_H = 12$ (respectively, $\bar{x}_L = 4$) euros. The reservation utilities of high (low) skilled workers are $u_H = 6$ and $u_L = 0$, respectively. Workers' types are private information, but a worker can signal her type by taking an action $y \in \mathbb{R}_+$ at cost $c(y, H) = y/4$ and $c(y, L) = y$, respectively.

(a) Compute the *pooling* perfect Bayesian Nash equilibria (PBNE), i.e., the PBNE in which y is not used as a signal.

(b) Compute the most efficient *separating* PBNE, i.e., the PBNE in which a worker signals her type. Which of the two PBNE would each type of worker prefer?

9. In a competitive labor market there are three types of workers, $t \in \{1, 2, 3\}$, which are present in equal proportions. A worker's type is his private information. A worker of type t generates a expected revenue $x_t = t$. The workers' reservation utilities are $u_1 = 1/2$, $u_2 = 1$ and $u_3 = 22/10$. Thus, the (expected) revenue of a firm depends on its labor force according to the formula

$$\bar{x}(L_1, L_2, L_3) = L_1 + 2L_2 + 3L_3,$$

where L_t is the number of workers of type t in the firm's labor force. Before entering the market each worker chooses his level of education, which is observed by the firms. The cost of acquiring a level of education y for a worker of type t is

$$C_t(y) = \frac{y}{t^2}.$$

(a) Determine whether the *fully separating* perfect Bayesian Nash equilibria (PBNE), and identify the most favorable to the workers. (Hint: start by arguing that $y_1 = 0$, and find the levels of education $0 < y_2 < y_3$ that will form a PBNE.)

(b) Find, if possible, a PBNE in which $y_1 = y_2 = 0$, and $y_3 > 0$, that is, in which the firm treats differently only the type 3 workers. Compute the most favorable PBNE for the workers of type 3. Compare the workers' utilities with those of part (a).

10. A population of workers and firms are randomly matched to bargain a labor contract. Competitive pressure forces firms to offer each worker a wage equal to the expected revenue generated by the interaction, which depends on the worker's ability, $t \in \{H, L\}$ and her level of education $y \in \mathbb{R}_+$, according to function $X(y, t) = \alpha_t + \beta\sqrt{y}$, where $\alpha_H = 2$, $\alpha_L = 1$, and $\beta \in \mathbb{R}_+$. The level of education is a worker's decision whose cost depends on her ability according to the function $c(y, t) = c_t y$, where $c_H = 1$ and $c_L = 4$. Firms make wage offers $w \in \mathbb{R}_+$ after observing the worker's level of education, but not her ability, which is private information. Both workers and firms are risk neutral, and the reservation utility of all workers is zero. Identify the most efficient pooling and separating perfect Bayesian Nash equilibria for $\beta = 0$ and $\beta = 1$.